STUDY ON VIBRATION OF U-SHAPED FLEXIBLE JUMPERS

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Abstract
During a free standing hybrid riser (FSHR) operation, because of vessel movement, a disturbance load is transmitted to the rigid risers through the U-shaped flexible jumpers from the vessel. This disturbance significantly affects the stability and fatigue of the entire system. To determine the effect of this disturbance, the dynamic load transmission characteristics of the disturbance through U-shaped flexible jumpers must be studied. To fulfil this aim, a lumped-mass flexible pipeline dynamic model is built. At the same time, the Newton-β iterative method and the Houbolt difference formula is employed for the time domain discrete calculation. To proof the theoretical model, the calculating results are verified with finite element software simulation. The disturbance transmission characteristics via U-shaped flexible jumper are studied from two aspects: 1. The relationship between the input disturbance and the output load; 2. The linear superposition characteristic of the output load. It is found that the frequency characteristics of the output load are the same as the input disturbance. On the other hand, after the transmission of input disturbance through the flexible jumper, the oscillation frequency and the delay time of the output show a linear superposition characteristic, while the output load amplitude exhibits a nonlinear superposition phenomenon. These results provide a basis for predicting the dynamic characteristics of the output load only by analysing the characteristics of the input disturbance.

Keywords: Dynamic load, Free standing, Hybrid riser, Lumped-mass method, U-shaped, WIV.
1. Introduction

A riser system is an important and fundamental link for offshore oil and gas transportation, regarded as an umbilical cord between surface vessels and a reservoir. Traditional riser systems such as a top tensioned riser (TTR) is directly connected to the surface TLP or spar platforms as shown in Fig. 1. When the platform is excited with horizontal displacement or heave motion, the tension characteristics of the risers change. In extreme sea conditions, when the surface platform has a large heave motion, it may cause the riser tension to increase extremely and suddenly even resulting in the breakage of the risers.

To avoid this accident, the free-standing hybrid riser (FSHR) has been developed, which separates the surface production vessel from the tension riser, and only uses hoses connecting the surface production vessel and the tension riser, as shown in Fig. 2. The design avoids the problem from the displacement of the surface ship.

![Top tensioned risers](image)

Fig. 1. Top tensioned risers.

However, by analysing the structure of FSHR, vibration can be generated at the flexible jumpers end connecting to the surface ship or the platform by the surface vessel motion, and the vibration can transmit to free standing rigid risers through flexible jumpers and affect stability and fatigue properties of rigid risers, although the risers are separated from the surface ship or the platform. Therefore, the
disturbance generated by the motion of surface vessel and transmitting through flexible jumpers is one of the main sources impacting the stability and fatigue characteristics of FSHR, which is the main point of this paper.

Fig. 2. Free-standing hybrid risers.

At present, existing works are on the FSHR overall dynamic response, without studying how the disturbance generated and how it is transmitted to the free standing rigid riser.

Chaudhuri et al. [1] studied the dynamic characteristics of FSHR using the finite element software FLEXCOM. A comparison of the time history and frequency spectral characteristics between the wave and the flexible tube bending moment are involved in the study and several questions related to the disturbance transmission characteristic of a flexible jumper are listed. However, no further analysis conducted on these questions.

Chen [2] obtained the tensile strength of a flexible jumper at the rigid riser connection point in the FSHR, but did not discuss the relationship between the input disturbance and the output load at both ends of flexible jumper or the impact on the stability and fatigue characteristics of the FSHR since the disturbance transmission through the jumper.

Kang [3] used spectral analysis to analyse the fatigue life of the rigid riser at different points, but only the fatigue life and not the disturbance source characteristics of the fatigue load.
Sun [4] noted that wave-induced vibration (WIV) and vortex-induced vibration (VIV) were two major sources affecting FSHR fatigue life. OrcaFlex was used for full-coupled time-domain analysis of the fatigue life of risers based on WIV and VIV. Unfortunately, the study only investigated the overall system dynamic response, not the dynamic characteristics of the fatigue load source or the transmission of fatigue loads.

While there are few studies on the dynamic load transmission characteristics of the U-shaped flexible jumper in FSHR, the researches of other shapes of pipeline structures on loads transfer characteristics may be used in establishing pipeline dynamic model in this paper and serve as an approach on studying load transfer characteristics of U-shaped flexible jumper.

At present, most of the research on the load transmission characteristics of pipeline structures are based on static problems, such as the anchoring force transmission of the anchoring system [5, 6], the axial pressure transfer of the coiled tubing [7-9] and the static load transmission characteristics of the pile foundation [10].

There are few studies on the dynamic load transmission, especially the dynamic load transmission characteristics of U-shaped pipeline connecting two floating bodies. Now that there are only a few studies on the dynamic transmission characteristics of J-shaped steel catenary risers (SCR, Fig. 3). Although the boundary conditions of a J-shaped pipeline and U-shaped pipeline are different, the model establishing methods are applicable.

Bai and Huang [11] established a static model of SCR based on the assumption of a large deformation in bending and extension as well as curvilinear beam model. And static characteristics, such as the static configuration, top tension, bending moment of touch down point (TDP) and effective tension are analysed.

Guo et al. [12] investigated the characteristics of large-amplitude vibration of a steel catenary riser. A nonlinear model of a SCR was established based on Kane’s method combined with Lagrangian-strain and Newmark Average Acceleration Method to obtain the dynamic response of the riser under different top end excitation.

Li and Li [13] established a nonlinear dynamic model of SCR based on three-dimensional lumped-mass method and calculated the static and dynamic results with Newton-Raphson and Newton-β methods respectively. This paper also analysed the effective tension of the joint, the shear force and the bending moment under vibrating top tension in different frequencies and different amplitude.

