SINGLE TRANSMIT FUZZY QUEUING MODEL WITH TWO-CLASSES: EXECUTION PROPORTIONS BY RANKING TECHNIQUE

USHA PRAMEELA K.1,*, PAVAN KUMAR2
1Department of Mathematics, Koneru Lakshmaiah Education Foundation (KLEF), Vaddeswaram, Guntur, AP-522502, India
2Department of Mathematics, Koneru Lakshmaiah Education Foundation (KLEF), Vaddeswaram, Guntur, AP-522502, India
*Corresponding Author: kushaprameela@gmail.com

Abstract
This paper proposes a single transmit fuzzy queuing model with two classes. Here we determine the execution proportions by applying the ranking technique. The entry and administration rate are spoken to be pentagonal, heptagonal and octagonal fuzzy numbers. The left and right positioning technique is adopted. The main intention is to evacuate the fuzziness before the exhibition measures are processed by utilizing the regular queuing hypothesis. Three Numerical examples are exhibited to show the validity implementation of the methodology. Ultimately sensitivity analysis has been applied to model variables.

Keywords: Execution proportions, Fuzzy numbers, Ranking technique, Single transmit fuzzy queue.
1. Introduction

As proved by the enormous number of references found [1-21] fuzzy set hypothesis is setup and developing examination discipline. The utilization of it as a procedure for demonstrating and dissecting choice frameworks is important to analyst’s underway administration. The reason being its capacity to quantitatively and subjectively model issues which include unclearness and imprecision. Queuing models have a wide application in administration associations. One of such application zones is genuine circumstances having a strategy of two class administration channels. Various sorts of lining models have been investigated in [1] going from lining models having steady fresh qualities to fuzzy qualities. In this way, Zadeh [2, 3] standard depicted certain models as possibility having articulations. This makes fuzzy queuing models more viable than the traditional queuing models in numerous genuine circumstances.

As fuzzy numbers don’t frame a characteristic straight request, similar to genuine numbers, a key issue in operationalizing fuzzy set hypothesis is the manner by which to look at fuzzy numbers. Different methodologies have been produced for positioning fuzzy numbers. In the current research, the regularly utilized strategy is to develop appropriate maps to change fuzzy numbers into genuine numbers purported defuzzification. These genuine numbers are then thought about. Then again, the transformation of fuzzy queues to crisp queues has additionally been widely examined in writing various strategies and methodologies that have been placed being used. One of such techniques is the positioning strategy [4]. Another technique that dates further over into writing is the strong positioning strategy [1], which was likewise contemplated by [5-7]. The creators here received this strategy with single channel commitment queuing models, while examinations [8, 9] embraced demand queuing models with fuzzy lines.

Kao et al. [10] proposed a general method to develop the participation elements of the presentation estimates M/F/1/∞, F/M/1/∞, F/F/1/∞ and FM/FM/1/∞ lines, where F and FM signify the fuzzy time and exponential time. Likewise, Ke et al. [11] figured the enrolment capacity of the framework with the properties of the retrial lining model and furthermore a fuzzy parameter portrayal of the entry rate and administration rate. Despite the fact that the creator Kao et al. [10] considered fuzzy line models. In any case, the procedure for processing the exhibition proportions of the line received a nonlinear parametric programming approach. This approach is progressively convoluted to embrace in contrast with the left and right technique to take up in this paper. This technique is short, advantageous and adaptable to utilize. Nonetheless, most past investigations have not considered two classes [12] of entry rates and two exponential administration rates under the arrival arrangement of first started things out service. Henceforth, this paper deals with one of the queuing models with the technique for transformation from fuzzy to crisp qualities known as the left and right positioning strategy in application to three sorts of participation capacities; pentagonal, heptagonal and octagonal enrolment capacities.

