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IMPROVING THE PERFORMANCE OF PRACTICAL BLOCK LENGTH OF POLAR CODE

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Abstract

The recent Forward Error Correction (FEC) code, polar code has been opulent material for researchers because of its uncomplicated encoder and decoder structure and good Bit Error Rate (BER) performance for a sizeable block length. Lately, it is introduced as a candidate for the fifth generation of mobile communication (5G) for control channels. It is known that for control channels used short block lengths, but the performance of polar code in short block lengths is very poor. In this work, we proposed a model to enhance the BER performance in short block lengths by using a list decoder concatenated with a Cyclic Redundancy Check (CRC) code, simulate the model in the Matlab program. When using the Successive Cancellation List (SCL) decoder, the results show that the BER decreases as the list size increases, but at the expense of the increase in the latency. For block length=512, we get a 1dB gain from list size=32 while the latency increases from 1.1 ms for list size=1 to 98 ms for list size=32 and a spreading process adds to improve both the BER performance and latency.

Keywords: Cyclic redundancy check (CRC) code, Polar code, Successive cancellation list (SCL) decoder, Spreading process.

1. Introduction

Polar code is the latest type of channel codes, was invented by Arikan [1]. This code caused a revolution in this field where it was able to reach the Shannon capacity limit with a less complicated encoder and decoder construction for the Binary Discrete Memoryless Channel (B-DMC) if the block size was large. The name polar comes from the channel polarization theory means that we can get N channels from N independent copies of B-DMC (W), which was the basis for polar code works. The channel polarization has two phases (channel combining and channel splitting).

The polar code design to work on a specific channel does not work on another channel, then we can say the code construction is a very important issue for selecting the frozen bits and information bits. And this process depends on the main properties of the channel (channel capacity and Bhattacharyya Parameter).

There are two methods of polar encoding, a straightforward and butterfly structure which represents a less complex encoding method. The first decoding process of polar code is the successive cancelation decoding, which introduced with polar code by Arikan [1]. The performance of polar coder was weak in short block lengths when used a SC decoder.

The current paper focuses on the enhancement of the BER performance for the practical block length of polar code with a little latency. Proposed a successive cancelation list (SCL) decoder and process a group of codewords to choose the right codeword to enhance the BER performance. To decrease the latency, we added the spreading process to our model that enhances the BER and latency.

2. Literature Review

Gamage [2], conducted a study and investigation for the candidate coding schemes for 5G technology on the LDPC code, turbo code, convolutional code, and polar code in the AWGN channel and multipath fading channel. The results show that the successive cancelation (SC) decoder BER performance was weak but with less complexity, while the SCL decoder has a good BER but with high complexity.

Huilgol [3], studied the fifth-generation mobile system, its characteristics, and it is demanding. For channel coding, this work focused on the polar code. It investigated and enhanced the code work in short block lengths using CRC aided-SCL decoder.

Emilia [4], compare the polar code performance with a Reed-Muller code, and the study was on the frozen bits selection and its effect on the code performance. The study found that a little change in the election of the frozen bits would lead to a massive performance change in the polar code.

3. Polar Coding

The name polar comes from the channel polarization theory, which was the basis for polar code work and represents the backbone of the polar code.

3.1. Channel polarization

Channel polarization means that we can get N channels from N independent copies of B-DMC (W), where$\{W_N^{(i)}: 1 <= i <= N\}$, i is channel indices, as N closes to infinity some channels will be reliable in which $I(W_N^{(i)}) = 1$ or close to 1, where

$I(W_N^{(i)})$ is the channel capacity, and some channels will be noisy channels, where $I(W_N^{(i)})$ is closed to zero, approaches $1-I(W)$ [2, 5]. These channels (i) called polarized channels as shown in Fig. 1.

