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ELICITATION OF STAGE DISCHARGE RELATIONSHIP FOR MULTI-GATE CONTROLLED BARRAGE

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Abstract

The aim of this research was to investigate, in the laboratory, several scenarios for closed gates and the manner in which symmetry and asymmetry affect the barrage discharge equation with multi-sluice gates. Using the Buckingham Theorem theory, a flow equation was derived, connecting the flow level and the volume of flow through the gates, in the case of all open gates, the symmetrical and asymmetric opened gates. Besides, this formula was suggested for the boundary between the conditions of free and submerged flow through various cases of opened gates, which was derived using dimensional analysis. The statistical analysis program (SPSS) was employed using the laboratory measurements to calculate the values of the parameters. The inferred formulae were confirmed using the laboratory data of the flow, measured employing the standard laboratory means. The relative error percentage of the values tested was extracted with precision. The findings of the mean absolute and mean relative errors of 3.9% and 0.9%, respectively, were recorded for a variety of cases and conditions. The inferred formulae can be applied to a wide range of tests.

Keywords: Asymmetric open gates, Barrage, Free flow, Submerged Flow, Symmetric, Sluice gate.

1. Introduction

A barrage is a kind of hydraulic installation built across rivers, whose function is to funnel volumes of water flow into artificial irrigation canals and waterway networks. Barrages principally function by raising the water levels to feed the main channels of the irrigation networks. Besides, some barrages are utilized as diversion amenities during the times of flood waves. From the information given above, these hydraulic installations, which control the flow calibration, appear to be necessary and significant in terms of design, management, and operation [1].

As this type of hydraulic installation is very significant and exerts an extensive impact, Abdullah et al. [2], worked on a paper concerned with the distribution of such types of hydraulic installations in Iraq, which in reality happens to be among the first installations used in this country. Generally, barrages involve several gates which enable the flow rates to be measured and the water level to be controlled. In some instances, a few gates are closed, as in the case of the low spending seasons; they are also helpful in preventing debris like wood pieces, which are carried by the flow, from clogging the openings to the gates. Therefore, closing some gates and opening a few others more widely will ensure the anticipated flow of the debris. Similarly, during the times of the regular periodic or emergency maintenance of some of the gates, the flow conditions can be either symmetric or asymmetric.

The designer and operator of such kinds of hydraulic installations require a drain-attributable formula which must be inferred for the symmetric or asymmetric conditions, as this is a fundamental aspect for them to use. By inferring a relationship between the discharge and water level height near the arches, the designer and operator can effectively know the volume of water rerouted [3].

In 1950, after performing several laboratory experiments, Holdhusen et al. [4] determined the coefficient of discharge of a vertical gate under conditions of free and submerged flows. The curved graph he thus produced, indicating the limits of the free and submerged water flow, continues to be in use until today.

Using the analytical method and applying the momentum equation, Rajaratnam and Subramanya [5], also conducted laboratory experiments. Momentum equation was used to identify a general formula to enable calculation of the flow via the vertical gates.

In fact, Negm et al. [6], explained the impact of various shapes of the sill beneath the gates influenced the coefficient of discharge by altering the inclination of the sill under the gates, at the front end and back of the gates during submerged flow. The formula they proposed to predict the coefficient of discharge (CD).

A laboratory study was conducted, as well as the momentum and energy equation were used on the radial gates to evaluate the (CD) under such gates [7].

In the work on the Kut Barrage, Maatooq and Saeed, presented a calibration of (CD) by giving the empirical equations for both the free and submerged flow [8].

Sauida [9] did an investigation for submerged flow, to derive an equation to assess the (CD) for the vertical sliding gates of an arch, for this scenario and multiple others, for both symmetrical and asymmetric openings.

Nago and Furukawa [10] explored the influence exerted by the sudden lateral expansion on the width of the channel and the vertical dip, as well as the effect of both cases of sudden down lift and lateral expansion unitedly on the value of the (CD).

Also, Ferro [11] investigated the relationship between the flow of a stream that simultaneously flowed below and above a sluice or a broad-crested gate.

