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A SMARTPHONE INDOOR LOCALIZATION BASED ON AFFINITY PROPAGATION CLUSTERING AND KULLBACK-LEIBLER MULTIVARIATE GAUSSIAN

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Abstract

Recently Location-Based Services (LBS) has gained a substantial interest to create smart space where significant utilize is provided to improve life services. LBS is used to provide a varied range of navigation services such as emergency healthcare and security services LBS represents the main goal in pervasive computing. Due to the wide spread of Wireless Local Area Network (WLAN), many researchers are investigating this technology with different algorithms to develop an LBS system with high accuracy. Unfortunately, the Received Signal Strength (RSS) of WLAN has a big limitation, the multimodal distribution of the signal overtime. To overcome this issue, we propose different algorithms in fingerprinting-based localization, in offline phase, an affinity propagation clustering is proposed with different schemes to reduce complexity of searching space and lessen the power consumption during online estimation. To increase the accuracy of the proposed system and reduce the localization distance error, different Access Point (APs) selection approaches is used. In online phase, we propose Kullback-Leibler Multivariate Gaussian (KLMVG) model incorporates with Probabilistic Neural Network (PNN) scheme. The proposed method was compared with nearest neighbour method which indicate the integrated system outperform the nearest neighbour estimation in term of localization distance error.

Keywords: Affinity propagation, Fingerprinting, Indoor localization, Probabilistic neural network, Wi-Fi.

1. Introduction

Recently, real-time categorizing the location of the object takes a wide range in hot topic research. Location based services is predictable to growth from of $935.05 million in 2014 to almost $4.42 billion in 2019 in research [1]. In general, The Global Positioning System (GPS) is considered as the greatest positioning system, nevertheless, it is impossible to use inside the building due to the absence the Line-of-Sight (LOS) which is considered as an obligated condition between the GPS satellite and the receiver. Therefore, the location based on smartphones can play a significant role in a lot of computing applications, besides many other technologies, for example, Radiofrequency Identification (RFID), Bluetooth, Wi-Fi, Zigbee, ultrasound, magnetic field variation, and light-emitting diode. Due to the existing of WLAN infrastructure and low cost, Wi-Fi is considered as the one of the most used technology [2, 3].

Indoor positioning system (IPS) is categorized into two major types: the fingerprinting localization based on collected data and the log-distance propagation model. The log-distance propagation model is classified into two main types: lateration and angulation methods. The main idea of indoor positioning system based on lateration estimation is using signal measurement and geometry to estimate the location between the access points (APs) and smartphone by using different algorithm. Nevertheless, the log-distance propagation model is suffering from major disadvantages, such as the absence the LOS due to the movement of people, furniture, and the presence of walls, which cause multipath signal. In addition, the accuracy of the signal drops dramatically if the coordinate of one of the AP is not accurately calculated. Due to these drawbacks, the estimation the location of the object has become more difficult. Thus, the indoor positioning system based on fingerprinting method has become more proposed as an alternative technology [4].

IPS based on fingerprinting-based technique is categorized into two main phases: the offline and online phase. during the offline phase, the building will be divided into a specific set of rectangular gird points, then the RSS of the APs at each grid is recorded and stored as a fingerprinting radio record [5-7]. During the online phase, the smart phone is collected the RSS data in interest and then the data is directed to the server to determine the object locations in the area of interest by comparing it with the database as shown in Fig. 1 [8, 9]. The K-Nearest Neighbors (KNN) formula is one of the pioneer algorithms that is used to estimate the location by using the squared Euclidean formula to estimate the similarity between the online measurement and offline measurement. To increase the accuracy and decrease the complexity in recent localization technique more task was added such as, AP selection schemes, where three schemes were used, random selection, AP strongest RSSI, and Fisher criterion. Furthermore, the Area of Interest had been divided into RPs sub-area, which proves an efficient metric to use.