Zhang et al. [14] established the catenary equation of catenary based on the slender rods model and solved the dynamic equation with Newton-β numerical method. The dynamic characteristics of the riser under oscillating shear flow and general shear flow were compared. By obtaining the dynamic results of each point on a flexible pipeline, the dynamic load transmission characteristics of on the pipeline can be determined.
This paper is organized as follows in the first section, studies on the dynamic characteristics of FSHR and dynamic load transmission characteristics of pipeline structures are reviewed. In the second section, a U-shaped flexible jumper static model is developed based on the slender rod theory and the catenary equation, according to the boundary conditions of the U-shaped flexible jumper. The dynamic model of the U-shaped flexible jumper is established as well, based on the lumped mass method. In the third section, the Newton-β iterative method has been employed to improve the dynamic model, and the Houbolt difference formula used to solve the dynamic model and to compute the time domain discrete solution using MATLAB as a calculator only. In the fourth section, the established model is validated with hydrodynamic finite element software simulation. Finally, the dynamic load transmission characteristic is analysed. In order to determine the mechanism of dynamic load transmission characteristics of U-shaped flexible pipeline, we analyse the pipeline based on the whole FSHR system starting with the condition under a simple regular wave input, progressing to a complex irregular wave input. The disturbance load transmission characteristics of a U-shaped flexible jumper are studied from two aspects: Firstly, the time history relationship between input disturbance and output load of flexible pipeline are studied. Secondly, the linear superposition characteristics of the output load are studied.

2. Theoretical Model

To study the dynamic load transmission characteristics of a U-shaped flexible jumper, a mechanical model of the flexible pipeline must be established. As the flexible pipeline is in relatively small bending stiffness, it can be regarded as a catenary line and solved with a catenary model. According to the boundary condition of the U-shaped flexible jumper in FSHR, the U-shaped pipeline static model is based on the slender rod theory and catenary equation with the dynamic theoretical model based on the lumped-mass method.
2.1. Static model

The following assumptions are made according to both actual flexible pipeline material characteristics and the condition using catenary equation model and lumped-mass model:

1. The elastic effect of the cable is negligible, regardless of the catenary plastic deformation.
2. Catenary material, self-weight, etc. are evenly distributed and isotropic.
3. The line density is large enough.
4. The catenary and water flow are located in the same vertical plane, regardless of the three-dimensional deformation.
5. Under a constant and mono-directional water flow rate less than 3 m/s.

2.1.1. Establishment of mathematical model

By assuming the catenary line as a large deformed beam element, the catenary equation is established. And a small section of catenary line is selected to perform force analysis, as shown in Fig. 4.

Assuming the tangential and normal dynamics of the section to be $F$ and $D$, the static equilibrium equation in the tangential direction of the section can be written as:

$$ (T + dT - \rho g A - \rho g A dz) \cos(d\phi) + F ds = T - \rho g A + \omega ds \sin\phi $$  \hspace{1cm} (1)

Similarly, the static equilibrium equation in the normal direction can be obtained as follows:

$$ (T + dT - \rho g A - \rho g A dz) \sin(d\phi) = D ds + \omega ds \cdot \cos\phi $$  \hspace{1cm} (2)

The two equations as simplified as:

\[
\begin{align*}
\frac{dT - \rho g A dz}{ds} &= (\omega \sin\phi - F)ds \\
\frac{(T - \rho g A) d\phi}{ds} &= (\omega \cos\phi + D)ds
\end{align*}
\]  \hspace{1cm} (3)

where $T$ is the upper tension of the catenary section, $\rho$ is material density of the catenary line, $A$ is equivalent cross-sectional area of the catenary line, $\omega$ is the catenary line density, $s$ is the catenary line length, $\phi$ and $d\phi$ are the value of the tension inclination angle and the tension inclination angle at the lower end of the section respectively.
To aid the solution of this nonlinear equation, the following assumptions are made:

1. The dynamic effects of the fluid are negligible;
2. The elastic effects of catenary are negligible.

The equations can be simplified as:

\[
\begin{align*}
\frac{dT}{ds} &= \omega \sin \varphi \\
Td\varphi &= \omega \cos \varphi ds
\end{align*}
\]

For brevity, the derivation process is omitted. The static model based on the catenary equation is:

\[
\begin{align*}
T &= T_0 + \omega \cdot z \\
s &= \frac{T_0 \cos \varphi}{\omega} (\tan \varphi - \tan \varphi_0) \\
x &= \frac{T_0 \cos \varphi}{\omega} \left[ \ln \left( \tan \varphi + \sqrt{\tan^2 \varphi + 1} \right) - \ln(\tan \varphi_0 + \sqrt{\tan^2 \varphi_0 + 1}) \right] \\
z &= \frac{T_0 \cos \varphi}{\omega} \left( \sqrt{\tan^2 \varphi + 1} - \sqrt{\tan^2 \varphi_0 + 1} \right)
\end{align*}
\]

where \(T_0\) is the initial tensile force at end of the catenary, \(A\) is the tensile force at the end of the catenary, \(B\), \(\varphi_0\) is the initial inclination angle at the \(A\) and \(\varphi\) is the inclination angle at \(B\). The equation contains eight variables: \(\varphi_0, \varphi, \omega, s, T, T_0, x, z\).

### 2.1.2. Solving the static model

According to the static model above, two steps are taken to determine the static characteristics of the catenary, which is different from the I-shaped flexible pipeline calculation: Step 1. To obtain both tension value and inclination angle at the end of catenary; Step 2. To determine the overall geometry or shape of the catenary and tension of all points on catenary.

**Step 1:**

According to the characteristics of a flexible jumper, the boundary conditions for the catenary solution can be obtained as follow:

1. \(A\) end of the catenary model is the end of input movement disturbance, thus the end is flexible and would have static/dynamic tension or movement accordingly;
2. \(B\) end of the catenary model is connected to the rigid risers, which is a stable connection point without much displacement, thus the \(B\) end is regarded as fixed point and a ball hinge connection with three degree of freedom;
3. Conforming to the actual flexible jumper design, the outline parameters of the pipeline can be obtained, such as the wet weight per unit length, \(\omega=1900\) N/m, overall length, \(s=600\) m, vertical distance of the endpoints, \(z=150\) m, lateral distance of the endpoints, \(x=50\) m.

Now that the four variables of the catenary are settled, the other four can be figured out with the equations group containing four equations.