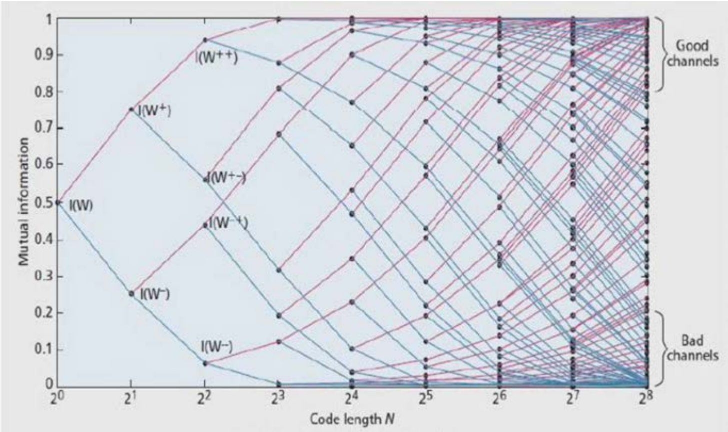

Fig. 1. Evolution of channel polarization.

The information bits will be sent on the reliable channel (the channel which has a less Bhattacharyya Parameter Z (W) and on noisy channels. The frozen bits sent to reach the symmetric capacity [3], this can create codes with a low error rate. Channel polarization has two phases (channel combining and channel splitting) [6].

3.1.1. Channel combining

The channel combining means combining N independent channels at the transmitter in a recursive manner to form channel vector W_N, where $N=2^n$, ($n>0$) is an integer value [1]. The recursion begins at stage (n=0) where $W=W_1$.

At the first stage (n=1), two independent copies of W_1 are combined to form W_2. As in Fig. 2 [7, 8]. The conditional probability of the resulting channel can be as Eq. (1):

$$W_2(y_1, y_2|u_1, u_2) = W(y_1|u_1 \oplus u_2)\, W(y_2|u_2) \tag{1}$$

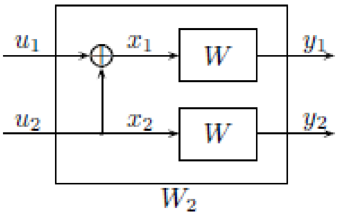

Fig. 2. Channel W_2.

In the next stage of recursion, two independent copies of W_2 combined to form W_4 shown in Fig. 3.

The capacity of the resultant channel W_2 represents the sum of the capacities of two combined channels, as shown in Eq. (2).

$$I(W2) = 2\, I\,(W) \tag{2}$$

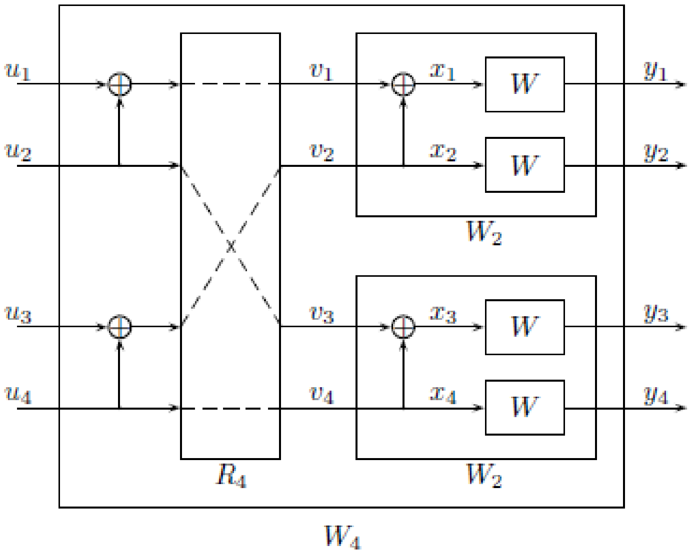

Fig. 3. The channel *W*4.