Problem statement

In this paper, to create the fingerprinting-database the RSS value were recorded into four different orientation (45°, 135°, 225°, and 315°) within 100-time sample in the offline phase. In general the conventional localization methods try to select the AP based on fingerprinting database, In this paper, three different methods were applied to choose the APs first: random selection without consideration to the

characteristics or distribution module of the Aps, secondly: Strongest APs Scheme were the selection based on the strongest APs power that can cover the largest area, thirdly, Fisher criterion to choose the optimum APs based on the discrimination ability of the RSS from the APs by using his metric property. Then, we proposed affinity propagation clustering method to utilize and minimize the number of reference points (RPs) that the proposed method have to go through which minimize the searching space. After that, we formulate Kullback-Leibler Multivariate Gaussian (KLMVG) model incorporates with Probabilistic Neural Network (PNN) scheme to obtain high localization accuracy. The proposed method outperformed both the nearest neighbour method and kernel estimation in term of localization distance error, robustness, and accuracy by using real database.

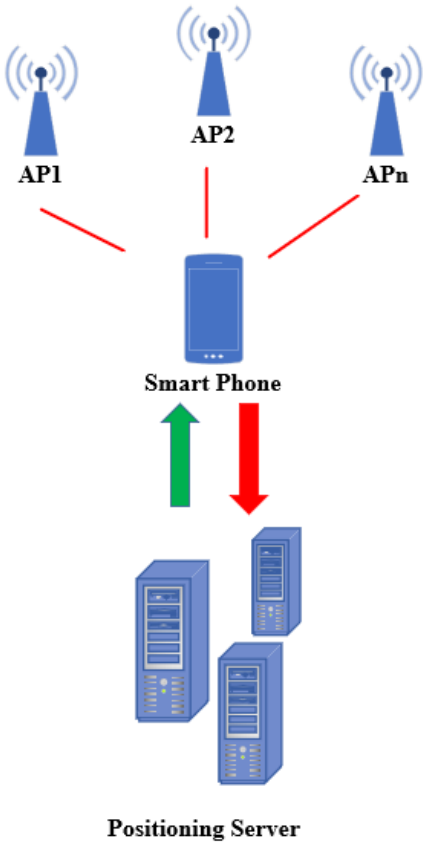

Fig. 1. Wi-Fi fingerprinting-based localization method.

2. Related Work

Many researchers focused on Wi-Fi signal in IPS using fingerprinting-based algorithms due to the fact that GSP can't be used inside the building. Many techniques were proposed to create a system that is working inside the buildings with low localization error, for instance, Log-Distance Path Loss (LDPL) model that is built a statistical relationship between the RSS value and the RF propagation function. Other approaches have been proposed such as Time of Arrival (TOA),

Time Difference of Arrival (TDOA), and Angle of Arrival (AOA) that are trade-off between cost and accuracy [10].

RADAR is an early approached that is used RSS from Wi-Fi to create an IPS, that is combine between the theoretical propagation and empirical fingerprinting approach. RADAR used both KNN and Weighted KNN (WKNN) [11]. A new version of KNN is proposed by Bahl and Padmanabhan [12] that is more efficient than traditional KNN , neural network, and probabilistic methods, that is relies on the decision tree of the radio map and take the average of the radio map reading as an alternative of taking the whole dataset to estimate the location of the object. Yim [13] proposed a modified KNN with Manhattan, Mahalanobis, and Euclidian distances and found that Manhattan equation outperformed the Euclidian and Mahalanobis distance, that is the localization distance error has decreased. Farshad et al. [14] used a probabilistic distribution measurement by using a framework from a Bayesian formula to measure the localization distance error. They were considered as a pioneer of using the probabilistic distribution measurement in IPS. Castro et al. [15] used the Kullback-Leibler Divergence (KLD) to estimate the localization distance error of the object, which shows uncertainty in how to quantify the localization object in varies space. Our implemented method is a natural way of smoothing the RSSI distribution overtime and trajectories, that doesn't need to recalibrate, which can be consider as general method to non-Gaussian distribution.

3. Kullback-Leibler Multivariate Gaussian and Probabilistic Neural Network Formulation

IPS fingerprinting-based localization technique was implemented, in a way that a person will hold a smartphone that have access to Wi-Fi to accumulate RSS values from the WLAN at different locations at Western Michigan University (WMU). However, the RSS distribution has a multimodal distribution due to many reasons, such as, reflections, scattering, multipath signals. In our research, a mobile robot was used to record the signal-to-noise ratio for one WLAN for a 35 min in a corridor. The mobile robot was programmed to stop every five minutes. different distribution was noticed by difference within 10 dBm, as shown in Fig. 2.