Maatooq [12] suggested ways of enhancing the empirical equations in order to estimate the (CD), surmised from the prior studies, as well as the likelihood of employing them to increasing accuracy of application of the flow in Kut Barrage.

To ensure that the discharge assessment has greater accuracy, Clemmens [13] explained the need to bypass the transition zone as a means to accomplish it.

Interestingly, Bijankhan et al [14]., accomplished a theoretical deduction of a curve to enable differentiation that permits the calculation of the max tail water depth, which is indicative of the demarcation line between the free and submerged flow.

In 1966, Henderson [15] investigated the relationship that exists between the (CD) and contraction coefficient (C_c).

These studies covered a few of the factors that affect the drainage beneath the gates, in the cases of both the free and submerged flows [16-21].

On the other hand, Shamkhi et al. [1] and Elsayed et al. [22] studied the impact of symmetrical and asymmetric flows and all the open gates, as well as investigated the manner in which the Froude number influenced the riverbed and the phenomenon of scouring, under a variety of scenarios. The study deduced an equation for the relationship of Froude number with scour length and depth. Also, it determined the worst and best scenarios for open gates arrangements.

In their study, Stefano et al. [23] examined the flow process in the weir types, with complex shapes, and the conclusions drawn are as mentioned.

This paper discusses devising equations to estimate the flow through the gates that are parallel in origin, having closed gates (symmetrically, asymmetrically, opening all gates) theoretically, as well as in the laboratory for the conditions of free and submerged flows.

Formulae of the equations for the theoretical estimation of the head-discharge flow, employing a dimensional analysis of the parameters which influence that flow.

In this study, using a laboratory channel, 510 experiments will be performed. Also, a general formula will be designed linking the discharge level, adopting the Buckingham Theorem theory, and re-confirming the deduced formula, using the experimental data gathered for the conditions of both submerged and free flow, besides the validation. The equations were proposed by linear regression analysis.

2. Dimensional Analysis

Using dimensional analysis, the relationship between the discharge and levels of the free and submerged flow states can be obtained, apart from finding a formula for the greatest depth of the free flow state [24, 25].

2.1. Dimensional analysis of free flow

Dimensional analysis for free flow, are given below [3]:

$$F1\ (Q, H_0, B, b, w, g, \mu, \rho)\ =\ 0 \qquad (1)$$

where function code $F1$; Q = discharge, B =flume width, b= opened gate width are independent factors μ, w, ρ and g and the inferred dimensional variables are as:

$\pi\ 1= \frac{H_0}{W}$, $\pi\ 2= \frac{Q^2}{W^5\, g}$, $\pi\ 3 = \frac{B}{W}$, $\pi 4= \frac{b}{W}$, $\pi 5= \frac{W^{1.5}\, g^{0.5}}{W}$

$\frac{\pi_2^{\frac{1}{3}}}{\pi_4^{2/3}} = \frac{y_c}{W}$, where ($y_c$) is the critical depth of the flow.

$\frac{\pi_5 \pi_2^{0.5}}{\pi_3} = Re$, where ($Re$) is Reynolds number, which can be ignored.

Equation (1) is expressed as:

$$\frac{y_c}{w} = f_2(\frac{{H_0}^f}{w}, \frac{b}{w}, \frac{B}{w}) \qquad (2)$$

The equation stated below indicates the free flow:

$$\frac{y_c}{w} = A(\frac{H_0}{w})^{b_f} (\frac{b}{w})^{c_f} (\frac{B}{w})^{d_f} \qquad (3)$$

2.2. Dimensional analysis of the boundary between the free and submerged flows

Dimensional analysis to distinguish the boundary between the free and submerged flows, are given below [3]:

$$y_f^* =\ f1\ (y_j, b, B, Q, w, g, \rho, \mu) \qquad (4)$$

where, $f1$ function code y_f^* = max depth of tail-water that permits the free flow, y_j = jet flow depth (c_c*w), independent factors μ, w, ρ, and g. Equation (4) is expressed, as:

$$\frac{y_f^*}{w} = f_2(\frac{y_j}{w}, \frac{b}{w}, \frac{B}{w}, \frac{y_c}{w}, Re) \qquad (5)$$

Ignoring the effect of the Re and considering the critical depth as a function of (H_0),

$$\frac{y_f^*}{w} = \alpha C_c (\frac{H_0}{w})^{\beta}, (\frac{b}{w})^{\theta} \qquad (6)$$

where α, θ and β are considered constants. The separation point between the free and submerged flows will be adopted when the effect of the flow depth is felt at the upstream depth, where it altered about 3%-5%.