3.1.2. Channel splitting

In this phase, splitting the combined channel W_N into N channels. The created channels are indicated as W_N^i where1 <= i <= N, so the created transition probability of the channel shown in Eq. (3) [9].

$$W_N^i(y_0^{N-1}, u_0^{i-1}|u_1) = \sum \frac{1}{2^{N-1}} W_N(y_0^{N-1}, u_0^{N-1}) \tag{3}$$

3.2. Polar code construction

The Polar Code is unlike other types of channel coding, it is a channel-specific design [1], which means the polar code designed to work on a specific channel it may not work on another channel. So, the code construction is an important issue.

The code construction means the process of selecting the frozen bits and information bits. This process happens depending on several channel properties. There are two main properties for any channel:

3.2.1. Channel capacity, *I*(*W*)

The symmetric capacity represents the highest data rate at which reliable communication can happen.

3.2.2. Bhattacharyya parameter, *Z*(*W*)

It is an upper limit of the probability that an error occurs when sending 0 or 1 via the channel W [10, 11]. The Bhattacharyya Parameter for the polarized channels in a general B-DMS channel defined in Eqs. (4) and (5) [5, 12, 13, 14].

$$Z\left(W_N^{2i}\right) \leq 2Z\left(W_{N/2}^{i}\right) - (Z(W_{N/2}^{i}))^2 \tag{4}$$
$$Z\left(W_N^{2i}\right) = (Z(W_{N/2}^{i}))^2 \tag{5}$$

It is worth mentioning that these two parameters, means $I(W)$ and (W) have values between (0 and 1), as one of them is close to 0 the other close to 1 [4].In this work, we adopt Bhattacharyya Parameter code construction.

3.3. Polar encoder

There are two methods for polar encoding [4]:

3.3.1. Straight forward

In this way, the codeword of length N generated from multiplying the block of length N (information bits plus frozen bits, which are set to zero before encoding) by a generator matrix G [15].

$$x = u.G \quad (6)$$

where u represents the information bits and G is the generator matrix.

3.3.2. Butterfly structure

It is the recursively way of polar encoding shown in Fig. 4. It is considered the basis for polar code and polarization [4]. 12 XOR operations for 8 bits encoder shown in Fig. 4. Generally, for any number of bits encoder, the number of XOR operations is $N/2\ Log_2N$ [16]. This encoding method used in this work. The frozen bits value must equal to zero before encoding.

The polar codeword identified by a parameter vector (N, K, A, u_{A^c}) [1], where N is the codeword length, K is the information length. A is the information bits set and u_{A^c} is the frozen bits vector.

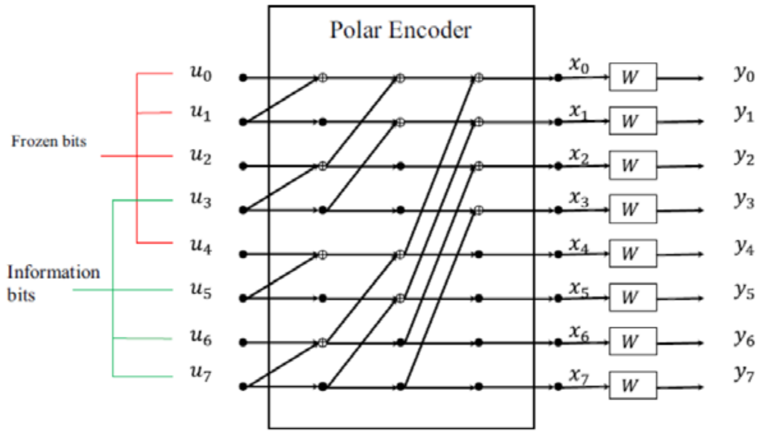

Fig. 4. Block diagram of 8-bit polar encoder.