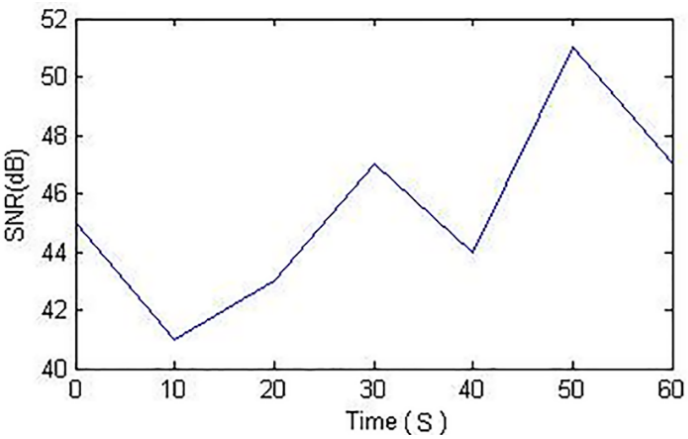

Fig. 2. Signal-to-noise ratio of received strength signal indicator variations over time.

To reduce the effect of the multimodal distribution and reduce the effect of the person that held the mobile phone which can be an obstacle for the signal. Different orientations (45°, 135°, 225°, and 315°) was used to record the signals of different APs to create fingerprinting radio map. The RSS were collected within time sample as $\{q_{i,j}^{(\circ)}(\tau), \tau = 1, \cdots, t, t = 100\}$, where t stands for the time sample and (°) represents the orientation direction of the person that held the mobile phone. The average and the covariance matrix of the RSS data were calculated in four directions. The radio map database is represented by $Q^{(\circ)}$:

$$Q^{(\circ)} = \begin{pmatrix} q_{1,1}^{(\circ)} & q_{1,2}^{(\circ)} & \cdots & q_{1,N}^{(\circ)} \\ q_{2,1}^{(\circ)} & q_{2,2}^{(\circ)} & \cdots & q_{2,N}^{(\circ)} \\ \vdots & \vdots & \ddots & \vdots \\ q_{L,1}^{(\circ)} & q_{L,2}^{(\circ)} & \cdots & q_{L,N}^{(\circ)} \end{pmatrix} \tag{1}$$

where $q_{i,j}^{(\circ)} = \frac{1}{q}\sum_{\tau=1}^{t} q_{i,j}^{(\circ)}(\tau)$ and t = 10. The time sample were arbitrarily chosen from 100-time sample. where L stands for the number of APs and N stands for the number of the reference points. The variance vector is the variance for AP i at RP j with orientation (°) is calculated as:

$$\Delta_j^{(\circ)} = \left[\Delta_{1,j}^{(\circ)}, \Delta_{2,j}^{(\circ)}, \Delta_{3,j}^{(\circ)}, \ldots\ldots \Delta_{L,j}^{(\circ)}\right] \tag{2}$$

where

$$\Delta_{i,j}^{(\circ)} = \frac{1}{t-1}\sum_{\tau=1}^{t}\left(q_{i,j}^{(\circ)}(\tau) - q_{i,j}^{(\circ)}\right)^2 \tag{3}$$

Thus, the fingerprinting database is $x_j, y_j, q_j^{(\circ)}, \Delta_j^{(\circ)}$ with $q_j^{(\circ)}$ defined as:

$$q_j^{(\circ)} = \left[q_{1,j}^{(\circ)}, q_{2,j}^{(\circ)}, q_{3,j}^{(\circ)}, \ldots\ldots\ldots\ldots, q_{L,j}^{(\circ)}\right] \tag{4}$$

the RSS measurement of online phase is represented by:

$$p_r = \left[p_{1,r}, p_{2,r}, \ldots\ldots\ldots, p_{L,r}\right] \tag{5}$$

3.1. Affinity propagation clustering in offline phase

Due to the multipath channel, scattering and reflection, RSS is varied overtime which can lead the RSS deviate from the signals stored in the radio map. To reduce the effect of this variations, affinity propagation is proposed and performed to divide the radio map into small regions of RPs that are similar reading of RSS measurements.

