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EVALUATION OF GENE EXPRESSION PROGRAMMING TO PREDICT THE LOCAL SCOUR DEPTH AROUND A BRIDGE PIER

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Abstract

One of the main reasons for bridge failure is the local scouring around the piers. In this way, the precise prediction of a permissible depth of scouring is pivotal to ensure safety and to keep successful maintenance. The main objective of the current study is to predict local scour around the piers by developing new empirical equations using two effective approaches, i.e., gene expression programming (GEP) and artificial neural networks (ANN). Various important parameters were used to derive the empirical equations such as pier shape, flow depth, flow intensity, pier width, and flow direction angle (attack angle). All these parameters were determined from the dimensional analysis of the problem. The two relationships were developed based on 729 data points from the numerical models, which were divided into two sets, training, and validation (test). Moreover, three statistical indexes (i.e., RMSE, R^2 and MAE) were used to identify the performance of the two approaches and their new empirical equations. The results of the comparison indicated that the ANN model (RMSE=0.102, R^2 = 0.94 and MAE=0.076) is performed better than the GEP model (RMSE= 0.124, R^2=0.90 and MAE=0.103) slightly. The latter is preferred on account of its ability to produce explicit and compressed arithmetic expressions. Furthermore, the sensitivity analysis results show that the index of flow depth/width ratio (y/b) has the significant influence on local scour depth predictions compared to other input variables.

Keywords: Artificial neural network (ANN); Bridge piers; Gene expression programming (GEP); Scour depth prediction; Statistical modelling.

1. Introduction

The issue of the rivers' flow is one of the most important parameters related to river management, that including sediment transport, flooding, and riverbed deformation and scouring. However, there are three types of scouring that could happen with the flow at the bridge places: namely, local scour, general scour, and contraction scour [1].

Riverbed deformation is of prime importance to hydraulic engineers and infrastructure designers, as the existence of hydraulic structures like bridges causes contraction in the stream of flow and scouring around piers and abutments. Several researchers have studied extensively the scouring problem from different perspectives under various conditions, some of these studies having evolved techniques and methodologies to analyse scour depth around the piers through experimental testing [2-5]. Most of these studies have been implemented on relatively large bridges, because they are highly expensive and need more maintenance. Recently, Computational Fluid Dynamics (CFD), which solves and discretizes Navier-Stokes and mathematical continuity equations, has been utilized in a wide range of flow process numerical simulations [6-11]. Large bridges are very expensive, this justifying accurate scour depth prediction, for both economic and safety purposes as an unexpected or excessive depth of scour may cause costly bridge construction or bridge failure [12, 13]. Several approaches have been evolved to predict the necessary scouring depth, nevertheless when its mechanism is complicated, and making it tricky to obtain, then a general empirical equation is suggested to estimate the scouring depth based on different field variables like pier shape, flow intensity, flow depth, pier width and flow direction angle (attack angle). Most predictive models were specified from the previous studies have been evolved to utilize a traditional regression-based mechanism using experimental and field data [14, 15]. Recently, Mohamed et al. [16] mentioned that Colorado State University (CSU) [17] have presented feasible predictions, while Jain and Fischer [18] and Melville and Sutherland [19] models over-predict the depth of scour, these results were dependent on the comparison of the scour equation of the bridge pier using both field and laboratory data. Some of the above studies were evolved their equations utilized dimensional analysis followed by an analysis of non-linear regression, however, this approach is not very precise and contains lengthy calculations, because this technique is presently less attractive. large number of recent studies are emerged the use of artificial intelligence (AI) methods, so that simulating can be carried out easily and accurately [20-23].