Solving the equation using MATLAB:

\[\varphi_0 = -89.266^\circ, \ \varphi = 86.413^\circ, \ T_0 = 341.540\ \text{kN}, \ T = 606.040\ \text{kN}\]

Solving using OrcaFlex hydrostatic simulation:
\[ \varphi_0 = -89.219^\circ, \varphi = 88.664^\circ, T_0 = 325.453 \text{kN}, T = 600.112 \text{kN} \]

There is close agreement between the theoretical calculation with MATLAB and OracaFlex hydrodynamic simulation.

**Step 2:**

For \( \omega = 1900 \text{ N/m}, T_0 = 341.540 \text{ N}, \varphi_0 = -89.219^\circ, \) and \( s = 600 \text{ m} \), the overall geometry of catenary is solved, as shown in Fig. 5. Moreover, the static tension of all points is obtained as well, which will be the basis for dynamic model calculation.

![Fig. 5. Geometry of the catenary.](image-url)

### 2.2. Dynamic model

The catenary equation can only solve the catenary static characteristics. To solve the dynamic characteristics, a dynamic model needs to be established. And the lumped-mass method is used to establish the dynamic model. The principle of the lumped-mass method is to decompose a flexible catenary into \( n \) segments, each of which consists of two parts: a massless segment with length and a mass concentrated node with mass equal to the adjacent segments. Since the elasticity of the catenary pipeline is not considered as noted in the assumption, the spring element is not used between adjacent segments. Figure 6 shows the schematic diagram of lumped-mass method [15].

By Newton's second law:

\[ F_{IF} = F_{EF} \]  

![Fig. 6. Lumped-mass method schematic diagram.](image-url)
2.2.1. Inner force

\[ F_{IF} = (M + m) \cdot a \]  

(7)

where \( M \) is the mass of the catenary at the node and \( m \) is the additional mass of the catenary at the node. The force is analysed at \( j \)-node in \( x \)-direction, as shown in Fig. 7.

Fig. 7. X-direction normal and tangential force analysis at \( j \)-node.

Node \( j \) moves in the \( x \)-direction. The normal inertial force of the \( j \) node is expressed as \( F_{nxj} \), and the component forces in the \( x \) and \( z \) directions are \( X'_{xj}, Z_{xj} \), respectively.

\[
\begin{align*}
F_{nxj} &= m_{nj} \cdot \ddot{x}_j \cdot \sin \varphi_j \\
X'_{xj} &= F_{nxj} \cdot \sin \varphi_j = m_{nj} \cdot \ddot{x}_j \cdot \sin^2 \varphi_j \\
Z_{xj} &= -F_{nxj} \cdot \cos \varphi_j = -m_{nj} \cdot \ddot{x}_j \cdot \sin \varphi_j \cdot \cos \varphi_j
\end{align*}
\]  

(8)

where \( m_{nj} = \rho \cdot \frac{D_c^2 \pi}{4} \cdot l \cdot C_{jm} \)

\( m_{nj} \) Normal additional mass at \( j \)-node.

\( D_c \) Equivalent cross-sectional diameter.

\( C_{mn} \) Normal added mass coefficient at \( j \)-node.

\( \rho \) Fluid density.

Similarly, the force at node \( j \) is resolved in \( z \)-direction, as shown in Fig. 8.

\[
\begin{align*}
F_{tzj} &= m_{tj} \cdot \ddot{x}_j \cdot \sin \varphi_j \\
X''_{xj} &= F_{tzj} \cdot \sin \varphi_j = m_{tj} \cdot \ddot{x}_j \cdot \sin^2 \varphi_j \\
Z''_{xj} &= -F_{tzj} \cdot \cos \varphi_j = -m_{tj} \cdot \ddot{x}_j \cdot \sin \varphi_j \cdot \cos \varphi_j
\end{align*}
\]  

(9)

where \( m_{tj} = \rho \cdot \frac{D_c^2 \pi}{4} \cdot l \cdot C_{jm} \)

\( m_{tj} \) Normal additional mass at \( j \)-node.

\( C_{mt} \) Normal added mass coefficient at \( j \)-node.

Similarly, the force at node \( j \) is resolved in \( z \)-direction, as shown in Fig. 8.
Fig. 8. Z-direction normal and tangential force analysis at j-node.

Node j moves in the z-direction. The normal inertial force of the node j is expressed as $F_{nzj}$, and the component forces in the x and z directions are $X'_{zj}$, $Z'_{zj}$ respectively.

$$\begin{align*}
F_{nzj} &= m_{nj} \ddot{z}_j \cos \phi_j \\
X'_{zj} &= -F_{nzj} \sin \phi_j = -m_{nj} \ddot{z}_j \sin \phi_j \cos \phi_j \\
Z'_{zj} &= F_{nzj} \cos \phi_j = m_{nj} \ddot{z}_j \cos^2 \phi_j
\end{align*}$$  \hspace{1cm} (10)

Node j moves in the z-direction. The tangential inertial force of node j is expressed as $F_{tzj}$, and the component forces in the x and z directions are $X''_{zj}$, $Z''_{zj}$ respectively.

$$\begin{align*}
F_{tzj} &= m_{nj} \ddot{z}_j \cos \phi_j \\
X''_{zj} &= -F_{tzj} \sin \phi_j = -m_{nj} \ddot{z}_j \sin \phi_j \cos \phi_j \\
Z''_{zj} &= F_{tzj} \cos \phi_j = m_{nj} \ddot{z}_j \cos^2 \phi_j
\end{align*}$$  \hspace{1cm} (11)

Applying Newton’s second law of motion in the x-direction:

$$F_{IF-xj} = M_j \ddot{x}_j + X'_{xj} + X''_{xj} + X'_{zj} + X''_{zj}$$  \hspace{1cm} (12)

Then,

$$\begin{align*}
F_{IF-xj} &= (M_j + m_{nj}) \ddot{x}_j + m_{nj} \ddot{z}_j \cos \phi_j + m_{nj} \ddot{z}_j \sin \phi_j \cos \phi_j - \delta_j
\end{align*}$$  \hspace{1cm} (13)

