LINEAR AND NON-LINEAR MODELLING OF SPACECRAFT RELATIVE MOTION IN THE PROXIMITY OPERATION AT LOW EARTH ORBIT

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Abstract

This paper provides a closed-form relative motion dynamics model with an extensive survey of the current literature with prominence approaches between various and their contribution to the overall field of study. Hence, difficulty is seen in the formation of a benchmark to determine the dynamics models that are applicable for deep space mission scenario and in the evaluation of the efficacy of newly developed relative motion solutions. In addition, illustrates the steps followed in finding an approximate model for a linear and nonlinear system using CW Equation and which make three explicit assumptions: (1) considering two spacecraft model which obey Keplerian motion, (2) close proximity operation between the chief and deputy, and (3) locating chief in near circular orbit. The method has been found to be useful in obtaining linearized approximation whose accuracy is more than that of the traditional approach of linearization about the origin.

Keywords: Clohessy-Wiltshire equation, Close proximity operation, Low Earth orbit, Relative dynamic motion.
1. Introduction

A relative dynamics model is encompassing in any orbiting satellite motion. Relative motion plays an important role in earth satellites (as at 1960). Many researchers have shown a keen interest in contributing their investigations which were commenced during space explorations.

In detail, a worthy consideration of relative motion might not have happened for celebrated endeavours of the space age. For an instance, the Apollo Space Mission and ISS (International Space Station) astronauts could not have travelled safely and returned from a deep space mission without the competence (consideration of space environment, radiation effect, relative dynamical states) of docking and rendezvous on their spacecraft, as like moon and ISS [1]. It envisaged key application of relative satellite motion.

As a matter of terminology, the relative dynamics model yields the definition of the motion of a spacecraft, denoted as the deputy, with respect to a reference spacecraft, denoted as the chief. A point to note is that, without loss of generality, the chief could also define an ideal reference orbit or trajectory instead of the physical orbit of a spacecraft. It is important to note that this survey addresses a relative motion with respect to a closed chief orbit about Earth in the context of the two-body problem [2].

The present technology shows the success rate of relative satellite motion in a practical approach (Imani and Mohsen 2013), which includes the formation of flying, multiple spacecraft with the accurate control to find their relative motion (position state) with the convinced data from the space environment [3-5]. With this assumption, a spacecraft attempts Rendezvous and Proximity Operation (RPO) in an autonomous mode [6-8].

Particularly, the autonomy system is always dependent on the on-board system which includes guidance algorithms, navigation filters and control systems and can be operated without any direction from ground station on earth [9].

The group of arranged models in a relative satellite state is the chief problem of the near circular orbit. The particular designed the model is allowable in permissible near circular (non-circular); therefore, the chief eccentricity would be considering a value between 0 and 1 (International Space Station, eccentricity is 0.00158).

With the assumption of a two-body orbit propagation, the near circular chief model is more complex than the circular chief problem which means an angular velocity of satellite, acceleration and the Earth’s centre distance to the orbit is too longer to be a constant [10-12]. A selected model of Non-linear motion equation is used for an on-board system of ISS.

2. Problem Statement

Although classified as various models existing for the circular chief problem of relative satellite motion, none has been stated in a high degree of operational efficiency of relative orbital elements (ROEs) for the non-circular (near circular) chief problem [13, 14]. Particularly, identification of the relative motion is not confined to the non-zero eccentricity of the chief satellite’s orbit.

Thus, it is still functional enough to facilitate autonomous guidance and manoeuvre systems [15, 16]. In this current research, attempts have been made to
answer the two-following questions namely, what evidences realize an operational efficacy of the roe? How to preserve those strategies, while reducing the eccentricity constraint?

3. Close Proximity Operation of ISS

The proposed designed model computes with a set of relative motion equation in a real-time space application (ISS missions). The model, specifically defined as the orbital element of chief orbit, provides a complete initialization of describing a relative motion between spacecraft’s or satellites is observed from the well-known method of CW equation the relative motion between the chief and deputy. The dynamic model is simplified by assuming that the target is as a near circular orbit (0.00158 eccentricities). Figure 1 represents the small eccentricity orbit model of earth orbit.

In MATLAB/Simulink, multiple iterations have been taken up to the level of accuracy in the control model. Initializing the orbital elements of ISS mission is shown in Table 1. Previous research has engrossed in the consideration of a J2 effect and atmospheric drag on a proposed dynamic model of satellite relative motion. The aim of this work is to find the CW solution for small eccentricity at LEO. The generated results can be validated with the real-time space mission data. Table 2 shows an integrated CW equation result in predefined orbital parameters.