In general, in K-mean clustering the K initials are randomly chosen, while in affinity propagation the cluster will be generated without needing the K initial values the exemplars such as in K-mean algorithm. in other words, all the RPs are treated equally as a potential exemplar known as preference.

After that, to measure the similarities the real messages will be recursively transmitted between the RPs until the cluster are created by using the initialization independent property. The pairwise similarity $s(i,j)$ measures the suitability of the candidate RP to be an exemplar in the clustering of the radio map. Every RP has a

noise measurement that can be indicate by δ_j, which is considered as Gaussian distributed to measure the pairwise similarity, Euclidean distance was used to define the similarity between the RSS of RPs.

$$s(i,j)^{(\circ)} = -\left\|\Psi_i^{(\circ)} - \Psi_j^{(\circ)}\right\|^2, \forall i, j \in \{1,2,.....,N\}, j \neq i, \circ, \in 0 \tag{6}$$

All the RPs in the radio map are treated equally to be an exemplar, a common preference number is used to generate a median number of clusters in radio map that is defined as:

$$p^{(\circ)} = \gamma^{(\circ)}.median\left\{s(i,j)^{(\circ)}, \forall i, j \in \{1,2,.....,N\}, j \neq i\right\} \tag{7}$$

where γ represents the numbers of clusters that are determined [16]. The main core of operation in affinity propagation algorithm is the transmission signals between messages between RPs, the responsibility message r(i,j) from the RP to the candidate exemplar is defined as:

$$a(i,j)^{(\circ)} = \min\left\{0, r(i,j)^{(\circ)} + \sum_{i^{'} \neq i,j} \max\left\{0, r(i^{'}, j^{'})^{(\circ)}\right\}\right\} \tag{8}$$

The messages between pairs are send recursively until the best candidate of exemplars of the radio map clustering are set. This process will be done in the offline phase. Let £ be the set of RP *j* that been chosen as exemplar, Let *S* represents the set of RPs within RP *j*. in this paper, a different coarse localization was proposed to match the cluster that online RSS measurements belongs to.

Criterion I: Cluster matching using the RSS measurement exemplar. In this criterion the negative measurement of the Euclidean measurement will be calculated between the online measurements and the exemplar to find best exemplar that the online phase measurement belongs to.

$$s(i,j)^{(\circ)} = -\left\|\Psi_i^{(\circ)} - \Psi_j^{(\circ)}\right\|^2 \tag{9}$$

Criterion II: Clustering matching using the average RSS measurement: in this criterion an average RSS measurement is used to match best cluster that online RSS measurement by calculating the average of all cluster members, which can lead to more accurate measurement, it was computed using Euclidean distance which is shown as follows:

$$s(i,j)^{(\circ)} = -\left\|\Psi_r^{(\circ)} - \Psi_c^{(\circ)}\right\|^2 \tag{10}$$

where

$$\Psi_r^{(\circ)} = \Psi_c = \frac{1}{\left|C_j^{(\circ)}\right|} \sum_{i \neq i,j} \Psi_k^{(\circ)} \tag{11}$$

The pseudo-code for Affinity Propagation Clustering In Offline Phase is provided in Algorithm (1)

Algorithm 1: Affinity Propagation Clustering In Offline Phase
Input: Ψ, s
Output: exemplars for each data object in Ψ
1: repeat
2: for each $(\Psi_i^{(\circ)}, \Psi_j^{(\circ)})^2$ Ψ^2 do
3: compute $p^{(\circ)}$ by using eq. (7);
4: for each $(\Psi_i^{(\circ)}, \Psi_j^{(\circ)})^2$ Ψ^2 do
5: compute $a(i,j)^{(\circ)}$ by using eq. (8);
6: until all $r(i,j)^{(\circ)}$ and $a(i,j)^{(\circ)}$ are not updated
7: for each Ψdo
8: get an exemplar;

3.2. PNN-KLMVG Ips method

PNN is a supervised feed-forward algorithm that proceeds from Bayesian decision network. The input signal is trained and compared with the database of the radio map. The first layer consists of the numbers that is representing the input vector, whereas the second layer or the training patters layer consists of neuron for each feature of the training database of the radio map.