2.3. Submerged flow dimensional analysis

Dimensional analysis for submerged flow, are given below [3]:

$$F\ (Q, H_0, y3, y_f^*, B, b, w, g, \mu, \rho) = 0 \qquad (7)$$

Are independent factors μ, w, and g

$\frac{\pi_2^{1/3}}{\pi_4^{2/3}} = \frac{Q^{2/3}}{wb^{2/3}g^{1/3}} = \frac{y_c}{w},\ \pi_1 - \pi_6 = \frac{H_0 - y_3}{w},$

$\omega(\pi_6 - \pi_7) + (\pi_1 - \pi_6) = \varpi \frac{y_3 - y_f^*}{w} + \frac{H_0 - y_3}{w},$

$\pi_5 \pi_2^{0.5} \pi_3^{-1} = Re,$

where *Re* ignored. So, the derived expression is:

$$\frac{y_c}{w} = a_1 (\frac{H_0}{w})^{b1} (\frac{b}{B})^{c_1} (\frac{H_0 - y_3}{\varpi(y_3 - y_f^*) + (H_0 - y_3)})^{e_1} \qquad (8)$$

Based on certain physical facts, which incorporate the situation of assuming that the backflow level of the flow nears the value of the upstream level of the flow, then the value of (y_c / w) tends to zero, because the value of Q tends to zero. Also, when the value of the depth of the backflow at the gates is directed from the value of the maximum depth of the free flow, then the equation returns to the free flow formula (Eq. (3)). So, the following sub-dimensional formula was deduced:

$$DRF = (\frac{H_0 - y_3}{\varpi(y_3 - y_f^*) + (H_0 - y_3)})^{e_1} \qquad (9)$$

where *DRF* is *DRF* function.

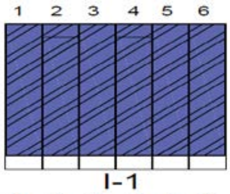
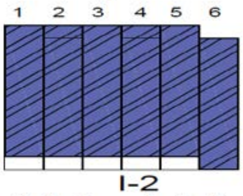
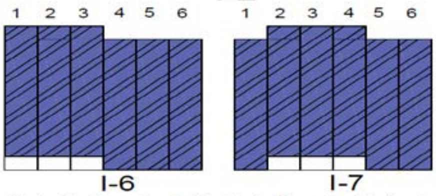
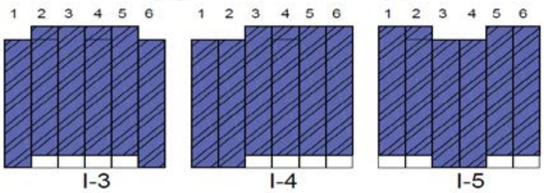
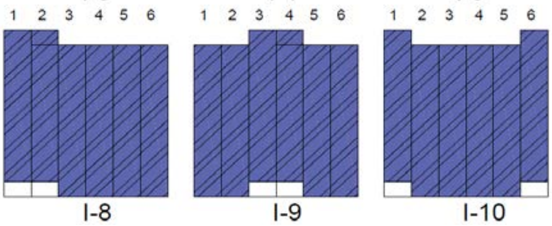
3. Materials and Methods

Experiments were conducted in the laboratory of the Irrigation and Drainage department of the Technical Institute in Al-Kut, using channel with dimensions of 0.5, 0.5, 20 width, height, and length, respectively, as depicted in Fig. 1.

Fig. 1. Laboratory flume and model of barrage.

Water from a fixed upper tank supplies the flume, and it receives the water via a 6-inch pump tube. This is found at the water outlet from the upper tank (V-notch) at a 90º.In the channel are the three-point carriages which are used to measure the height of the point, with 0.5 mm accuracy. The flow depth at the downstream is assessed by a vertical sliding gate fixed at the end of the channel. Ten scenarios of and opening the gates were employed, as revealed in Table 1.