Inductive technique methods based on AI, are utilizing extensively to model complex response functions including analysis of scour due to their non-linear model structures, and its ability to catch the relationship of cause-and-effect from these operations. These (AI)-based technologies are containing artificial neural networks (ANNs), adaptive neuro-fuzzy inference systems (ANFIS), genetic algorithms (GA), genetic programming (GP), and Gene Expression Programming (GEP). For hydraulic design problems as in the case of nonlinear and highly complex reaction functions, ANN has been reported to supply rationally good results [24]. Gene Expression Programming (GEP) has recently been recognized as superior to many of the other available methods because of coding ease, quick calculations, and simple modeling. Many studies over a range of engineering fields, have reported that GEP is also more precise and workable than other previously recommended approaches.

In the previous studies, it was shown that, in general, the mathematical analysis of scour depth builds on artificial intelligence technologies and GEP in specific, but that these have not been widely carried out, implying that there is an important need to implement this study. So, the main aim of this research is to evolve a new formula of scour depth by utilized GEP as a function to the main influencing parameters (flow intensity, the ratio of flow depth, pier shape, the ratio of pier width and angle of attack) depend on the data computed from numerical models, as well as an examination of the performance of the suggested GEP model in comparison to ANN model is conducted.

2.Materials and Methodology

2.1. Dimensional Analysis

Figure 1 shows the local scouring depth around the pier of the bridge under a steady state of flow, above a bed of non-cohesive and uniform sediments and with clear water states, based on a number of variables: fluid parameters, flow parameters, bed sediment parameters, flume geometry, pier parameters and time. Depth of scouring (ds) in straight channels having homogeneous sediment can be written as follows [19]:

$$ds = f(\rho, \nu, V, y, g, \rho_s, d_{50}, \sigma g, V_c, B, b, L, K_s, K\theta, t) \tag{1}$$

where *ds* represents the maximum depth of scour, ρ fluid density, ν fluid kinematic viscosity, *V* approach velocity, *y* flow depth, *g* acceleration of gravitational, ρ_s the density of sediment, d_{50} the median size of sediment, σg standard deviation of the distribution of the particle size, *Vc* critical mean velocity, *B* width of the channel, b pier diameter, *L* pier length, K_s pier shape factor, $K\theta$ correlation coefficient of flow alignment and t flow duration.

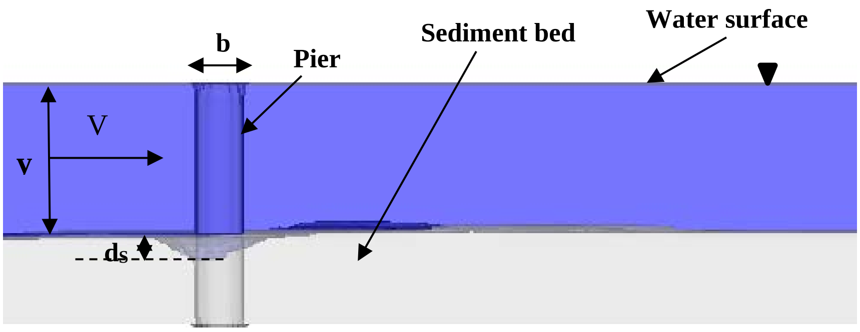

Fig. 1. Sketch of the local scouring about a bridge pier (circular shape).

The mechanism of local scour about a pier of the bridge can be better understood if suitable dimensionless variables that describing the issue are specified, so some of these variables are disregarded in the current study because of the large quantity of data. One layer of sediment with the median size $d_{50} = 0.385\ mm$) was suggested during this research, with a good uniform gradient, and the flow time duration was introduced to be (t=30 min.). in other words, the parameters d_{50}, σg and t are fixed and can be neglected from Eq.(1). additionally, in Eq. (1), the dimensional analysis of the fifteen independent variables is decreased to five dimensionless variables by utilized Buckingham's theorem. Then chosen V, ρ and b were chosen as repeated parameters. Therefore, the equation that depicts the effect of these parameters on the depth of scour can be written as follows:

$$\frac{d_s}{b} = f\left(K\theta \;, \frac{y}{b} \;, K_s \;, \frac{B}{b} \;, \frac{V}{V_c}\right) \tag{2}$$