This prompt acquiring diverse execution measures as far as crisp qualities for the fuzzy queuing model with two classes of landing rates and blend of exponential administration rates. The primary thought of this paper is to acquire the exact crisp values from the fuzzy values and then applying within the queuing performance formulas. Fuzzy Queuing models are also studied by Mueen et al. [12], Julia Rose
The design of this paper pursues: Section 1 contains an introductory outline, Section 2 describes some basic definitions, section 3 clarifies fuzzy queuing model, Section 4 depicts the plan of ranking technique, Section 5 presents numerical examples, Section 6 gives the results and discussion, section 7 analyses sensitivity analysis, section 8 prompts limitations whereas section 8 finishes up the article.

2. Basic Definitions

2.1. Fuzzy set

A Fuzzy set can be characterized as: \( \tilde{A} = \{ \langle x, \mu_{\tilde{A}}(x) \rangle : x \in X \} \) and \( X \) be a non-void set, \( \mu_{\tilde{A}}(x) \in [0, 1] \) is the membership of \( x \in X \) in \( \tilde{A} \).

2.2. Infinite framework limit

If the arrival rate is not affected by the number of customers being served and waiting, i.e., systems with large population of potential customers (unlimited capacity).

3. Fuzzy Queuing Model with Two Classes

Consider a solitary channel fuzzy queueing model with two classes FM/F (H1, H2)/1/FCFS without any priorities in entry rates, where FM signifies the fuzzy landing rates as a Poisson procedure, while F(H1, H2) indicates the fuzzified hyper exponential administration time rates with two classes in a First Come First Serve (FCFS) way, with infinite framework limit and population size.

In this model, clients arrive in gatherings by a solitary channel spoken to by \( \tilde{\lambda}_1 \), \( \tilde{\lambda}_2 \), \( \tilde{\mu}_1 \) and \( \tilde{\mu}_2 \) separately. Let \( \tilde{\lambda}_1 \) \((w)\), \( \tilde{\lambda}_2 \) \((x)\), \( \tilde{\mu}_1 \) \((y)\), and \( \tilde{\mu}_2 \) \((z)\) at that point be the fuzzy sets spoken to by four sets as in Eq. (1), Eq. (2), Eq. (3) & Eq. (4):

\[
\tilde{\lambda}_1 = \{ (w, \tilde{\lambda}_1 (w)) | w \in W \} \\
\tilde{\lambda}_2 = \{ (x, \tilde{\lambda}_2 (x)) | x \in X \} \\
\tilde{\mu}_1 = \{ (y, \tilde{\mu}_1 (y)) | y \in Y \} \\
\tilde{\mu}_2 = \{ (z, \tilde{\mu}_2 (z)) | z \in Z \}
\]

where \( W \), \( X \), \( Y \) and \( Z \) are crisp universal gathering of the landing rate and administration rate. Moreover, let \( f(w,x,y,z) \) mean the specific arrangement of interest. Consequently \( w, x, y \) and \( z \) are fuzzy numbers and allegedly \( f(w,x,y,z) \) are fuzzy numbers.

Let \( L_q^{(1)} \) and \( L_q^{(2)} \) represent the current equation in the traditional single queuing model as in Eq. (5) & Eq. (6):

\[
L_q^{(1)} = \frac{\lambda_1 (\rho_1^2 + \rho_2)}{1 - \rho_1^2 - \rho_2} \\
L_q^{(2)} = \frac{\lambda_2 (\rho_1^2 + \rho_2)}{1 - \rho_1^2 - \rho_2}
\]

The stability steady state is \( \rho \equiv \rho_1 + \rho_2 < 1 \) and \( 0 < \rho < 1 \). Other exhibition estimations are characterized by Eq. (7), Eq. (8) & Eq. (9)
\[Wq^{(i)} = \frac{Lq^{(i)}}{\lambda_i}\]  
(7)

\[Ws^{(i)} = Wq^{(i)} + \frac{1}{\mu_i}\]  
(8)

\[Lq^{(i)} = \lambda_i WS^{(i)}; \quad i = 1, 2\]  
(9)

4. Ranking Technique

In this section, the way to change over the fuzzy numbers into crisp numbers is explained. Three sorts of fuzzy numbers; pentagonal, heptagonal and octagonal fuzzy numbers are executed with the left and right ranking technique \([2, 3]\) which is addressed by \(F(R) \rightarrow R\).