3.4. Polar decoder (successive cancelation, SC Decoder)

The first decoding process of polar code is the successive cancelation decoding, which introduced with polar code by Arikan [1] in his research paper. After the data u_1^N encoded to x_1^N, and x_1^N sent over the channel W_N, let the received codeword be y_1^N which represents the channel output. This represents a copy of the data which has been affected by the noise. Here, the decoder will generate an estimation $\hat{u}_1^N$ of u_1^N, by the knowledge of A, $u_A{}^c$ and y_1. The frozen bits will be decoded directly without errors because the decoder knows their values in advance. As shown in Eq. (7):

$$\hat{u}_i \triangleq \begin{cases} u_i, & if\ i \in A^c \\ L_i(y_1^N, \hat{u}_1^{i-1}), & if\ i \in A \end{cases} \quad (7)$$

where i represents channel indices from 1 to N, L_i is the likelihood ratio.

The information bits are estimated using the following equations [1, 17, 18]

$$\hat{u_i} = \begin{cases} 0, if\ L_N^{(i)}(y_0^{N-1}, \hat{u_0}^{i-1}) \geq 1 \\ 1,\ otherwise \end{cases} \quad (8)$$

Likelihood ratio is

$$L_N^{(i)}(y_0^{N-1}, \hat{u_0}^{i-1}) = \frac{W_N^{(i)}(y_0^{N-1}, \hat{u_0}^{i-1} | u_i = 0)}{W_N^{(i)}(y_0^{N-1}, \hat{u_0}^{i-1} | u_i = 1)} \quad (9)$$

The decoding process will be one bit at a time in a certain order from top to bottom, which means that it will start from u_1^N and ends with $\hat{u}_N$. The decoding begins by calculating the likelihood ratios of the received bit y_0^{N-1} [19].

$$L_i(y_i) = \frac{W(y_i|0)}{W(y_i|1)} \quad (10)$$

After that, calculate the likelihoods through XOR operation, from right to left shown in Fig. 5. Likelihood is achieved at the left end of the graph, take the decision, it is spread from left to right to decode the remainder bits. Likelihoods are calculated through f and g functions shown.

$$f(L_a, L_b) = \frac{L_a L_b + 1}{L_a + L_b}, \quad g(L_a, L_b, \hat{u_s}) = \begin{cases} L_a . L_b & if\ \hat{u_s} = 0 \\ L_a / L_b & if\ \hat{u_s} = 1 \end{cases} \quad (11)$$

$\hat{u_s}$, is the predetermined bit [19].

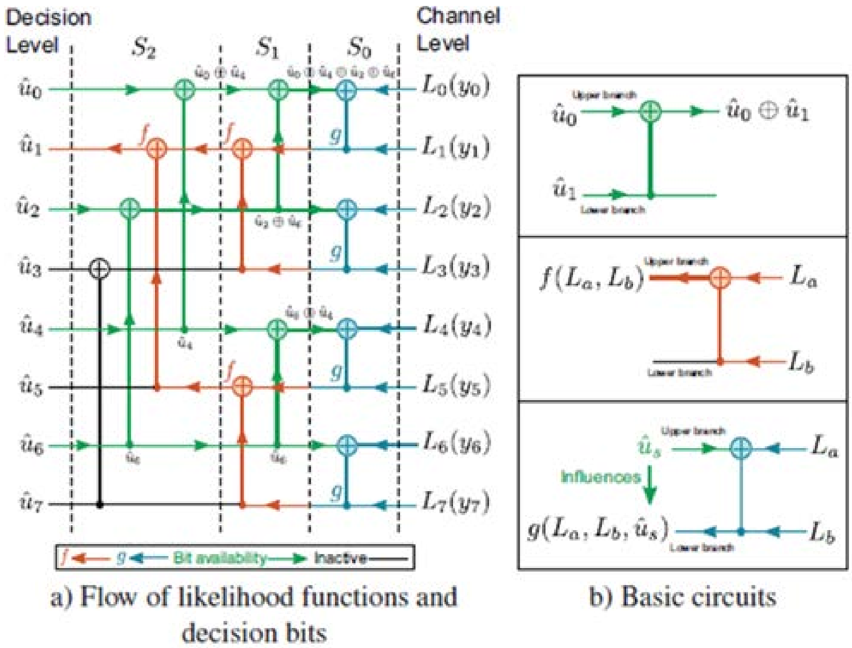

Fig. 5. SC decoder structure for N=8 Bits (polar code decoder) [19].