The dot products are used to compute the similarity/dissimilarity between the input pattern and the training data. Radial basis function (RBF) is used an in-activation function to find the similarity with the input vector [17].

The Wi-Fi signal distribution of the radio map represents the conditional probability distribution of the object position that is indexed by ℓ where $q_\ell = q(S_j \mid \{x_\ell, y_\ell\})$. Due to the important role of the (APs), S represents the multivariate random variable. The Bayes conditional probability is given by:

$$P(\{x_i, y_i\}/S_j) = \frac{q_\ell P(\{x_i, y_i\})}{\sum_{j=1}^{m} P(S_j/\{xj, y_j\}) P(\{x_j, y_j\})} \tag{12}$$

$P(\{x_i, y_i\})$ represents the probability of the RSS data of the class $\{x_i, y_i\}$.where input vector S_j related to class $\{x_i, y_i\}$ if

$$P(\{x_i, y_i\}/S_j) > P(\{x_j, y_j\}/S_j) \;\; \forall j = 1,2,3, \ldots . m \tag{13}$$

and $P(\{x_i, y_i\}/S_j)$ represents the conditional probability function (PDF) of S_j. since prior probability $P(S_j/\{xj, y_j\})$ is undefine, a kernel-based approach is used to calculate it.

$$P(p,q) = \frac{1}{\sqrt{2\pi\sigma}} exp(-\frac{(p-q)^2}{2\sigma^2}) \tag{14}$$

If the p=q the kernel function will be equal to 1 and it will start to lessening if the value between signals becomes larger. Figure 3 shows the architecture of probabilistic neural network with layers.

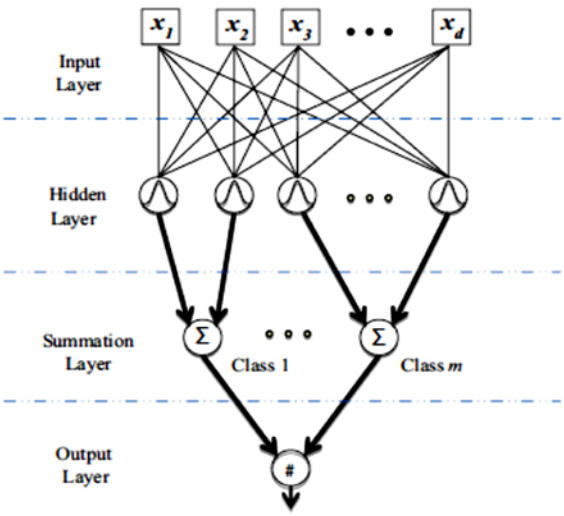

Fig. 3. Probabilistic neural network architecture.

i. *Input Layer:* the test point is represented by this layer. The number of neurons is equal to the number of APs (variables) that is required to describe the form of the input.

ii. *Pattern layer*: This layer represents the learning set of the hidden layer, that each input has a neuron that could estimate the similarity with the present input vector. In general, PNN uses the pattern layer to estimate the likelihood of the input vector by estimating the PDFs to locate the class that the input belongs to [18].

$$p(S|\{x,y\}) = \prod_{j=1}^{J} p\big(S_j\{x,y\}\big) \tag{15}$$

The probability kernel is implemented by using either a weighted scheme that allows to estimate the probabilities from the training dataset of the radio map or a regression scheme. A kernel approach using KL_{MVG} is proposed, this model exploit the interdependencies between the radio map, such that the geometry is quantified to estimate the correlation among the radio maps. KL_{MVG} is used to measure the dissimilarity between the input vector points and the radio map:

$$KL_{MVG}(p||q_j{}^{(\circ)}) = \frac{1}{2}\begin{pmatrix} (\mu_{q,j}^{S} - \mu_p^{S})^T(\Sigma_{j,q}^{s})^{-1} \\ (\mu_{q,j}^{S} - \mu_p^{S}) + tr(\Sigma_R^{s}(\Sigma_{j,q}^{s})^{-1} \\ -I) - ln\left|\Sigma_p^{s}(\Sigma_{j,q}^{s})^{-1}\right| \end{pmatrix} \tag{16}$$