To begin assigning the innovation for this system, we consider the following cases:

4.1. Pentagonal fuzzy number (Case 1)

Let a convex pentagonal fuzzy number \(\tilde{A}(z) = \tilde{A}(a_1, a_2, a_3, a_4, a_5; w)\). Then the Left and right ranking index is portrayed Eq. (10)

\[R(\tilde{A}) = \int_0^w \frac{L^{-1}(z) + R^{-1}(z)}{2} \, dz\]  
(10)

where,

\[L^{-1}(z) = [w(b-a) + a] + \frac{1}{2}[w(c-b) + b]\]

\[R^{-1}(z) = \frac{1}{2}[w(d-c) - d] + [w(e-d) - e]\]

From Eq. (10), after simplification, we obtain Eq. (11)

\[R(\tilde{A}) = \frac{w(2a_1 + 3a_2 + 2a_3 + 3a_4 + 2a_5)}{4}\]  
(11)

4.2. Heptagonal fuzzy number (Case 2)

Let a convex heptagonal fuzzy number \(\tilde{A}(z) = \tilde{A}(a_1, a_2, a_3, a_4, a_5, a_6, a_7; w)\). Then the Ranking Index is portrayed by

\[R(\tilde{A}) = \int_0^w \frac{L^{-1}(z) + R^{-1}(z)}{2} \, dz\]  
(12)

Proceeding in the same way as in case-1, we obtain Eq. (12)

\[R(\tilde{A}) = \frac{w(2a_1 + 7a_2 + 7a_3 + 7a_4 + 7a_5 + 7a_6 + 2a_7)}{54}\]  
(12)

4.3. Octagonal fuzzy number (Case 3)

Let a convex octagonal fuzzy number \(\tilde{A}(z) = \tilde{A}(a_1, a_2, a_3, a_4, a_5, a_6, a_7, a_8; w)\).

Then the Ranking Index is portrayed by

\[R(\tilde{A}) = \int_0^w \frac{L^{-1}(z) + R^{-1}(z)}{2} \, dz\]  
(13)

Proceeding in the same way as in case-1 and case-2, we obtain Eq. (13)

\[R(\tilde{A}) = \frac{w(3a_1 + 6a_2 + 4a_3 + 5a_4 + 5a_5 + 4a_6 + 6a_7 + 3a_8)}{36}\]  
(13)
5. Numerical Examples

In this section 5, numerical examples are presented for each case. Consider a Sequential construction system of gadget accepting two kinds of entry Clients $\tilde{\lambda}_1$, $\tilde{\lambda}_2$ and the administration time spoke to as a blend of the Exponential conveyance $\tilde{\mu}_1$, $\tilde{\mu}_2$ individually. All the parameters are fuzzy. The administration needs to register the mean of line as well as system Length and average waiting time of line and system for each class.

5.1. Example 1

Accept that both landing rate with two classes and administration rates are pentagonal fuzzy numbers in a First Come First Serve (FCFS) way, with infinite framework limit and population size, characterized as:

$\tilde{\lambda}_1=[2, 4, 6, 8, 10; 1]$, $\tilde{\lambda}_2=[3, 5, 7, 9, 11; 1]$

$\tilde{\mu}_1=[12, 14, 17, 19, 22; 1]$, $\tilde{\mu}_2=[13, 15, 18, 21, 24; 1]$

According to the above Eq. (11), the ranking index of $\tilde{\lambda}$

$R(\tilde{\lambda}_i) = R(2, 4, 6, 8, 10; 1) = \frac{w(2a_1 + 3a_2 + 2a_3 + 3a_4 + 2a_5)}{4} = 18$

$& R(\tilde{\lambda}_2) = R(3, 5, 7, 9, 11; 1) = \frac{(2+3+3+7+9+11+10)}{4} = 21$