where $\frac{d_s}{b}$ represents the scour depth ratio, $\frac{y}{b}$ the ratio of flow depth, $\frac{B}{b}$ the ratio of pier width, and $\frac{V}{V_c}$ the intensity of flow. Dimensionless variables in Eq. (2) were used for the two models GEP and ANN, as input and output variables. Only one output parameter was used (dependent), namely, the ratio of scour depth ($\frac{d_s}{b}$), all other parameters were used as input variables (independent). 729 data points were obtained from the numerical model of the scour depth of bridge pier with different pier shapes [25], they were divided into two sets (randomly): i.e., 85% used to train the model, while the other 15% were used to test (validate) model. Table 1 shows the limitations of the variables used. This data group were simulated by applying GEP and ANN to evolve a mathematical model to predict the maximum depth of scour ratio (ds/b) around the bridge pier and to examine the most appropriate methods to predict the depth of scouring using three important statistical indexes: RMSE, $R^2$and MAE. The summary of the methodology for this study is shown in Fig. 2.

Table 1. Minimum and maximum values of parameters used in the training and testing models.

Parameters	**Data Limits**	
	Minimum	**Maximum**
V/V_c	0.55	1.00
y/b	0.20	2.95
b/B	0.11	0.15
Ks	0.71	1.26
Kθ	1.00	1.68
ds/b	0.00	1.88
y: cm	5.00	15.00
b: cm	5.08	6.85
B: cm	45.60	45.60
V: cm/s	18.00	32.80

2.2. GEP and ANN

GEP is an evolved genetic programming, improved by Ferreira [26]. It is a research mechanism based on artificial intelligence and computer programs such as logical terms, decision tree and math data. It is an extension of genetic programming (GP) optimized by Koza [27] that includes both simple linear chromosomes of fixed length (genomes), similar to those used in Genetic Algorithms (GAs), and branching structures of different sizes and shapes formulated as expression trees (ETs). In the form of a phenotype, like trees analysis in genetic programming. In its current form, it merges the benefits of its predecessors, GP, and GA, while removing some of the limitations of these technologies [28-30]. The main objective of this system is to generate a mathematical equation that can be certified to the presented dataset to generate a GEP model.

The GEP operation for this equation involves metaphorical regression across most GA genotypes. The GEP operation begins with the random obstetrics of chromosomes for a specific single number (initial population). Each of these single chromosomes is then assessment by utilizing a certain function (the function of fitness) versus a set of fitness statuses. Then, the selection of chromosomes is relying on the value of fitness: so, the chromosomes that have a better 'goodness of fit' have a bigger probability of being chosen for the new next obstetrics. After chromosomes are chosen, any adjustments made by genetic factors are reproduced: insertion sequence switching (IS), mutation, inversion, gene switching, root insertion sequence switching (RIS), single or double / recombination and gene crossover. In this context, the present paper was used GeneXproTools 5.0 to develop GEP-based approaches, generate a straightforward, mathematical expression of the scour model of piers bridge.