Table 1. Cases of the open gates of the barrage.

No. of open gates	Sketch and Symbols
Six	
Five	
Three	
Four	
Two	

The model was manufactured from plastic material with CNC technology. The sill under the barrage body has a 0.1 inclination to the upstream and a 0.05 downstream part of the barrage. The plastic gates with dimensions of 0.6 cm thickness and 50 cm height for the gates and supports to the iron framework, for controlling the gates, as depicted in Fig. 1.

Parallel flow below the gates is performed using the dimensions of the gate openings (*b*w*) multiplied by the open gates No. and the cases of symmetrical and asymmetric gates, and the free and submerged flows, as revealed in Fig. 2.

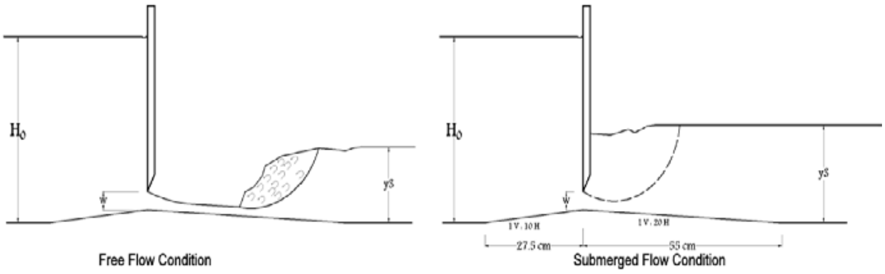


Fig. 2. Schematic sketch of the free and submerged flow.

First, the free-flowing condition in each test is applied for a specified discharge and the flow depth is recorded at the origin. Next, a submerged flow was achieved, in which the tail gate was gradually altered until the water depth at the head of the origin started to show a response to the alteration in downstream. This aspect has been considered as the commencement of the transition to the submerged flow. This data is applied according to the dimensional analysis mentioned earlier. After a function (*Ln*) for it is taken and converted into a linear relationship, and the constants required in the equation are identified.

4. Results and Discussions

4.1. Cases of free flow

The performance of the drainage equations deduced to calculate the flow rate through the parallel sluice gates, under conditions of free flow, must be evaluated. Accordingly, Eq. (3) was depicted along with the experimental data related to the parallel dike gates, with the symmetric/asymmetric closed gates. For six gates, the dispersion of the data observed is shown in Fig. 3. A slight scattering was noted in the data of the directions of the symmetric flow, as evident from Figs. 4(a) and 4(b).

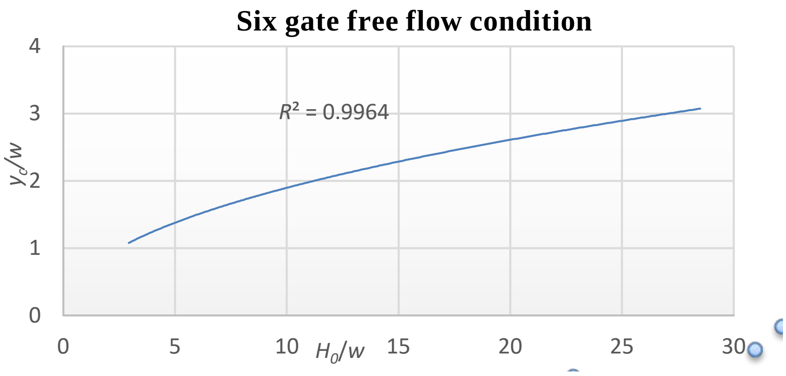


Fig. 3. y_c/w versus H_0/w for six gates of the barrage.