Continuing similarly, we get the ranking index of $\tilde{\mu}$

$R(\tilde{\mu}_1) = R(12, 14, 17, 19, 22; 1) = 50.25$

$R(\tilde{\mu}_2) = R(13, 15, 18, 21, 24; 1) = 54.5$

The positioning of class one and class two are

$R(\tilde{\lambda}_1) = 18$

$R(\tilde{\lambda}_2) = 21$

$R(\tilde{\mu}_1) = 50.25$

$R(\tilde{\mu}_2) = 54.5$

From Eq. (5) & Eq. (6)

$L_{q(1)} = \frac{\lambda_1 (\frac{\tilde{\mu}_1 + \tilde{\mu}_2}{\tilde{\lambda}_1 + \tilde{\mu}_2})}{1-p}$

$L_{q(2)} = \frac{\lambda_2 (\frac{\tilde{\mu}_1 + \tilde{\mu}_2}{\tilde{\lambda}_1 + \tilde{\mu}_2})}{1-p}$

where $\rho_1=\tilde{\lambda}_1 / \tilde{\mu}_1$, $\rho_2=\tilde{\lambda}_2 / \tilde{\mu}_2$

The exhibition measures are

$L_{q(1)} = \frac{18(\frac{18}{50.25} + \frac{21}{54.5})}{1-0.996} = 0.996$

$L_{q(2)} = \frac{21(\frac{18}{50.25} + \frac{21}{54.5})}{1-1.162} = 1.162$

$R(1) = \tilde{\lambda}_1; R(2) = \tilde{\lambda}_2; R(1) = \tilde{\mu}_1; R(2) = \tilde{\mu}_2.$
From Eq. (7), Eq. (8) & Eq. (9)

The performance measures for pentagonal fuzzy number are
\[ L_q^{(1)} = 0.996, \quad W_q^{(1)} = 0.0553, \quad W_s^{(1)} = 0.0752, \quad \text{and} \quad L_s^{(1)} = 1.353 \]
\[ L_q^{(2)} = 1.162, \quad W_q^{(2)} = 0.0553, \quad W_s^{(2)} = 0.077, \quad \text{and} \quad L_s^{(2)} = 1.617 \]

5.2. Example 2

Accept that both landing rate with two classes and administration rates are heptagonal fuzzy numbers in a First Come First Serve (FCFS) way, with infinite framework limit and population size, characterized as:
\[ \tilde{\lambda}_1 = [2, 4, 6, 8, 10, 12, 14; 1], \quad \tilde{\lambda}_2 = [3, 5, 7, 9, 11, 13, 15; 1], \]
\[ \tilde{\mu}_1 = [16, 19, 22, 25, 29, 31, 33; 1], \quad \tilde{\mu}_2 = [18, 21, 24, 28, 30, 32, 35; 1]. \]

According to the above Eq. (12), the ranking index of \( \tilde{\lambda} \) is
\[ R(\tilde{\lambda}_1) = (2, 4, 6, 8, 10, 12, 14; 1) = \frac{w(2a_1+7a_2+7a_3+7a_4+7a_5+7a_6+2a_7)}{54} = 8 \]
\[ & R(\tilde{\lambda}_2) = R(3, 5, 7, 9, 11, 13, 15; 1) = \frac{w(2a_1+7a_2+7a_3+7a_4+7a_5+7a_6+2a_7)}{54} = 9 \]

Proceeding similarly, we get
\[ R(\tilde{\mu}_1) = R(16, 19, 22, 25, 29, 31, 33; 1) = 25.092 \]
\[ R(\tilde{\mu}_2) = R(18, 21, 24, 28, 30, 32, 35; 1) = 27.24 \]

The ranking of class one and class two are
\[ R(\tilde{\lambda}_1) = 8 \]
\[ R(\tilde{\lambda}_2) = 9 \]
\[ R(\tilde{\mu}_1) = 25.092 \]
\[ R(\tilde{\mu}_2) = 27.24 \]