### 2.2.2. External force

External force analysis at node j:

$$\begin{align*}
F_{EF-xj} &= T_j \cos \phi_j - T_{j-1} \cos \phi_{j-1} - f_{dxj} \\
F_{EF-zj} &= T_j \sin \phi_j - T_{j-1} \sin \phi_{j-1} - f_{dzj} - \delta_j
\end{align*}$$  \hspace{1cm} (14)

where $T_j$ is the catenary tension at node j, which can be obtained by the catenary equation. The variables $f_{dxj}$, $f_{dzj}$ in the equation can be calculated as:

$$\begin{align*}
f_{dxj} &= -\frac{1}{2} \rho D \left(C_n |u_{nj}| u_{nj} \sin \phi_j + C_t |u_{tj}| u_{tj} \cos \phi_j \right) \\
f_{dzj} &= -\frac{1}{2} \rho D \left(C_t |u_{tj}| u_{tj} \sin \phi_j - C_n |u_{nj}| u_{nj} \cos \phi_j \right)
\end{align*}$$  \hspace{1cm} (15)
where \( u_{nj} = -(\dot{x}_j - c_j)\sin\phi_j + \dot{\phi}_j \cos\phi_j, \quad u_{ij} = -(\dot{x}_j - c_j)\cos\phi_j + \dot{\phi}_j \sin\phi_j \).
\( c_j \) is the water flow rate.

### 2.3. Dynamic equation

Supposing

\[
\begin{align*}
M_1 & = M_j + m_{nj}\sin^2\phi_j + m_{ij}\cos^2\phi_j \\
M_2 & = (m_{ij} - m_{nj})\cdot\sin\phi_j \cdot \cos\phi_j \\
M_3 & = M_j + m_{ij}\cos^2\phi_j + m_{nj}\sin^2\phi_j
\end{align*}
\]  

(16)

According to the force analysis above and the Newtonian equilibrium equation at node \( j \):

\[
\begin{align*}
M_1 \cdot \ddot{x}_j + M_2 \cdot \ddot{\phi}_j & = F_{xj} \\
M_3 \cdot \ddot{\phi}_j + M_2 \cdot \ddot{x}_j & = F_{\phi j}
\end{align*}
\]  

(17)

\( \ddot{x}_j \) and \( \ddot{\phi}_j \) can be solved as:

\[
\begin{align*}
\ddot{x}_j & = \frac{T_j(M_2\cos\phi_j - M_2\sin\phi_j) - T_{j-1}(M_3\cos\phi_{j-1} - M_2\sin\phi_{j-1})}{M_1 M_3 - M_2^2} + \frac{M_2 f_{axj}}{M_1 M_3 - M_2^2} + \frac{M_2 f_{axj} + \delta_j}{M_1 M_3 - M_2^2} \\
\ddot{\phi}_j & = \frac{T_j(M_2\sin\phi_j - M_2\cos\phi_j) - T_{j-1}(M_3\sin\phi_{j-1} - M_2\cos\phi_{j-1})}{M_1 M_3 - M_2^2} - \frac{M_1 f_{azj}}{M_1 M_3 - M_2^2} - \frac{M_1 f_{azj} + \delta_j}{M_1 M_3 - M_2^2}
\end{align*}
\]  

(18)

### 3. Solving the Dynamic Equation

In the previous section, the basic dynamic model of the U-shaped pipeline has been obtained. The dynamic model equation can be solved in this section by the Houbolt finite difference formula and the Newton-\( \beta \) iterative method.

#### 3.1. Time step dispersion of dynamic equation

Assuming

\[
\begin{align*}
R_j & = \frac{M_2\cos\phi_j - M_2\sin\phi_j}{\lambda}, \quad S_j = \frac{M_2\sin\phi_j - M_2\cos\phi_j}{\lambda} \\
P_j & = \frac{M_2\cos\phi_{j-1} - M_2\sin\phi_{j-1}}{\lambda}, \quad Q_j = \frac{M_2\sin\phi_{j-1} - M_2\cos\phi_{j-1}}{\lambda} \\
U_j & = \frac{M_2(f_{axj} + \delta_j) - M_2 f_{axj}}{\lambda}, \quad V_j = \frac{M_2 f_{axj} - M_2 f_{axj}}{\lambda}
\end{align*}
\]  

(19)

The dynamic equation can be expressed as:

\[
\begin{align*}
\dot{x}_j & = R_j T_j - P_j T_{j-1} + U_j \\
\dot{\phi}_j & = S_j T_j - Q_j T_{j-1} + V_j
\end{align*}
\]  

(20)

To conduct a time step dispersion of the dynamic equation, the Houbolt finite difference formula is employed. If the current time step is \( n \), the time step formula for calculating the next \( n+1 \) is:
\[
\begin{align*}
\dot{x}_{j}^{n+1} &= \frac{1}{\Delta t^2} (2 \times x_{j}^{n+1} - 5 \times x_{j}^{n} + 4 \times x_{j}^{n-1} - x_{j}^{n-2}) \\
\ddot{x}_{j}^{n+1} &= \frac{1}{6\Delta t} (11 \times x_{j}^{n+1} - 18 \times x_{j}^{n} + 9 \times x_{j}^{n-1} - 2 \times x_{j}^{n-2}) \\
\dot{z}_{j}^{n+1} &= \frac{1}{\Delta t^2} (2 \times z_{j}^{n+1} - 5 \times z_{j}^{n} + 4 \times z_{j}^{n-1} - z_{j}^{n-2}) \\
\ddot{z}_{j}^{n+1} &= \frac{1}{6\Delta t} (11 \times z_{j}^{n+1} - 18 \times z_{j}^{n} + 9 \times z_{j}^{n-1} - 2 \times z_{j}^{n-2})
\end{align*}
\] (21)

The formula above is the velocity and acceleration difference formula. The displacement difference formula is:

\[
\begin{align*}
\tilde{x}_{j}^{n+1} &= \frac{5}{2} x_{j}^{n} - 2 x_{j}^{n-1} + \frac{1}{2} x_{j}^{n-2} + \frac{\Delta t^2}{2} (R_{j}^{n+1} \omega_{j}^{n+1} - P_{j}^{n+1} \omega_{j-1}^{n+1} + U_{j}^{n+1}) \\
\tilde{z}_{j}^{n+1} &= \frac{5}{2} z_{j}^{n} - 2 z_{j}^{n-1} + \frac{1}{2} z_{j}^{n-2} + \frac{\Delta t^2}{2} (S_{j}^{n+1} \omega_{j}^{n+1} - Q_{j}^{n+1} \omega_{j-1}^{n+1} + V_{j}^{n+1})
\end{align*}
\] (22)