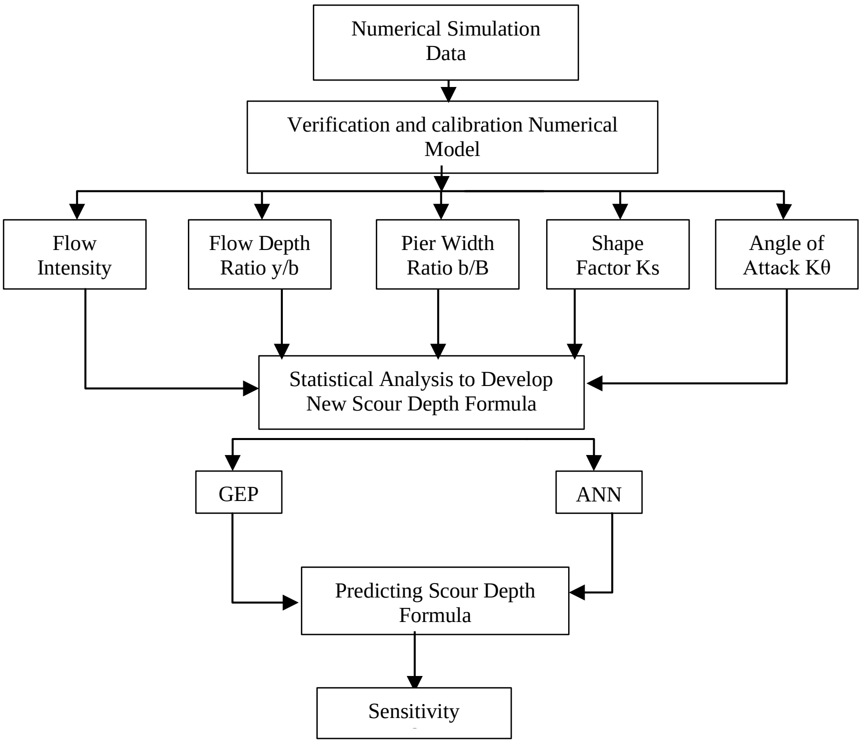

Fig. 2. Research methodology flowchart.

On the other side, artificial neural network (ANN) is defined as a flexible mathematical structure capable of characterizing non-linear and complex relationships between input and output data sets. ANN models are effective and useful, especially with problems where it is difficult to describe the properties of processes using physical equations. ANNs can be used to estimate local scouring depth by constructing a multi-layered, feed-forward network. A stochastic selecting between input and output parameters is supplied three basic layers of neurons; the Input layer, hidden layer, and output layer, each neuron is function as a separate computational component. There is a very high degree of freedom in ANNs that could be related to their architecture, and this advantage provides strength to the model. Before implementation, a neural network learns by data groups, and this gives the network's input and output, couple and values for connection weights, centres, or biases.

2.3. Modeling of the Depth of Local Scouring

More than 700 data groups for scour depth of bridge pier, were computed from the computational fluid dynamics numerical model of the bridge pier by Flow-3D program [25]. This information is represented in Eq.(2), the variables $K\theta, \frac{y}{b}$, K_s , $\frac{B}{b}$, $\frac{V}{V_c}$ are used as independent input parameters, while the ratio of local scour depth ($\frac{d_s}{b}$) is considered as a dependent output variable. GEP was used to evolve a model of the output parameter ($\frac{d_s}{b}$) based on the numerical output data.

These data were split into two groups: one for the training process and another group for the validation/testing process. 620 data points (85%) were selected (randomly) and utilized for calibration (training) to construct the GEP model. Also, another data (109) observation were used for GEP model validation (testing), which represented approximately 15%. After that, different variables for the model building were identified, these reported in the following points:

Step one: GEP model starts with an initial set of individuals. An individual population consists of chromosomes of specific length which may be multi-gene or single. For the initial population, any volume of the population can be utilized, but the chromosomes within the zone (30 -100) have given more accurate results as mention in the previous study [26]. After sufficient attempts to find the optimum number of the population wanted to make suitable (good) results, the size of the chosen population for this study was 75 chromosomes.

Step two: each chromosome is assessment, their fitness computed by the values of one statistical index, in this study RMSE is used.

Step three: for each individual gene in the chromosome, the group of the function (F) and the group of the terminals (T) are known. This model is designed by utilized essential computational process and power, thus giving F = {+, -, *, /, power}. The terminal groups, within the independent parameter and random numerical constant coefficient, let $T = \{ K\theta , \frac{y}{b} , K_s , \frac{B}{b} , \frac{V}{V_c}, ? \}$ where '?' represented the random numerical constant (RNC).