The performance measures are
\[ L_q^{(1)} = 7.825, \text{ and} \]
\[ L_q^{(2)} = 8.803 \]

Likewise, from Eq. (5) - Eq. (9), the performance measures for heptagonal fuzzy number are as follows:
\[ L_q^{(1)} = 7.825, \quad W_q^{(1)} = 0.978, \quad W_s^{(1)} = 1.0178, \quad \text{and} \quad L_s^{(1)} = 8.136 \]
\[ L_q^{(2)} = 8.803, \quad W_q^{(2)} = 0.978, \quad W_s^{(2)} = 1.014, \quad \text{and} \quad L_s^{(2)} = 9.126 \]

5.3. Example 3

Assume that both arrival rate with two classes and service rates are octagonal fuzzy numbers in a First Come First Serve (FCFS) way, with infinite framework limit and population size, defined as:
\[ \tilde{\lambda}_1 = [4, 6, 8, 10, 12, 14, 16, 19; 1], \quad \tilde{\lambda}_2 = [3, 5, 7, 9, 11, 13, 15, 18; 1], \]
\[ \tilde{\mu}_1 = [20, 22, 25, 27, 30, 33, 37, 41; 1], \quad \tilde{\mu}_2 = [21, 24, 28, 31, 34, 38, 40, 42; 1]. \]

According to the above Eq. (13), the ranking index of \( \tilde{\lambda} \) is
\[ R(\tilde{\lambda}_1) = R(4, 6, 8, 10, 12, 14, 16, 19; 1) = \frac{w(3a_1 + 6a_2 + 4a_3 + 5a_4 + 5a_5 + 4a_6 + 6a_7 + 3a_8)}{36} = 11.08 \]

& \[ R(\tilde{\lambda}_2) = R(3, 5, 7, 9, 11, 13, 15, 18; 1) = \frac{(3\cdot 3 + 6 + 5 + 4 + 7 + 5 + 9 + 5 + 11 + 6 + 15 + 6 + 3 + 19)}{36} = 10.08. \]

Proceeding equally, we get

\[ R(\tilde{\mu}_1) = R(20, 22, 25, 27, 30, 33, 37, 41; 1) = 29.27 \]

Similarly, we obtain

\[ R(\tilde{\mu}_2) = R(21, 24, 28, 31, 34, 38, 40, 42; 1) = 36.58 \]

The ranking of class one and class two are

\[ R(\tilde{\lambda}_1) = 11.083 \quad R(\tilde{\lambda}_2) = 10.08 \]

\[ R(\tilde{\mu}_1) = 29.27 \quad R(\tilde{\mu}_2) = 36.58 \]

The performance measures are \( L_q(1) = 0.674 \) and \( L_q(2) = 0.613 \).

As before, the execution proportions for octagonal fuzzy number are

\[ L_q(1) = 0.674, \quad W_q(1) = 0.060, \quad W_s(1) = 0.094, \quad L_s(1) = 1.041. \]

\[ L_q(2) = 0.613, \quad W_q(2) = 0.060, \quad W_s(2) = 0.094, \quad L_s(2) = 0.947. \]

6. Results and Discussion

The obtained outcomes are given in Tables 1 and 2, which clarify various estimations of each class for a wide range of membership functions considered (pentagonal, heptagonal, and octagonal fuzzy numbers):

**Table 1. Performance measures- \( W_s^{(1)}, W_s^{(2)} \) and \( L_s^{(1)}, L_s^{(2)} \).**

<table>
<thead>
<tr>
<th>Membership Function</th>
<th>( W_s^{(1)} )</th>
<th>( W_s^{(2)} )</th>
<th>( L_s^{(1)} )</th>
<th>( L_s^{(2)} )</th>
</tr>
</thead>
<tbody>
<tr>
<td>Pentagonal</td>
<td>0.0752</td>
<td>0.077</td>
<td>1.353</td>
<td>1.617</td>
</tr>
<tr>
<td>Heptagonal</td>
<td>1.017</td>
<td>1.014</td>
<td>8.136</td>
<td>9.126</td>
</tr>
<tr>
<td>Octagonal</td>
<td>0.094</td>
<td>0.094</td>
<td>1.041</td>
<td>0.947</td>
</tr>
</tbody>
</table>