where S represents the RSS values of different APs at known locations, where j represents the location of the cell in radio map

$$S_j{}^{(\circ)} = \{\mu_j{}^{(\circ)}, \Sigma_j{}^{(\circ)}\} \tag{17}$$

where $\mu_j{}^{(\circ)}$represents the mean of J_{th} of the RSS values measurement and $\Sigma_j{}^{(\circ)}$ is the covariance matrix of input data, and $|\Sigma|$ represents the determinant of Σ. The KL_{MVG} is formulated as a probability kernel-based approach to estimate the PDF of the dataset of the radio map and the test point (TP) from the online by measuring the likelihood between them. The distribution of the RSS is defined as [19]:

$$D(p, q_\ell) = exp\left(-\frac{KL_{MVG}(p||q_j^{(\circ)})}{2\sigma^2}\right) \tag{18}$$

iii. ***Output Layer***: The neuron of the output layer has to be equal to the existing classes. Every-single neuron is connected to all the neuron in the hidden layer. The output determines which classes that the input belongs to, which has been presented as probabilistic output that is proportional to density function. Nevertheless, to estimate the location with high proficiency with low localization error, the class was performed as weighted kernel regression, the location of the sample distribution of RSS was estimated as:

$$(\bar{x}, \bar{y}) = \frac{\sum_\ell (x_\ell, y_\ell) k(p, q_\ell)}{\sum_\ell k(p, q_\ell)} \tag{19}$$

3.3. AP selection schemes

AP selection is made in both online and offline phases. Nevertheless, if the APs selection subset is made during the offline phase using the RSS of the fingerprinting data, then the RSS characteristics of the online phase will be useless and may lead to high localization distance error. However, the APs selection subset is selected from the online phase, it can exploit the RSS characteristics of the online and offline phase. There is another way to exploit the characteristics of RSS measurement of the offline and online data map, by using the RPs of coarse localization phase, which means that the RPs of the offline phase, and RSS from the online phase can have very similar characteristics.

- Random Selection Scheme:

  Under this scheme, the APs will be arbitrarily selected, without consideration to the characteristics or distribution module of the APs, Random selection scheme has less complexity and calculation, and it needs to generate only once.

- Strongest APs Scheme [20]

  The previous studies advocate that the APs that were selected based on the strength of the RSS can lead to a better localization distance error. The perception is based on that the strongest APs power can provide a better coverage. However, this scheme may not be suitable and is not always render to a better location.

- Fisher criterion [21]

  The fisher criterion is used to measure the discrimination ability of the RSS from the APs by using his metric property, the metric of the statistical properties of the fingerprinting radio map is used to select the APs during the offline phase. A score is determined for every AP as:

$$\zeta_i = \frac{\sum_{j=1}^{N} (q_j^{i^{(o)}} - \bar{q}_i)^2}{\sum_{j=1}^{N} (\Delta_j^{i^{(o)}})} \tag{20}$$

$$\bar{q}_i = \frac{1}{N} \sum_{j=1}^{N} q_j^i \tag{21}$$

The proposed method shown in Fig. 4.

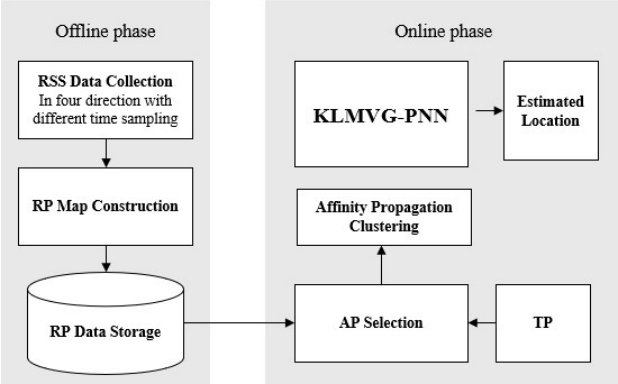

Fig. 4. The Wi-Fi-based fingerprinting architecture.