4. Simulation of the Improved Polar Code

Two stage were simulating of polar code in this work:

The first stage was an investigation of it is the performance with a Successive Cancelation (SC) Decoder, notice that its poor performance in small-sized blocks compared with other FEC codes as shown in Fig. 6.

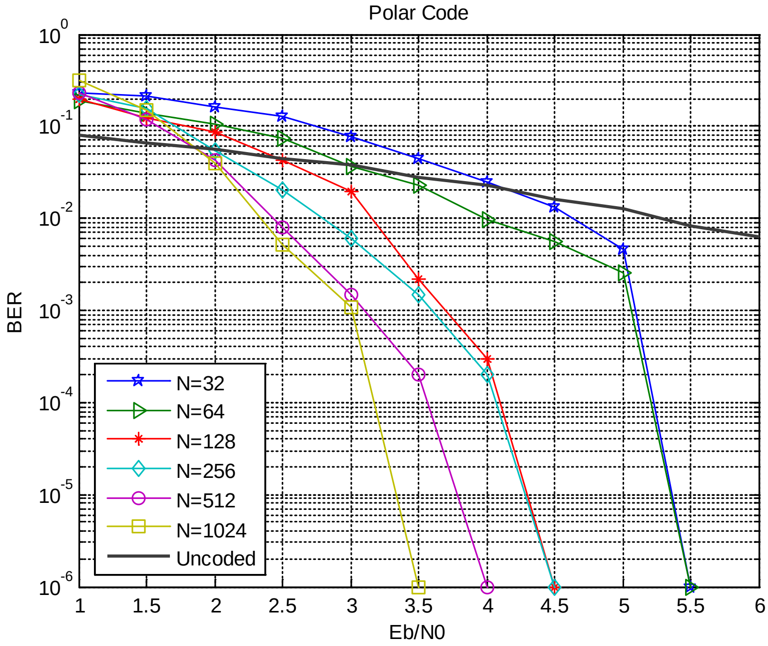

Fig. 6. BER for different block lengths with SC-decoder.

The second stage was to enhance the BER performance using a successive cancellation list (SCL) decoder of polar code concatenated with a cyclic redundancy check (CRC) code, to achieve it we used the algorithm which proposed by Tal and Vardy in [20]. To enhance the BER performance with less latency, we added a spreading process to our model. In the following, an explanation for our model.

A random signal, as shown in Fig. 7, was generated of k-bits block size, encoded using CRC encoder was parity bits added by multiplying the information block by the generator matrix of size (*k*, CRC polynomial size), and appended at the end of the resulting codeword. At the code construction stage, the resulting bits are set on the indices of the good channels (have less Bhattacharyya values) and on the bad channels, frozen bits are set. The *N*-bits codeword encoded using the polar encoder and the level of the resulting codeword bits converted to bi-polar. The resulting bits are multiplied by a spreading sequence and sent via an AWGN channel. At the receiver end, the received likelihoods of the signal despreaded then decoded using SCL decoder, where a group of codewords sent to the CRC detector to choose the codeword, which achieves the CRC check.

The SCL decoder regarded as an extension of the original SC decoder. The principle working of SCL decoder differs from SC decoder. In SC decoder, the received bit guessed to be either 0 or 1, but in SCL decoder, the decoding path doubled to zero and one at every decoding phase *i*ε (*A*: the information bits set). This means rather than keeping the path that owns the most likelihood, multiple paths are retained with their calculated likelihoods after each decoding phase, and the path that has the highest probability at the end of the decoding process will then be selected. One of the disadvantages of this way is, the complexity will increase, because of the huge number of paths after the decoding process. So, a certain number of paths that have the highest likelihood are retained at each phase rather than retaining all paths.