**Table 2. Performance measures- \( W_q^{(1)}, W_q^{(2)} \) and \( L_q^{(1)}, L_q^{(2)} \).**

<table>
<thead>
<tr>
<th>Membership Function</th>
<th>( W_q^{(1)} )</th>
<th>( W_q^{(2)} )</th>
<th>( L_q^{(1)} )</th>
<th>( L_q^{(2)} )</th>
</tr>
</thead>
<tbody>
<tr>
<td>Pentagonal</td>
<td>0.996</td>
<td>1.162</td>
<td>0.0553</td>
<td>0.0553</td>
</tr>
<tr>
<td>Heptagonal</td>
<td>7.825</td>
<td>8.803</td>
<td>0.978</td>
<td>0.978</td>
</tr>
<tr>
<td>Octagonal</td>
<td>0.674</td>
<td>0.613</td>
<td>0.060</td>
<td>0.060</td>
</tr>
</tbody>
</table>

The graphical representations of Tables 1 and 2 are demonstrated in Figs. 1 and 2 respectively.
From the outcomes shown in Tables 1 and 2 and its graphical representation demonstrated in Figs. 1 and 2, the positioning technique gives different arrangements of real values, for example, landing and administration rates for each class. In like manner, diverse execution estimations are gotten which are given and apparently converges between two classes in the entire framework. It is likewise observed from Tables that all execution proportions of class one is less than or equal to execution proportions of class two in the framework for all the three sorts of fuzzy numbers (i.e., pentagonal, heptagonal, octagonal).

7. Sensitivity Analysis

In this section to analyse sensitivity of the model a sensitive analysis is performed with different fuzzy numbers (pentagonal, heptagonal, and octagonal fuzzy numbers) based on estimations of each class. The objective of sensitivity analysis is to
determine variations between the two given categories while calculating their execution proportions. The obtained outcomes are given in below Table 3.

The sensitivity analysis of this model is analysed by varying i.e., decreasing or increasing the values of any one parameter \((\lambda_1, \mu_1, \mu_2)\) keeping all other parameters unaltered. For example in case of pentagonal fuzzy number when the values of the parameter \(\lambda_1\) is decreased by 0.5 i.e., when \(\lambda_1 = [1.5, 3.5, 5.5, 7.5, 9.5; 1]\) keeping \(\lambda_2, \mu_1, \mu_2\) as it is, then the performance measures \(L_q, L_s, W_q, W_s\) of class 1 are less than or equal to the performance measures of class 2, similarly when \(\lambda_1\) is increased by 0.5 i.e., when \(\lambda_1 = [2.5, 4.5, 6.5, 8.5, 10.5; 1]\) The same phenomenon is observed. In the case of heptagonal when \(\mu_2\) is decreased by 0.5 and increased by 0.5 the same result we got and finally in case of octagonal when \(\lambda_1\) is decreased and increased by 0.2 then also the results are almost the same. We also observed that the utilizing of more sorts of fuzzy numbers drives us to get all the more genuine information and adaptable decisions in the framework.