4. Experimental Evaluation and Implementation Results

This section provides experimental evaluation of the proposed algorithms. The RSS data were gathered at Western Michigan University (WMU) as shown in Fig. 5. A Huawei smartphone with operating system was used to collect RSS data in area of interest. The MAC address and the RSS value were collected using a smartphone. 1 second interval sample was used for 100 second on four directions (45°, 135°, 225°, and 315°) to overcome body-blocking influence. During the offline phase, 47 APs were observed in our recording within a grid of 1 m in our area of interest. 50 unknown location in different time was used as a testing point as evaluating point to assess our proposed system and estimate the localization distance error.

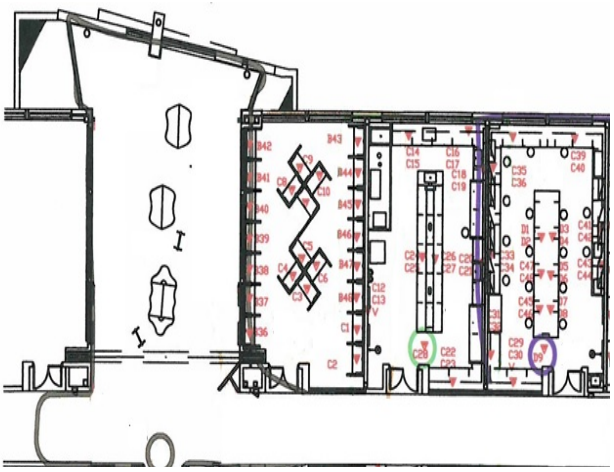

Fig. 5. 2D layout for the first floor of the college of engineering and applied sciences at WMU.

The variation of the RSS value is affected the proposed techniques and the number of recorded the APs and the reliability of the APs. To reduce the effect of RSS variation random RSS value was taken and then calculate the average to enhance the accuracy of the proposed systems. Furthermore, to reduce the effect of RSS variation, affinity propagation is used to create clusters on radio map with their

exemplars during the training phase. The preference value determine the number of clusters that will be created by affinity propagation that is experimentally set.

4.1. Results discussion

Figure 6. shows the localization distance error with different number of clusters regarding to the number of APs that was used. The number of clusters is created by the affinity propagation through the value of the preference that is experimentally set. The figure shows that at 22 APs the localization distance error was 1.45 m when 10 nearest numbers are used with 35 clusters, while it was 1.65 m when 15 clusters are used.

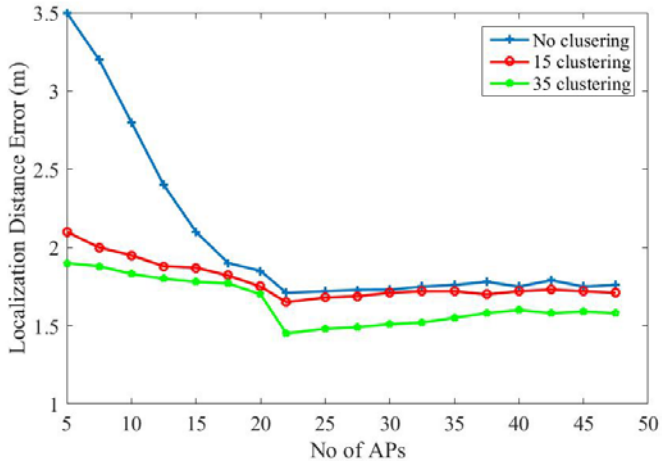

Fig. 6. Localization distance error when different number of clusters is used as a function of APs.

Figure 7 shows the localization distance error with different APs selectins schemes are used, the highest quality was obtained when random selection scheme is used especially when 22 APs were used, followed by fisher criterion and then strongest APs regarding to their RSS value.