### 3.2. Tension calculation

To more accurately calculate the tension for the next time step \( T_{j}^{n+1} \), the Newton-\( \beta \) iterative method is used, where \( T_{j}^{n+1} \) contains two items:

\[
T_{j}^{n+1} = \tilde{T}_{j}^{n+1} + \Delta T_{j}^{n+1}
\] (23)

\( \tilde{T}_{j}^{n+1} \) is the estimated value of the tension, which is replaced by the result of the previous time step; \( \Delta T_{j}^{n+1} \) is the tension correction value, which is the main part to be solved.

By geometric constraints, the sum of the squares of the coordinate differences between the ends of each segment is equal to the square of the length of each elastic elongation:

\[
(\tilde{x}_{j}^{n+1} - \tilde{x}_{j-1}^{n+1})^2 + (\tilde{z}_{j}^{n+1} - \tilde{z}_{j-1}^{n+1})^2 = \left[ \frac{\beta_{j}^{n+1}}{EA} \right]^2
\] (24)

The constraint equation can be expressed as:

\[
\phi_{j}^{n+1} = (\tilde{x}_{j}^{n+1} - \tilde{x}_{j-1}^{n+1})^2 + (\tilde{z}_{j}^{n+1} - \tilde{z}_{j-1}^{n+1})^2 - \left[ \frac{\beta_{j}^{n+1}}{EA} \right]^2 = 0
\] (25)

where \( \phi_{j}^{n+1} \) is the tension function of the catenary of the \( n+1 \) time step.

Since \( \phi_{j}^{n+1} \) is related to \( T_{j-2}^{n+1}, T_{j-1}^{n+1}, T_{j}^{n+1} \), extending \( \phi_{j}^{n+1} \) according to the Taylor formula:

\[
\phi_{j}^{n+1} = \phi_{j}^{n+1} + \frac{\partial \phi_{j}^{n+1}}{\partial T_{j-2}^{n+1}} \Delta T_{j-2}^{n+1} + \frac{\partial \phi_{j}^{n+1}}{\partial T_{j-1}^{n+1}} \Delta T_{j-1}^{n+1} + \frac{\partial \phi_{j}^{n+1}}{\partial T_{j}^{n+1}} \Delta T_{j}^{n+1} + \cdots = 0
\] (26)

As the estimated value is already closer to the true value, the higher-order term can be ignored, and a set of \( N \) linear equations for the tension correction \( \Delta T_{j}^{n+1} \) is obtained:
\[ E_j^{n+1} \Delta T_{j-2}^n - E_j^{n+1} \Delta T_{j-1}^n + \bar{G}_j^{n+1} \Delta T_j^n = -\bar{\phi}_j^{n+1}, \quad (j = 2, 3 \ldots N + 1) \] (27)

After coordinate transformation of the matrix:

\[
\begin{bmatrix}
\Delta T_1^{n+1} \\
\Delta T_2^{n+1} \\
\vdots \\
\Delta T_{N-1}^{n+1} \\
\Delta T_N^{n+1}
\end{bmatrix} =
\begin{bmatrix}
-F_1^{n+1} & G_2^{n+1} & 0 & \cdots & 0 & 0 \\
-E_2^{n+1} & F_3^{n+1} & G_4^{n+1} & 0 & \cdots & 0 \\
\vdots & \vdots & \vdots & \ddots & \vdots & \vdots \\
0 & \cdots & E_{N-1}^{n+1} & -E_N^{n+1} & \bar{G}_N^{n+1} & 0 \\
0 & \cdots & \cdots & \cdots & \cdots & \cdots \\
\end{bmatrix}
\begin{bmatrix}
\Delta T_1^n \\
\Delta T_2^n \\
\vdots \\
\Delta T_{N-1}^n \\
\Delta T_N^n
\end{bmatrix} + 
\begin{bmatrix}
-\bar{\phi}_1^{n+1} \\
-\bar{\phi}_2^{n+1} \\
\vdots \\
-\bar{\phi}_{N-1}^{n+1} \\
-\bar{\phi}_N^{n+1}
\end{bmatrix}
\] (28)

The equation for correcting the tension of the last node is:

\[ \Delta T_N^{n+1} = E_N^{n+1} \cdot (\bar{\phi}_N^{n+1}) - E_N^{n+1} \cdot (\bar{\phi}_N^{n+1}) + \bar{G}_N^{n+1} \cdot (\bar{\phi}_N^{n+1}) \] (30)

where

\[ \bar{\phi}_j^{n+1} = (\bar{x}_j^{n+1} - \bar{x}_{j-1}^{n+1})^2 + (\bar{z}_j^{n+1} - \bar{z}_{j-1}^{n+1})^2 \] (31)

\[ E_j^{n+1} = \frac{\partial \phi_j^{n+1}}{\partial x_{j-2}^{n+1}} \] (32)

\[ F_j^{n+1} = \frac{\partial \phi_j^{n+1}}{\partial x_{j-1}^{n+1}} \] (33)

\[ G_j^{n+1} = \frac{\partial \phi_j^{n+1}}{\partial T_j^n} \] (34)