Step four: In this step, the structural organization of chromosomes is symbolizing in order to compute the length of genes heads and their number. this starts by utilize an individual gene and increasing it gradually. Ferreira [26] reported that the success ratio could be raised when the number of genes increases from one to three in each chromosome. According to that, three genes were utilized in each chromosome in order to make it multigenic, while the used head value is 8.

Step five: This step involves choosing the link function. Because there are three genes, the output of the model can be created from three various subsets (Expression Trees). These subsets (ETs) are related by addition factors (+) to obtain the final output solution.

Step six: Finally, the group of genetic operators causing the variations and rates for their values were chosen. A set of all genetic terms like mutation, reversibility, transposition (RIS, IS, and genetic switching), Dc genetics and recombination (genetic recombination, one and two points) were used. A mutation rate equal to 0.044 of one and two points was used. The other genetic factor rates are illustrated in Table 2.

The GeneXproTools 5.0 was used to simulate the models after determined all parameters of the model. Also, RMSE was used to compute the maximum fitness and as a termination standard. the simulation model was operated for a certain generation number, then was stopped when it was no further development in the result values of statistical indexes or fitness function, or when the simulation attains maximum fitness function (up to 1000). The results of the final simulation utilized to predict the scouring depth ratio (ds/b) around the pier of the bridge and formulated it as an expression tree Eq.(3) is presented as follows:

$$\frac{ds}{\mathrm{b}} = d2 * \left(\frac{\frac{d1}{d0}+\frac{d4}{d1}}{d0+d1-7.24}\right) + (d0 * d2) * [2.72 * d3 + d1 + 2.72 * d4] + d2 \quad - \left[\left((d1^{d0} * d2) + \frac{d0}{d1}\right) * d2\right] \tag{3}$$

Table 3 illustrates the definition of all parameters were used in Eq. (3).

Table 2. The values of parameters used with the GEP model.

Parameters	Values	Parameters	Values
Population size	75	**Inversion rate**	0.1
Function group	+, -, *, /, power	**IS transposition rate**	0.1
Terminals group	$K\theta, \frac{y}{b}, K_s, \frac{B}{b}, \frac{V}{V_c}$, ?	**RIS transposition rate**	0.1
Random numerical constant (RNC)	05	**Gene transposition rate**	0.1
Type of (RNC)	Floating point	**One-point recombination rate**	0.1
RNC Range	[-10, 10]	**Two-point recombination rate**	0.3
Head Length	08	**Gene recombination rate**	0.3
Genes Number	03	**Dc-specific mutation rate**	0.044
Function of Linking	+	**Dc-specific inversion rate**	0.1
Fitness function	RMSE	**Dc-specific IS transposition rate**	0.1
Mutation rate	0.0441	**Random constant mutation rate**	0.01

Table 3. Definition of the corresponding variables used in GEP model.

Variable	Definition
d_0	V/V_c
d_1	y/b
d_2	b/B
d_3	K_S
d_4	Kθ
C1 (gene1)	-7.24
C4 (gene2)	2.72

For the second prediction model used in this study (ANN), the same dataset segmentation was used as for GEP. For more than 700 datasets, the neural network model was trained utilized by (85%) from the datasets (620). While, for validating(testing) the network prediction model, 109 remaining datasets (15%) were used. The ANN model was performed with Neural Power 2.5. A basic front feed kind mesh was trained to utilize the reverse diffusion method. For further training and validation of model, the data were normalized before entering into the program, and several experiments were performed to obtain the best ANN structure. For the hidden and outputs layers, the transformation of values through layers was simulated by utilizing the sigmoid activation function. The initial weights utilized are randomly created for the values close to zero in ANN simulating. In the current study, a neural network with a momentum factor of 0.4 and a learning rate of 0.1 was used.