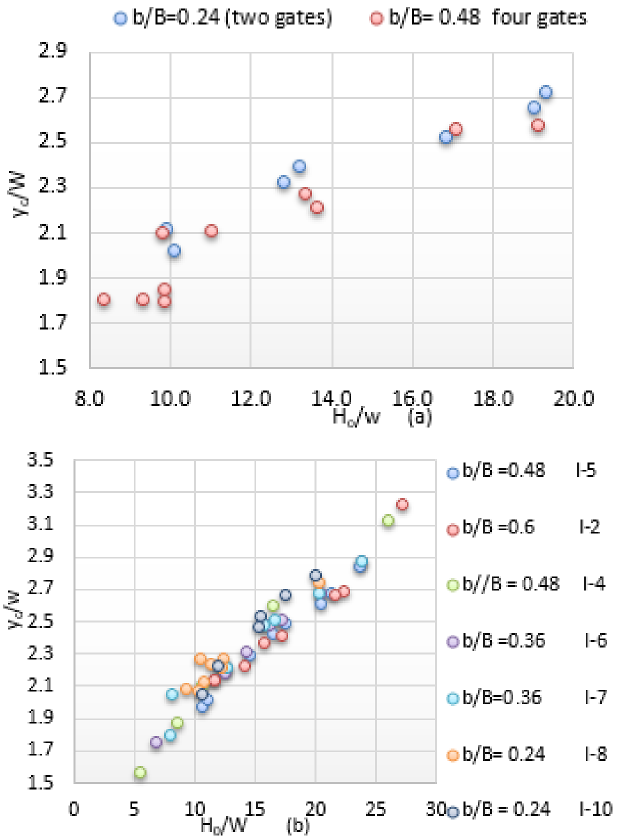


Fig. 4. (a) (y_c/w) versus (H_0/w) (Symmetric);
(b) (y_c/w) versus (H_0/w) (Asymmetric).

Experimental parameters were generated for each of the states of the symmetric and asymmetric gates, because these parameters are recalculated in the laboratory for the values beyond the range, and the measurements are listed below in Table 2.

Table 2. Hydraulic characteristics of barrage.

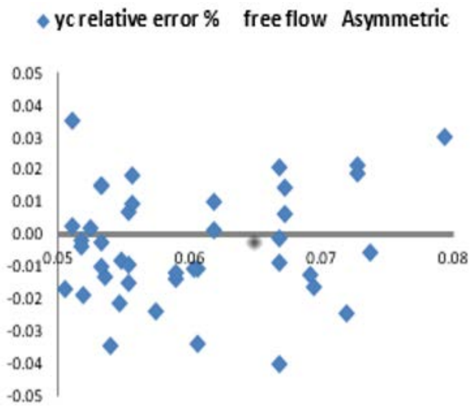
Type	Total width of opened gates	*Q l/s*		*H₀/w*		*Y3/w*
		free	Subm.	free	Subm.	Subm.
Six opened gates	0.72	7.3 -13.6	9-12.0	3.4-24.9	6.3-20.9	3.5-13.9
Symmetric	0.48	9.8-12.1	7-9.0	9.9-19.1	5.8-12.8	3.4-7.9
	0.24	4.6-5.3	4.1-7.1	9.9-19	10.3-16.3	3.6-9.3
Asymmetric	0.6	10.1-12.1	9.5-13.1	9.4-38.1	5.8-12.8	3.6-9.3
	0.48	6-10.1	6.1-10.2	2.9-23.7	7.7-16	3.8-10.6
	0.36	7-10.0	6.5-7.6	6.5-23.9	6.9-14.9	3.4-9.2
	0.24	5-7.0	4.1-6.8	9.4-20	8.5-15.6	3.3-10.4

The IBM SPSS Statistics V23 is used to analyse the equation via linear regression analysis. Table 3 lists the values of the coefficients and the R values for three states.

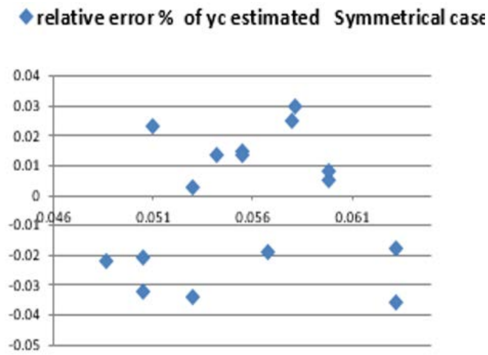
Table 3. Empirical coefficients of Eq. (3).