From

\[ x_j^{n+1} = \frac{5}{2} x_j^n - 2 x_j^{n-1} + \frac{1}{2} x_j^{n-2} + \frac{\Delta t^2}{2} (T_j^{n+1} T_j^{n+1} - T_j^{n+1} T_j^{n+1} + U_j^{n+1}) \]

and

\[ \Delta x_N = \Delta z_N \times \tan \psi_N = \Delta s \times \cos \psi_N, \quad \Delta z_N = \Delta s \times \sin \psi_N; \]

\[ \begin{bmatrix}
E_1^{n+1} \\
E_2^{n+1} \\
\vdots \\
E_N^{n+1} \\
G_1^{n+1}
\end{bmatrix} = \begin{bmatrix}
p_{n+1}^{n+1} (x_j^{n+1} - x_{j-1}^{n+1}) + \gamma_{n+1}^{n+1} (z_j^{n+1} - z_{j-1}^{n+1}) \\
p_{n+1}^{n+1} (x_j^{n+1} - x_{j-1}^{n+1}) + \gamma_{n+1}^{n+1} (z_j^{n+1} - z_{j-1}^{n+1}) + 2 l^{-2} \left( \frac{\gamma_{n+1}^{n+1}}{k e} \right) \\
\vdots \\
p_{n+1}^{n+1} (x_j^{n+1} - x_{j-1}^{n+1}) + \gamma_{n+1}^{n+1} (z_j^{n+1} - z_{j-1}^{n+1}) \\
p_{n+1}^{n+1} (x_j^{n+1} - x_{j-1}^{n+1}) + \gamma_{n+1}^{n+1} (z_j^{n+1} - z_{j-1}^{n+1})
\end{bmatrix} \] (35)

4. Example Calculation and Results Verification

With the improved theoretical model established above, the static and dynamic characteristics of the U-shaped pipeline is calculated in this section with MATLAB to solve the theoretical equations. By comparing the dynamic results of both ends of the catenary pipeline, the dynamic loads transmitting characteristic can be determined
primarily. To verify the theoretical mechanical model, results from the MATLAB calculating of theoretical model are compared with that from a hydro-simulation finite element software, Ocraflex. A catenary with the length of 600 m submerged entirely in water is used in the cases study. The linear density of the catenary is 1900 kN/m with an effective cross-section diameter of 244.5 mm (9 5/8).

4.1. Static results

In the static calculation, the spatial geometry and the tensile characteristics of the catenary pipeline are the main parameters to be obtained and discussed. As shown in Fig. 5 the geometry of the catenary pipeline is calculated in MATLAB under the theoretical model. It is shown that the catenary is in U-shaped with end points in expected position. As shown in Table 1 the results from MATLAB and the finite element software calculation results are compared. Additionally, it should be mentioned that the end of the catenary, A is the FPSO connecting end with a higher z coordinate; the B end is the rigid riser connecting end with a lower z coordinate relatively. The formula for calculating deviation is:

\[
\text{Deviation} = \frac{|\text{Theoretical model} - \text{Finite Element}|}{\text{Finite Element}}
\] (36)

As the deviation between the theoretical model and the finite element software is very narrow, less than 5\%, which means the theoretical model is acceptable.

<table>
<thead>
<tr>
<th>Items</th>
<th>Case 1</th>
<th>Case 2</th>
</tr>
</thead>
<tbody>
<tr>
<td>Wave Height/m</td>
<td>2</td>
<td>10</td>
</tr>
<tr>
<td>Amplitude in x-direction/m</td>
<td>0.455</td>
<td>1</td>
</tr>
<tr>
<td>Amplitude in z-direction/m</td>
<td>1.22</td>
<td>8.5</td>
</tr>
<tr>
<td>Wave/t</td>
<td>2 ( \pi )</td>
<td>2 ( \pi )</td>
</tr>
<tr>
<td>Motion of ( A/t )</td>
<td>2 ( \pi )</td>
<td>2 ( \pi )</td>
</tr>
<tr>
<td>Waveform</td>
<td>Wave Sine</td>
<td>Sine</td>
</tr>
</tbody>
</table>

4.2. Dynamic results

With the dynamic theoretical model programming in MATLAB to solve the equations only, the disturbance load transmitting characteristics through the catenary pipeline can be calculated. Given the input excitation in form of displacement at one end of the catenary, A, the tension oscillating characteristic at the other end, B, as the output can be obtained. Two cases with different wave heights are selected to get the tension oscillating characteristic at B end. In both cases, sine input waves are used at A, and the wave period is 2 \( \pi \). The FPSO motion data is obtained from finite element simulation as shown in Table 2. Under the regular sine wave, FPSO has the same frequency sinusoidal motion with the wave.
With the calculation of the disturbance input at A end, which has the same displacement as the FPSO, the disturbance output characteristic at B end is obtained. Figure 9 shows the output results both from the theoretical model and the finite element software under Case 1 and Case 2 conditions, respectively.

![Case 1](image1)

![Case 2](image2)

**Fig. 9. Comparison of output results in steady state using MATLAB (left), OrcaFlex (right).**

It is obvious that the waveforms of the output in time history from theoretical results and finite element results are in good agreement, especially while there’s a larger input load as in case 2. As shown in Table 3 the theoretical model result, which considered hydrodynamic factors, has good agreement with finite element calculation. The deviation is less than 1.5%, which validates the theoretical model. The deviation in Table 3 is calculated by the equation below:
Deviation = \frac{T_{AM} - T_{AO}}{\max T_0} \quad (37)

where, $T_{AM}$ is the magnitude of the tension curve calculated in MATLAB, $T_{AO}$ is the magnitude of the tension curve simulated in OrcaFlex, $\max T_0$ is the maximum tension value of the catenary line by OrcaFlex.

Table 3. Comparison of theoretical result and finite element result for tension at B (output).

<table>
<thead>
<tr>
<th>Tension/kN</th>
<th>Case 1</th>
<th>Case 2</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td>MATLAB</td>
<td>OrcaFlex</td>
</tr>
<tr>
<td>Max</td>
<td>345</td>
<td>327</td>
</tr>
<tr>
<td>Min</td>
<td>338</td>
<td>324</td>
</tr>
<tr>
<td>Amplitude</td>
<td>0.06</td>
<td>0.02</td>
</tr>
<tr>
<td>Deviation</td>
<td>1.2%</td>
<td>0.08%</td>
</tr>
<tr>
<td>Period</td>
<td>2 $\pi$</td>
<td>2 $\pi$</td>
</tr>
<tr>
<td>Waveform</td>
<td>Sine</td>
<td>Sine</td>
</tr>
</tbody>
</table>

Comparing the input and output’s dynamic characteristics, one U-shaped pipeline disturbance transmission characteristic can be preliminarily determined. The output tension at B end has the same frequency characteristics i.e. frequency and waveform, as input excitation at A end. The output tension fluctuation amplitude is 1.9% compared to the mean value of tension at B end. Even when the excitation wave height is 10 m and the amplitude of the FPSO heave motion is 8.5 m, the amplitude of the output tension is only 9.4% of the average tension at B. Predictably, the wave disturbance does not cause instability to free standing rigid riser because of a big amplitude output tension vibration. However, this conclusion is based on a single wave disturbance. To determine the performance of dynamic loading transmission on U-shaped pipeline in actual conditions, further study must be carried out.