3. Discussion of Results

The predicted scour depth obtained by the two models GEP and ANN are drawn versus the observed scour depth around the bridge pier. The performance of the two different methods was tested by calculated three statistical indexes; namely, RMSE, R^2 and MAE, as listed in Table 4. As noted from Table 4, the ANN model performed slightly better than the GEP model, with high-value predictions: i.e., R^2 = 0.94, RMSE=0.102 and MAE=0.076. While the GEP approach (Eq. 3) predictions produce R^2 = 0.90, RMSE=0.124 and MAE= 0.103. it is worth mentioned that these results are noted to be close to the ANN model indices. The training and testing results of scatter diagrams for the GEP and ANN models are shown in Figs. 3 and 4, respectively. These figures of predicted (training and testing) dataset against observed values is utilized to estimate the degree of convergence for the two models with measured data.

Additionally, it is clear from Fig. 5 that both models (GEP and ANN) are more precise at predicting local scour depth when ds/b is less than one, in comparison to it when ds/b is more than one. The main expected reason for the emergence of this phenomenon is due to limitations and assumptions in used numerical models. However, this could be happened when increasing the value of the depth of local scour (ds) with a constant diameter of the pier (b), there will be raise in the velocity of the flow, this driving to the creation of vortices which increase the turbulence of flow. This turbulence and vortices were not considered as operators, which may have an effect on the estimated depth of scour, resulting in a decrease in its predictive precise. One of the important features of GEP is that it gives an easy-to-use, an explicit empirical formula for its bridge scour model as noted by Eq. (3), which verified versus [19] laboratory data, this granting it a priority over the ANN method. The maximum scours depth (ds) around the pier (circular shape) computed from the GEP model, Eq. (3) is 3.6 cm, while the scouring depth measured from Melville experimental model is 4 cm giving an error rate ratio up to 10%.

Table 4. Summary of statistical indexes results for GEP and ANN.

Model	Training			Testing		
	RMSE	**R^2**	**MAE**	**RMSE**	**R^2**	**MAE**
GEP-Model	0.101	0.92	0.105	0.124	0.90	0.103
ANN-Model	0.091	0.952	0.052	0.102	0.940	0.076

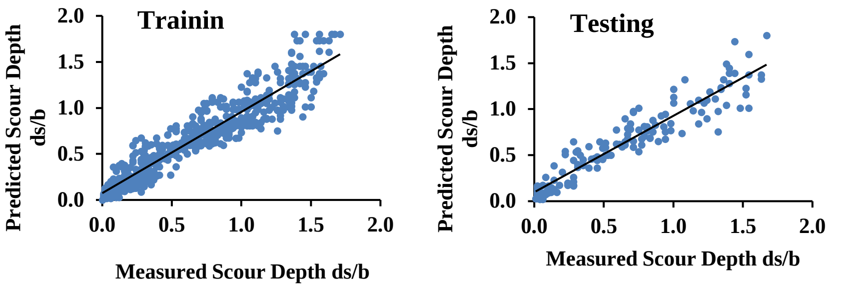

Fig. 3. Measured versus predicted scour depth ratios (ds/b) for the GEP model.

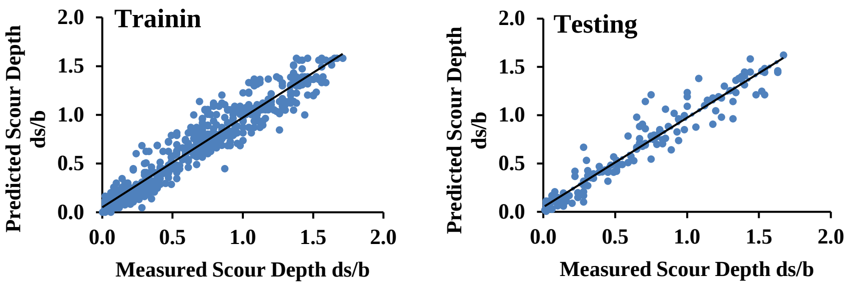

Fig. 4. Measured versus predicted scour depth ratios (ds/b) for the ANN model.