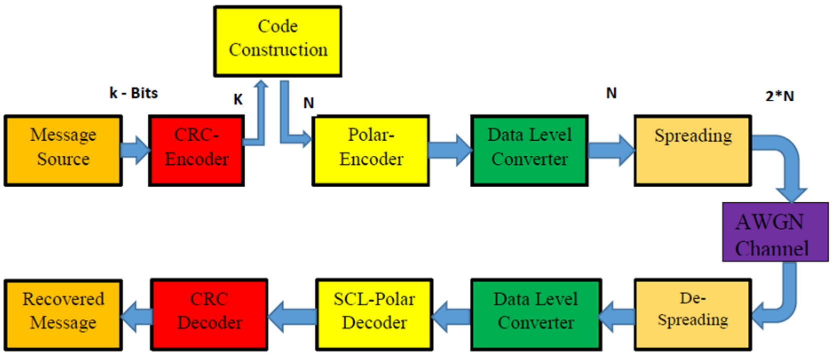

Fig. 7. A block diagram of improved polar code.

As we mentioned above, in this manuscript, we investigate and enhance the BER and latency performance of the polar, we used a Matlab simulation program. The channel we tested in this model was the AWGN channel.

4.1. Simulation results for SCL decoder plus CRC code

For SCL decoder, a group of lists used, *L*=2, 4, 8, 16, and 32 with block length *N*=512 and *N*=1024. The CRC size used in this work is 6.

It is concluded from Figs. 8 and 9, that as the list size increases, the BER performance will be enhanced. From Fig. 8, for *L*=32 the BER =10^{-6}at E_b/N_0= 2.2 dB, while in *L*=8 and 1 the BER = 10^{-6} achieved at E_b/N_0= 2.4 dB and 3.6 dB, respectively. This enhancement in BER was at the expense of the latency. As the list size increases the BER decreases but, the latency increase, this was observed from the results as shown in Tables 1 and 2. So, in this section there is no enhancement in the latency.

Table 1. Latency for different list sizes for block length N=512.

List size	*L*=1	*L*=2	*L*=4	*L*=8	*L*=16	*L*=32
m (s)	1.1	9.6	16.2	22.3	44.8	98

Table 2. Latency for different list sizes for block length N=1024.

List size	*L*=1	*L*=2	*L*=4	*L*=8	*L*=16	*L*=32
m (s)	1.28	1.27	22	25	41.9	164

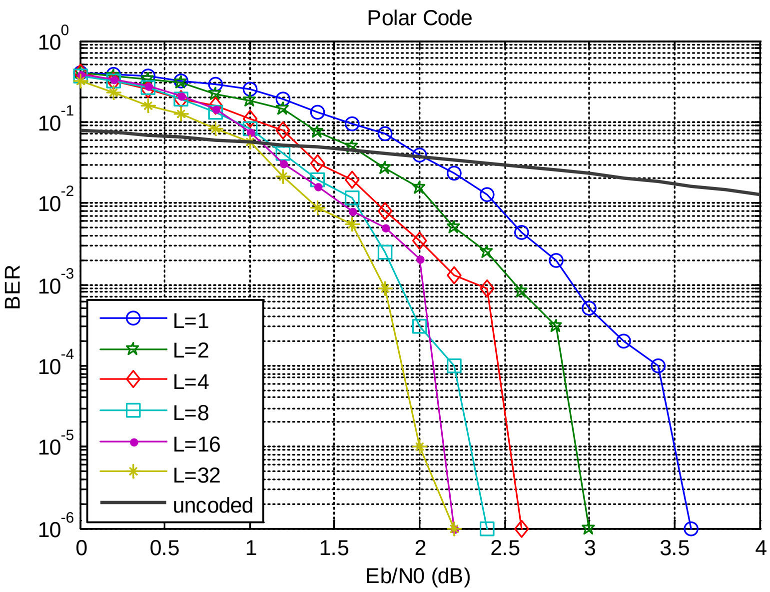

Fig. 8. BER performance for different list sizes of block length=512.