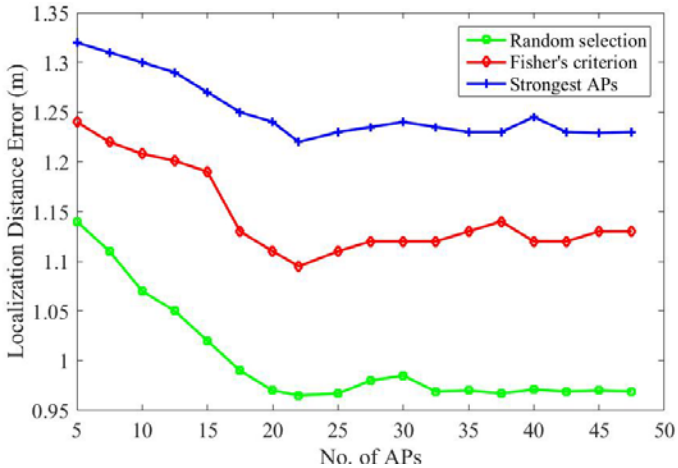

Fig. 7. The Localization distance error under different AP selection schemes.

4.2. Comparison to prior work

Different fingerprinting traditional algorithm was compared to our proposed algorithm, such as KNN and kernel algorithms. Figure 8 shows the cumulative distribution function (CDF) when 35 clusters are used with 22 APs and 20 nearest neighbor. Our proposed algorithm showed that our proposed algorithm is outperformed the other algorithms.

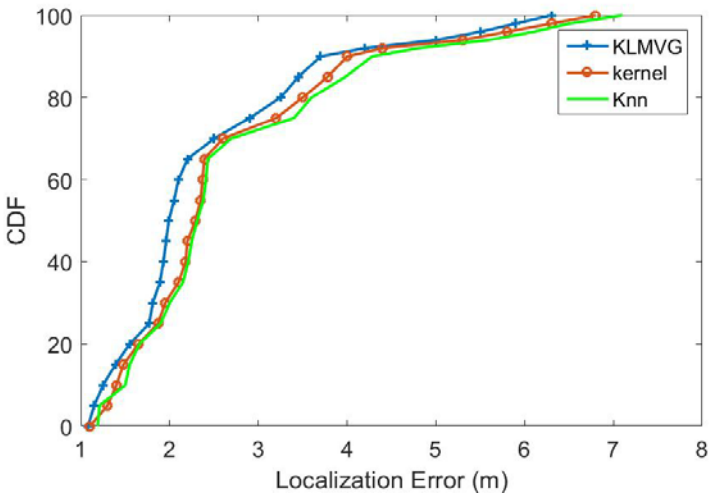

Fig. 8. Experiment results: The Cumulative Distribution Function (CDF) of localization error when using 20 nearest neighbours.

The Fisher criterion uses the variance in his calculation which leads to the fact that the APs with high variance has a less score to be selected and considered as reliable in his calculations. However, the Fisher criterion cannot be considered as a reliable source to choose a reliable APs due to the fact that the Fisher criterion is used during the offline phase only, which means, if one of the APs are not presented in the online phase, the fisher criterion cannot proceed in his calculation. A comparison was made in Table 1 of the implemented methods of wireless indoor localization such as KNN [16], kernel-based method [20], and KLMVG showed that the 90% error of the smallest distance error that were obtained with the implemented algorithms.

Table 1. Position error statistics.

Technique	Median (m)	Accuracy 90%
KNN	1.9	4.9
kernel	1.6	4.2
KLMVG	1.1	3.7

5. Conclusion

In this paper, we proposed two novel algorithms for wireless indoor localization, first we proved that using clustering algorithm will enhance the efficiency of the system and reduce the overlapping and the multimodal of the signal. Affinity

propagation clustering is used to reduce computational cost with different scheme to prevent choosing the fault cluster during the online phase. Furthermore, to reduce the affection of variation of the RSS, different APs selection methods was used to reduce the localization distance error, some of them during the offline phase, while other during the online phase. The other two proposed algorithms are an accurate based localization technique using Kullback-Leibler Multivariate Gaussian (KL_{MVG}) model incorporates with probabilistic neural network (PNN) scheme is proposed. Different approaches were proposed to compare with our proposed system such as KNN and PNN. Our proposed system shows outperformed results with error less than 1 m. our future work will investigate on using different clustering algorithm to prevent the overlapping clustering and investigate more efficient methods to reduce the multimodal distribution of the signal.

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