5. Dynamic Transmission Characteristics Discussion

In the previous section, the theoretical model has been verified by comparing the results with the finite element calculation. In addition, the primary dynamic characteristics of the U-shaped pipeline have been obtained. However, to understand the mechanism of dynamic load transmission characteristics, additional dynamic characteristics of catenary pipeline must be determined.

Dynamic characteristics of the vibrating disturbance load include the waveform, frequency and the wave amplitude. Since it is a load transmission process, the delay of the transmission process is one of the dynamic characteristics which should be taken into account. To determine the fundamental mechanism of disturbance transmission, the relationship between input wave and output loads is studied under regular input wave and irregular input wave. In other words, the research is conducted from only a single input wave to multi-wave, step by step. Furthermore, to get better understanding, the dynamic transmission characteristics are also studied based on principle of linear superposition. Figure 10 is the configuration of FSHR analysed in this section, which is also the same as the one in section 4.
5.1. Regular input wave

5.1.1. Single input wave

Waves with different wave heights and different frequencies are selected as the comparison cases in single input wave. Table 4 lists the parameters of the catenary pipeline and wave used as a case study.

<table>
<thead>
<tr>
<th>Catenary parameters</th>
<th>Outer diameters</th>
<th>Line density</th>
<th>Length</th>
<th>Wave type</th>
</tr>
</thead>
<tbody>
<tr>
<td>Case</td>
<td>SW1</td>
<td>SW2</td>
<td>SW3</td>
<td>SW4</td>
</tr>
<tr>
<td>Wave height /m</td>
<td>4</td>
<td>6</td>
<td>8</td>
<td>10</td>
</tr>
<tr>
<td>Wave period /s</td>
<td>2 \pi</td>
<td>3 \pi</td>
<td>4 \pi</td>
<td>5 \pi</td>
</tr>
</tbody>
</table>

Figure 11 shows the comparison between input excitation displacement at A end (z) and output loads/effective tension at B end (Effective Tension). Overall, the two dynamic curves are basically the same in terms of the vibration frequency and waveform. This indicates that frequency characteristics of input disturbance do not changed through the catenary. It can be found that the output loads in three cases
(wave height > 4 m), except for case SW1 (wave height = 2 m), have minimal change in the amplitude of the output tension, although with input wave height increasing significantly. This indicates that the change of the wave height has little effect on the amplitude of the output tension. In order to facilitate the observation of the delay characteristics in disturbance transmission, an enlarged view of the adjacent period contrast curve is added above the main curves. It can be found that although the period of the wave is increasing, except for case SW1, the delay of the other three cases has not changed, with all delays around 2 s. It indicates that the increasing of wave period cannot delay the dynamic loads transmission through U-shaped pipeline.

![Fig. 11. Input and output comparison under single input wave.](image)

### 5.1.2. Double input wave

According to the principle of wave linear superposition, waves can be regarded as a superposition of numerous sine waves. But it remains to be seen if the dynamic load transfer via U-shaped pipeline is consistent with this principle as well, whether the output dynamic characteristics such as waveform and frequency can be obtained under superposition of simple waves, and whether the complex input wave cases has the same behaviour as the single input wave cases. To get these answers, further study is required under multiple input waves. Firstly, the two double input wave’s case are analysed. Table 5 shows the catenary parameters and input wave parameters in double input wave cases.

<table>
<thead>
<tr>
<th>Case parameters</th>
<th>Outer diameters</th>
<th>Line density</th>
<th>Length</th>
<th>Wave type</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td>244.5 mm</td>
<td>1900 kN/m</td>
<td>600 m</td>
<td>Airy</td>
</tr>
<tr>
<td>Wave height /m</td>
<td>6</td>
<td>4</td>
<td>6</td>
<td>6</td>
</tr>
<tr>
<td>Wave Period /s</td>
<td>$2 \pi$</td>
<td>$2 \pi$</td>
<td>$3 \pi$</td>
<td>$2 \pi$</td>
</tr>
</tbody>
</table>
In this paper, two cases, one superimposed wave with the same frequency and different wave heights and the other superimposed wave with different frequency and the same wave heights are discussed. The results are shown in Fig. 12. In terms of frequency characteristics, the output tension waveform and frequency are the same as the input in both cases. In terms of the disturbance transmission delay, the results of multiple input wave’s case are the same as that of single input waves cases. In terms of the amplitude of the output tension, the output peak in case DW1 agrees well with that in case SW1. But compared with the lower frequency case SW2, a lower peak is obtained in case DW1 though an input wave superposition added, which indicates that the input wave frequency actually affects the output wave amplitude. Besides, the highest output peak in case DW2’s increases significantly, nearly by 15% and the second peak decreases slightly compared with that in case SW2. It is partly a normal phenomenon resulting from the superposition of double different frequency input wave, which causes the higher peak and the lower peak. But the superposed highest peak goes too high compared with the second peak referring to the two peaks of the input wave. This tremendous increase of the highest peak indicates that the superposition of different frequency input disturbance loads stimulates a nonlinear increase of disturbance when the disturbance transmits through the U-shaped pipeline.

Fig. 12. Input and output comparison under double input wave.

5.1.3. Triple input wave

In the cases of triple input waves, three comparison cases of superimposed waves are investigated: different wave heights with the same frequency, the same wave height with different frequencies and the same wave height with the same frequency. The catenary and wave parameters are shown in Table 6.