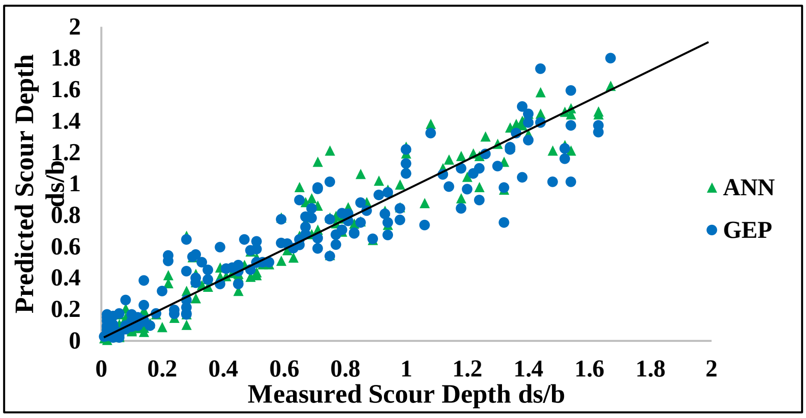

Fig. 5. Comparison between GEP and ANN models (testing data).

In summary, while the empirical equations are not as efficient as mathematical and analytical formulas, it still produces rationally acceptable results in addition to its easy used. Although the ANN model performs slightly superior to the GEP model in results of statistical indexes and scatters diagrams, it does not introduce any overt mathematical format. GEP can produce a compressed and explicit mathematic expression that could be useful to hydraulic engineers in the future.

4. Conclusion

The correct estimation of the depth for scour around bridge piers is difficult and complicated to measure. By using different modelling methods, this study aimed to evolve a new, more accurate empirical formula to compute the depth of local scour around bridges piers. The variables most important at predicting scouring depth were obtained by utilizing the dimensional analysis process. Five dimensionless variables were produced by dimensional analysis: pier shape factor (K_S), flow depth ratio (y/b), flow intensity (V/Vc), pier width ratio (b/B), and flow direction angle (K_θ). A functional relationship was generated utilizing GEP, its performance comparative to the ANN model. The ANN model provided smaller values for MAE (0.076) , RMSE (0.102) and a greater R^2 (0.94) value from the values computed utilizing the GEP formula (R^2 =0.90, RMSE=0.124 and MAE=0.103). According to the statistical indexes, GEP's performance for test results is slightly less than that of ANN as ANN provided lower values for RMSE (0.102) and MAE (0.076) and a higher value for R^2 (0.94). The ANN model is to some extent better than the GEP model, but in spite of this performance, it is not as superior as it does not provide any candid mathematical formula that is simple to use by bridge designers. The GEP model provides a clear and direct equation, the benefit of this making the GEP model more efficacious and unique. Therefore, it can be inferred that the GEP method is an effective simulating model for computing the depth of local scour, providing simple and easy to utilize empirical equations for simulated response functions.

Nomenclatures

B	Width of the channel, cm
b	Pier diameter, cm
ds	Maximum depth of scour, cm
d_{50}	Median size of sediment, cm
g	Acceleration of gravitational, m/s^2
K_s	Pier shape factor
L	Pier length, cm
t	Flow duration, sec
V_c	Critical mean velocity, cm/s
V	approach velocity, cm/s
y	Flow depth, cm

Greek Symbols

$K\theta$	Correlation coefficient of flow alignment
ρ	Fluid density, g/cm^3
ν	Fluid kinematic viscosity
ρ_s	Density of sediment, g/cm^3
σg	Standard deviation of the distribution of the particle size

Abbreviations

AI	Artificial Intelligence
ANFIS	Adaptive Neuro-Fuzzy Inference Systems
ANN	Artificial Neural Network
CFD	Computational Fluid Dynamics
CSU	Colorado State University

GA	Genetic Algorithm
GEP	Gene Expression Programming

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