<table>
<thead>
<tr>
<th>Table 3: Sensitivity analysis of execution proportions.</th>
</tr>
</thead>
<tbody>
<tr>
<td>(L_q^{(1)})</td>
</tr>
<tr>
<td>--------------------------------------------------------</td>
</tr>
<tr>
<td><strong>Pentagonal</strong></td>
</tr>
<tr>
<td>decrease (\lambda_1) values by 0.5</td>
</tr>
<tr>
<td>Increase (\lambda_1) values by 0.5</td>
</tr>
<tr>
<td><strong>Heptagonal</strong></td>
</tr>
<tr>
<td>-decrease (\mu_2) values by 0.5</td>
</tr>
<tr>
<td>Increase (\mu_2) values by 0.5</td>
</tr>
<tr>
<td><strong>Octagonal</strong></td>
</tr>
<tr>
<td>decrease (\lambda_1) values by 0.2</td>
</tr>
<tr>
<td>Increase (\lambda_1) values by 0.2</td>
</tr>
</tbody>
</table>

8. Limitations

One obvious limitation is that the arrival rate is not stationary. It is state dependent. Queuing model gives the steady state solution. Unlike the classic model that expects arrivals follow a Poisson process and exponentially distributed service times, the arrival rate in many real situations is more possibility than probabilistic.

There is only a single channel model. The mean arrival rate is less than the mean service rate, i.e., \(\lambda < \mu\). This model is valid for only 2 classes, not valid for more than 2 classes.
9. Conclusion

In this paper, the fuzzy set theory is shown to be a strong tool when dealing with real applications in queuing models with two classes such as the manufacturing production line. The ranking approach adopted is also seen to be effective when transforming fuzzy queues into crisp queues, thus evaluating the system by conventional performance measurements such as the expected queue length of customers in the queues and system for both classes of arrivals. Also, the computation of the expected waiting time of customers in the queue and in the whole system is obtained too. In this paper, sensitivity analysis is also performed to determine variations between the two given categories while calculating their execution proportions. It is observed in all the cases that the execution proportions of class one is less than or equal to execution proportions of class two in the framework. Additionally, the utilizing of more sorts of fuzzy numbers drives us to get all the more genuine information. Therefore the manager can take the best values and make optimal decisions. Another advantage of using the ranking method index is obtaining exact values inside the closed crisp interval, while also providing more than one solution of values in the queuing system with different types of membership functions.

This paper can be extended by considering the uncertainty of random variables i.e., probabilistic parameter can be considered in place of fuzzy numbers. Another possible area for future research work is to consider intuitionistic fuzzy numbers and neutrosophic sets. Researchers can also investigate the effectiveness of this approach to other queuing models and other types of linear membership functions.

Nomenclatures

<table>
<thead>
<tr>
<th>Symbol</th>
<th>Meaning</th>
</tr>
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<tbody>
<tr>
<td>(F(H_1,H_2))</td>
<td>Fuzzy administration rate with 2 classes. (classes/second)</td>
</tr>
<tr>
<td>(FM)</td>
<td>Fuzzy landing rate (landing/second)</td>
</tr>
<tr>
<td>(L_s)</td>
<td>Average no. of customers in a system. (customers/second).</td>
</tr>
<tr>
<td>(L_q)</td>
<td>Average no. of customers waiting in queue (customers/second).</td>
</tr>
<tr>
<td>(W_s)</td>
<td>Average waiting time of customers in system, (seconds).</td>
</tr>
<tr>
<td>(W_q)</td>
<td>Average waiting time of customers in queue, (seconds).</td>
</tr>
<tr>
<td>(X, W)</td>
<td>Set of inter arrival time</td>
</tr>
<tr>
<td>(Y, Z)</td>
<td>Set of inter service time</td>
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</tbody>
</table>

Greek Symbols

<table>
<thead>
<tr>
<th>Symbol</th>
<th>Meaning</th>
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<tbody>
<tr>
<td>(\mu_A(x))</td>
<td>Membership of x</td>
</tr>
<tr>
<td>(\lambda)</td>
<td>Average arrival, (seconds).</td>
</tr>
<tr>
<td>(\mu)</td>
<td>Average service rate,(seconds).</td>
</tr>
<tr>
<td>(\rho)</td>
<td>Stability steady state</td>
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Abbreviations

<table>
<thead>
<tr>
<th>Abbreviation</th>
<th>Meaning</th>
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<tbody>
<tr>
<td>FCFS</td>
<td>First Come First Serve</td>
</tr>
</tbody>
</table>

References