Type	*A*	*bf*	*cf*	*R%*
Six gates	0.6571	0.4605		99.8
Symmetrical	0.6629	0.443	-0.077	97.1
Asymmetrical	0.769	0.399	-0.059	97.8

As shown in Figs. 5(a) and 5(b), the estimated values of (y_c/w) calculated using Eq. (3) were compared with those recorded experimentally ($y_{c\ est.}/w$) along with the associated relative errors, i.e., (r %) = ((y_c − $y_{c\ est.}$)/$y_{c\ est.}$) x 100, which are recalculated. In the event of symmetric flow, the maximum and mean absolute relative errors of 3.9% and 0.9%, respectively were estimated. For the asymptomatic state, the values recorded were 3.8% and 0.7%. No statistically significant differences were found between the experimental coefficients assessed for both the symmetric and asymmetric cases. For the symmetric case, (y_c/w) correlates with (b/B) strongly (0.077), while for the asymmetric case (0.059) was found to fall between (y_c/w) and (b/B).



(a) Asymmetric case



(b) Symmetric case

Fig. 5. The relative error r% (y_c) estimation

4.2. The boundary between the free and submerged flows

The closed gates cause the state of the submerged flow to take place at a lower flow level, at the back of the origin. This may be possibly due to the flow momentum, as evident in Fig. 7 between the curve of the greater depth of free flow in both cases (the two gates and four gates), where it becomes evident that in the case of the two gates, the submerged flow commences at the level of ($y3/w$ = 1.9), while it is at the level of ($y3/w$ = 2.1) in the case of the four gates. It is crucial to confirm the boundary curve between the free and submerged water flows via the parallel sluice gates in order to record pertinent water measurements that will enable the analysis of the flow under the sluice gates; therefore, experiments were done to shed light on the condition. In this respect, for the dimensions of the parallel sluice gate, the free flow was calculated first.

Based on the dimensional analysis and the resulting Eq. (10) given below, we increased the tail gate of the channel little by little, to produce the condition of submerged flow.

To fulfil the requirements for Eq. (10), tests were done to identify the value of (C_c). The experimental results are as shown below, in Fig. 6, where a value of (C_c = 0.61) was used, corresponding to what was stated in the source [17].

$$\frac{y_f^*}{w} = \alpha C_c \left(\frac{H_0}{w}\right)^{\beta}, \left(\frac{b}{w}\right)^{\theta} \quad (10)$$

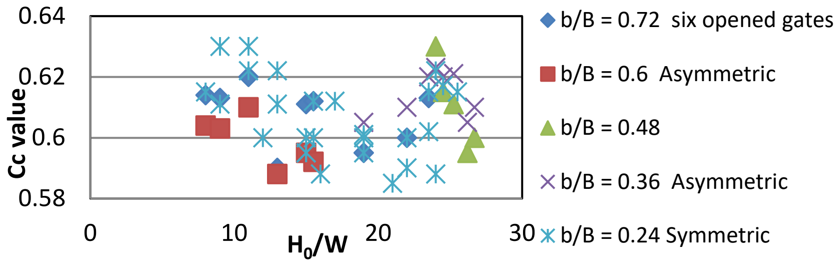


Fig. 6. The C_c versus H_0/w for various arrangements of open gates.

The coefficients (α, β,) are extracted after conducting several laboratory experiments, which come within the range stated in Table 2. So, the values of these parameters are listed in Table 4, which were calculated based on the experiences for the different gate cases.

Table 4. The empirical coefficients of Eq. (6).

Type	α	β	ϑ
Symmetric	3.3	0.367	0.160
Asymmetric	1.091	0.655	0.86

The value of this equation helps to determine the point at which the submerged flow commenced, in the model and for the variety of cases from the gate openings, as shown in Fig. 7. For the different states of the gates and different values of the discharges that have been attempted.