<table>
<thead>
<tr>
<th>Catenary parameters</th>
<th>Outer diameter/mm</th>
<th>Line density/kN/m</th>
<th>Length/m</th>
<th>Wave type</th>
</tr>
</thead>
<tbody>
<tr>
<td>Case</td>
<td>TW1</td>
<td>TW2</td>
<td>TW3</td>
<td></td>
</tr>
<tr>
<td>Wave height /m</td>
<td>4</td>
<td>6</td>
<td>8</td>
<td>6</td>
</tr>
<tr>
<td>Wave period /s</td>
<td>$2\pi$</td>
<td>$2\pi$</td>
<td>$2\pi$</td>
<td>$3\pi$</td>
</tr>
</tbody>
</table>

Table 6. Catenary and sea state parameters of triple input wave.
The comparison result of input and output is shown in Fig. 13. Overall, the output waveform and frequency characteristics of the three cases are basically the same as the input. Of these, case TW1 has the best overlap. In the other two cases, though the second peak changes with the third peak in case TW3, the highest peak of the output agrees well with that of the input and the peak times and frequency characteristics are highly consistent with the input as well. In terms of the disturbance transmitting delay, the case of multiple input waves does not change compared with the case of single input waves. In addition, the amplitude of the output tension in case TW1 is slightly increased compared with that of cases DW1. This is the normal phenomenon of load wave superposition. In case TW2 and TW3 with multi-frequency input, unexpected phenomenon appears in two of the peaks change with each other and Tremendous increase of the highest happens again, which is the same as case DW2. These phenomena indicate the nonlinear excitation effect of disturbance superposition on the disturbance load transmitting with the in consonant frequency input waves’ overlap.

Fig. 13. Input and output comparison under triple input wave.

5.2. Irregular input wave

The catenary pipeline disturbance transmitting characteristics are further analysed in the actual sea conditions. In this paper, the JONSWAP wave is employed. Parameters of the catenary and the irregular spectrum are shown in Table 7 and Fig. 14

<table>
<thead>
<tr>
<th>Table 7. Catenary and irregular wave parameters.</th>
</tr>
</thead>
<tbody>
<tr>
<td>Catenary parameters</td>
</tr>
<tr>
<td>---------------------</td>
</tr>
<tr>
<td></td>
</tr>
</tbody>
</table>

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Figure 15 shows the comparison of the input and output under the irregular wave. In actual sea conditions, the waveform and frequency of output tension are in good agreement with that of input. This result is the same as single input wave cases. In terms of the output tension, the peak times and frequency characteristics of input and output are consistent as well. Overall, the input and output disturbance amplitude values are basically consistent with the values of the single input wave, and no more than the double input wave cases, in which the nonlinear stimulation doesn’t happen significantly. The largest output amplitude occurs when the input disturbance reaches the maximum, as much as the largest output amplitude in the case of triple input waves superposition. At this point, the wave superposes the most complicated together and the tremendous nonlinear increase stimulated, as a result that there’s a ‘sudden increase’.
6. Conclusion

In order to study the continuous disturbance load transmitting characteristics of U-shaped flexible jumper in FSHR, an improved theoretical dynamic model is established based on the lumped-mass method. The Newton-β iterative method is used to correct the model to enhance the accuracy of the results and the Hublot finite difference method is employed to obtain the time domain results. The improved U-shaped catenary model is verified by finite element software. Based on the theoretical model, the disturbance load transmitting characteristics is studied from two aspects by analysing four major typical characteristics. It is found that the frequency characteristics of the output load are the same as that of the input disturbance in terms of load waveform and frequency. The conclusion can be drawn as follows:

- In terms of waveform and frequency, the output tension vibration characteristic is the same as the input wave frequency characteristic. No compounded excitation effect is found in these two characteristics, and the principle of linear superposition is applicable in the prediction of these two characteristics, even in irregular wave’s condition.

- In terms of the amplitude of the output tension, the peak time agrees well with that of the input wave. But a nonlinear superposition phenomenon, ‘superimposition of the excitation effects’, occurs when there’s a superposition of different frequency input waves. This nonlinearity increases the output vibration amplitude significantly and may lead the exchange of peaks with each other. The nonlinear increase should be checked carefully in the output tension amplitude analysis, especially in the complex sea condition.

- In terms of the disturbance transmission delay, despite different conditions, the delay is always around 2 seconds.

These results provide a basis for predicting the dynamic characteristics of the output load by analysing the characteristics of the input disturbance. Further research can be conducted to discuss factors influencing dynamic load transmitting characteristics on flexible pipeline.

Acknowledgement

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<table>
<thead>
<tr>
<th>Nomenclatures</th>
<th>Description</th>
</tr>
</thead>
<tbody>
<tr>
<td>$A$</td>
<td>Equivalent cross-sectional area of the catenary line, m$^2$.</td>
</tr>
<tr>
<td>$C_{mn}$</td>
<td>Normal added mass coefficient $^2$.</td>
</tr>
<tr>
<td>$D_c$</td>
<td>Equivalent cross-sectional diameter, m$^2$.</td>
</tr>
<tr>
<td>$M$</td>
<td>Mass of the catenary at the node, kg.</td>
</tr>
<tr>
<td>$m$</td>
<td>Additional mass of the catenary at the node, kg.</td>
</tr>
<tr>
<td>$T$</td>
<td>Upper tension of the catenary section, kN.</td>
</tr>
</tbody>
</table>

**Greek Symbols**

<table>
<thead>
<tr>
<th>Symbol</th>
<th>Description</th>
</tr>
</thead>
<tbody>
<tr>
<td>$d\phi$</td>
<td>Tension inclination angle at the lower end of the section, deg.</td>
</tr>
</tbody>
</table>
\[ \rho \] Fluid density, kg/m³
\[ \varphi \] Value of the tension inclination angle, deg.
\[ \omega \] Catenary line density, kg/m.

Abbreviations

- FSHR: Free standing hybrid riser
- SCR: Steel catenary riser
- TDP: Touch down point
- TLP: Tension leg platform
- TTP: Top tensioned riser
- VIV: Vortex-induced vibration
- WIV: Wave-induced vibration

References