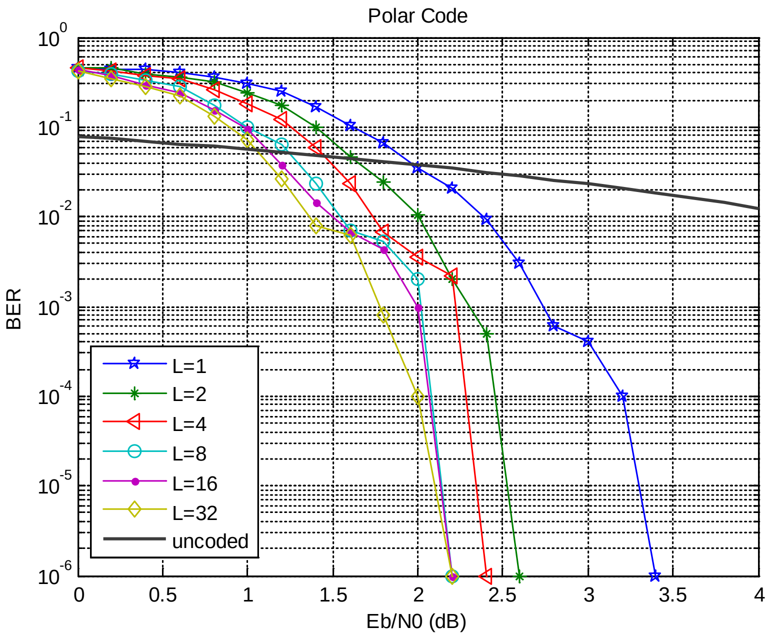

Fig. 9. BER performance for different list sizes of block length=1024.

4.2. Simulation results for SCL decoder plus CRC code with the spreading process

As seen from the results in Figs. 10 and 11, and Tables 3 and 4, we obtained a 0.2 dB gain from the spreading process with a little latency. If the comparison made between the results for block length = 512, it is seen that, for list size = 8 with spreading, a BER performance is close to that get from list size = 16. While the latency for SCL *(L*=8) with spreading is 2.68e-2 s, and the latency for *L*=16 without spreading is 4.48e-2 s, so the delay gain was 18 ms. The enhancement in the latency by adding the spreading process negatively affected the data rate.

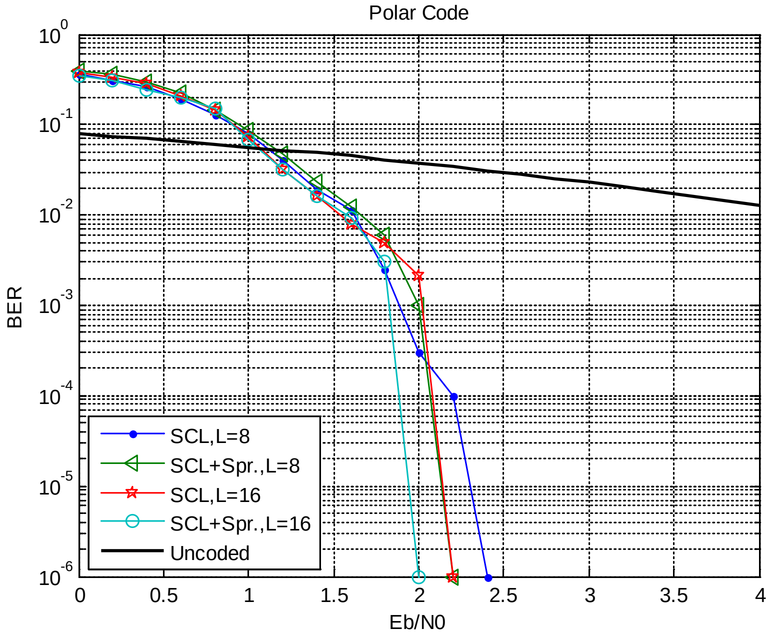

Fig. 10. BER for block length=512 with and without spreading.