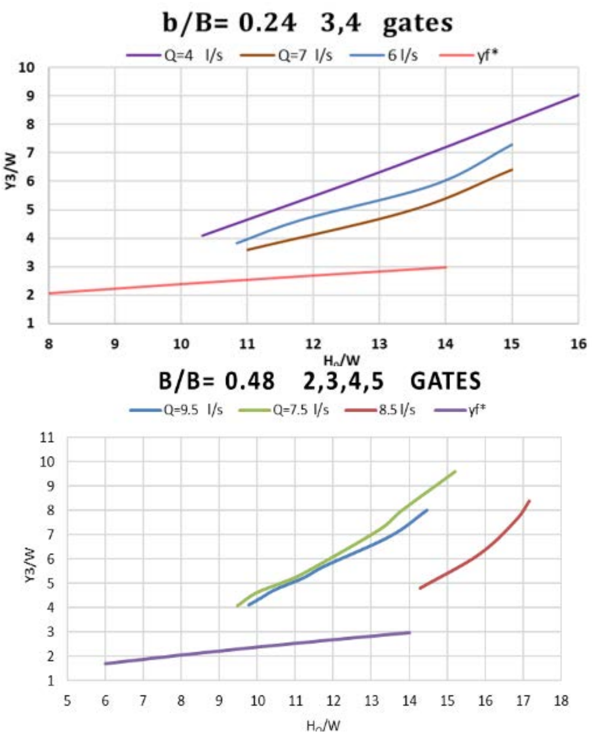


Fig. 7. Experimental data plotted on the characteristic condition curves for the symmetrical cases.

4.3. Submerged flow

This is the state that occurs after going beyond the depth of the flow at the back of the origin of the threshold value that was drawn from the previous case mentioned above. According to the dimensional analysis of the submerged flow and inferred Eq. (11), as shown below:

$$\frac{y_c}{w} = a_1(\frac{H_0}{w})^{b1}(\frac{b}{B})^{c_1}(\frac{H_0 - y_3}{\varpi(y_3 - y_f^*) + (H_0 - y_3)})^{e_1} \tag{11}$$

There is a gradual lifting of the tail gate to increase the level of flow below the origin, until there is an alteration in the level of the upper level of the origin, which implies that the state of the submerged flow has started to get realised.

Table 5 reveals the values of the coefficients deduced in the laboratory. The SPSS V24 program is used, apart from the value (*R*%) for each inferred equation, which shows the strength of this relationship.

Table 5. The empirical coefficients of Eq. (8).

Type	ϖ	*a1*	*b1*	*C1*	*e1*	*R%*
Six gates	0.1115	0.181	0.342		1.517	97.6
Symmetric	0.18	0.954	0.252	-0.107	0.836	92.5
Asymmetric	0.1118	0.125	0.406	-0.108	1.676	88.9

5. Conclusions

In this study the number of open gates was controlled. The degree of the influence exerted by the different factors and the direction of their effect on the values of the free and submerged flow conditions is also shown. Formulae of the deduced equations were done employing the theory (π) to procure a relationship between the level in the front of the arches and the drainage for the cases listed below:

- Free flow symmetric and asymmetric installations and all open gates.
- Boundary equation between free and submerged flow.
- Submerged flow symmetric and asymmetric installations and all open gates.

Identify the parameters of these formulae, using laboratory data, after which, confirm them by obtaining the relative error, the laboratory data and the inferred results were highly useful. So, the formulae presented were verified, and the values of (R%) for the cases of free flow were in the range of 97.8 -99.8%. In the cases of submerged flow, the values of (R) were in the range of 88.9 -97.6%.

The results inferred based on the laboratory and statistical evaluations were highly acceptable in estimating the expenditure and rates for the cases mentioned. It is significant to note that the transaction results are specific to the model mentioned above, and the magnitude of the expenses are listed in Table 2.

Nomenclatures

A	Constant parameter
a_1, b_1, c_1, d_1	Empirical coefficients
a_2, b_2, c_2, d_2	Empirical coefficients
B	flume width, m
b	Opened gates width, m
C_c	Contraction coefficient
DRF	Discharge reduction function
g	Gravitational acceleration, m/s^2
H_o	Upstream depth, m
Q	Discharge, m^3/s
y_c	Critical depth, m
y_f^*	Maximum downstream depth, for which the free flow takes place through parallel sluice gates, m
y_j	Jet flow depth, m
w	Openness height, m

Greek Symbols

μ	Water viscosity, m^2/s
ρ	Water density, kg//m^3
ψ	Functional symbol
ω	Empirical coefficient

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