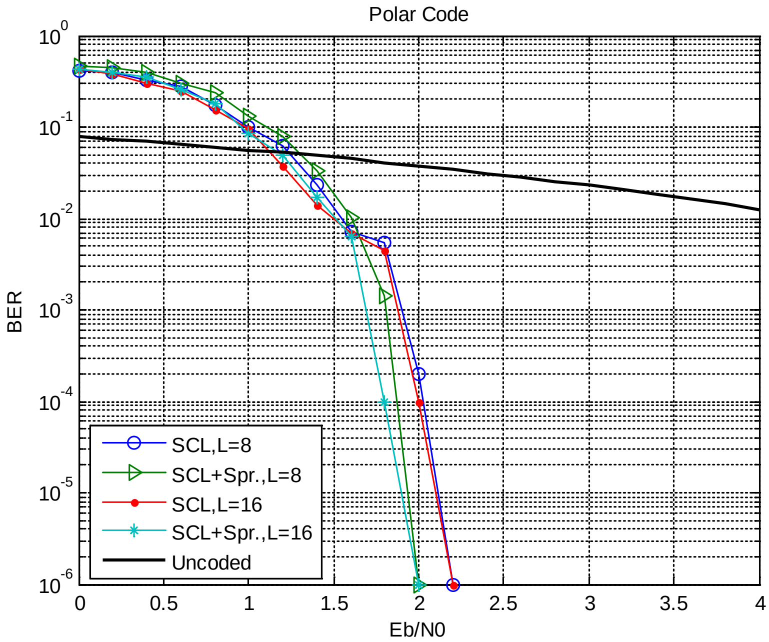

Fig. 11. BER for block length=1024 with and without spreading.

Table 3. Latency for SCL decoder with spreading for block length=512.

List size	Polar model	Latency (s)
***L*=8**	With Spr.	2.68e-2
	Without Spr.	2.23e-2
***L*=16**	With Spr.	5.77e-2
	Without Spr.	4.48e-2

Table 4. Latency for SCL decoder with spreading for block length=1024.

List size	Polar model	Latency (s)
***L*=8**	With Spr.	2.9e-2
	Without Spr.	2.5e-2
***L*=16**	With Spr.	5.4e-2
	Without Spr.	4.19e-2

5. Conclusion

The aim of this research paper was the investigation and enhancement of the BER of functional block length for polar code. From the results, it is noticed that its performance was weak in the small-sized block lengths, and for block length=512 we get a BER=10^{-6}at E_b/N_0= 3.6 dB. To enhance the performance, the SCL decoder used with a CRC code, it is observed that the BER enhances as the list size increases, we get BER=10^{-6} at E_b/N_0= 2.2 dB for list size=32, but at the expense of latency. To improve the performance of latency, we added a spreading process and obtained a 0.2 dB gain with little latency. If the comparison made between the results for block length = 512, it is seen that, for list size = 8 with spreading a BER performance is close to that get from list size = 16, while the latency for SCL (*L*=8) plus spreading equally to 2.68e-2 s and the latency for *L*=16 without spreading is 4.48e-2 s, so the delay gain was 18 ms.

Nomenclatures

A	Information bits set
G	Generator matrix
K	Information length
L_i	Likelihood ratio
u_0^{n-1}	Information and frozen bits vector
u_A^c	Frozen bits vector
$\hat{u}_i$	Estimated vector
$W_N^{(i)}$	Channel index
y_0^{n-1}	Channel output

Abbreviations

AWGN	Additive White Gaussian Noise Channel
BER	Bit Error Rate
BPSK	Binary Phase Shift Keying Modulation
B-DMC	Binary Discrete Memory less Channel
CRC	Cyclic Redundancy Check
FEC	Forward Error Correction
LDPC	Low Density Parity Check Code
SCL	Successive Cancellation List

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