![Fig. 1. Image of chief small eccentricity orbit around Earth.](image)

**Table 1. Chief orbital elements of ISS mission.**

<table>
<thead>
<tr>
<th></th>
<th>Radius at Apogee</th>
<th>Radius at Perigee</th>
<th>Inclination</th>
<th>Right Ascension of Ascending Node</th>
<th>Argument of Perigee</th>
<th>True Anomaly</th>
<th>Eccentricity</th>
</tr>
</thead>
<tbody>
<tr>
<td>1</td>
<td>505.5 km</td>
<td>505.5 km</td>
<td>97.5 deg.</td>
<td>17.562 deg.</td>
<td>135.571</td>
<td>0 deg.</td>
<td>0.00158 deg.</td>
</tr>
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</table>

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### Table 2. CW Solution of ISS mission orbital elements.

<p>| | | |</p>
<table>
<thead>
<tr>
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<th></th>
<th></th>
</tr>
</thead>
<tbody>
<tr>
<td>1</td>
<td>Maximum True Anomaly</td>
<td>0.00109 rad/s</td>
</tr>
<tr>
<td>2</td>
<td>Mean True Anomaly</td>
<td>0.00105 rad/s</td>
</tr>
<tr>
<td>3</td>
<td>Minimum True Anomaly</td>
<td>0.001102 rad/s</td>
</tr>
<tr>
<td>4</td>
<td>Mean Motion</td>
<td>0.001105 rad/s</td>
</tr>
<tr>
<td>5</td>
<td>Error at max. time true anomaly</td>
<td>0.32%</td>
</tr>
<tr>
<td>6</td>
<td>Error at min. time true anomaly</td>
<td>0.32%</td>
</tr>
<tr>
<td>7</td>
<td>Max. radius of chief</td>
<td>5172.6 km</td>
</tr>
<tr>
<td>8</td>
<td>Mean radius of chief</td>
<td>5167.584 km</td>
</tr>
<tr>
<td>9</td>
<td>Min. Radius of chief</td>
<td>5172.568 km</td>
</tr>
<tr>
<td>10</td>
<td>The required delta velocity</td>
<td>[0.017, -0.010, 0.000]</td>
</tr>
<tr>
<td>11</td>
<td>Final relative position using non-linear EOM</td>
<td>[0.161, -0.192, 0.000]</td>
</tr>
<tr>
<td>12</td>
<td>Final relative position using integrate of CW EOM</td>
<td>[0.000, 0.000, 0.000]</td>
</tr>
</tbody>
</table>

### 4. Close Proximity Operation of Spacecraft for Near Circular Orbit

Beneath the consideration of a circular reference orbit, the angular velocity of the chief frame is the constant chief mean motion, and equation (1) becomes autonomous and simpler. The following models have been developed using this fundamental assumption. But they are often implemented in cases where the chief eccentricity is very small but nonzero. Figure 2 shows the simulation model of the chief and deputy spacecraft.

![Simulation model of chief and deputy spacecraft.](image)

4.1. Linear models

In the framework of spacecraft rendezvous, Clohessy-Wiltshire have derived equations of a linear form and relative motion by retaining only the first-order terms in the Taylor expansion of the nonlinear differential gravity, neglecting all other perturbations in Eq. (1) [17]. These equations of motion, known as the Hill-Clohessy-Wiltshire (HCW) equations, have a simple closed-form state transition matrix (STM) solution characterized by decoupled in-plane cycloid motion and out-of-plane harmonic motion. The HCW equations have been used extensively since the 1960s for spacecraft close-proximity operations modelling, with flight heritage on the manned Gemini, Apollo, and Shuttle missions (NASA) [18-20] the XSS-10 and -11 on-orbit servicing technology demonstrations (Air Force Research Laboratory).
Laboratory) [21], the autonomous ETS-VII (Japanese Aerospace Exploration Agency) [22], and others.

Although the initial formulation considers rectilinear relative position and velocity, more recent investigations discussed [23-25]. Ichikawa and Ichimura [26] showed the possibility of formulation of an identical form of the HCW equations using a curvilinear relative state. This representation intrinsically captures the orbit curvature better, leading to improvements in the computation of the initial conditions and hence, in model accuracy. The results presented in [26] reveal delivery by the curvilinear HCW equation of position propagation errors that are orders of magnitude better than the rectilinear implementation when the relative separation is largely in the along-track direction, with perfect Keplerian relative motion prediction in the case of purely along-track separation.

Instead of a direct description of the relative motion through position and velocity components, [27, 28] have used a change in coordinates to establish a state built from the integration constants of the HCW solution. In their research studies, the authors demonstrate the improved geometric insight and guidance control simplicity afforded by working with these new HCW invariant parameters but do not develop a framework for including other perturbations. Additional examples of parameterizing the relative motion with invariant trajectory parameters have been provided [29] in the context of relative orbit transfer using the HCW equations as well as by Leonard et al. [30], who define a set of geometric relative orbital parameters that are analogous to the classical orbital elements.

The latter provide a detailed methodology for computing the geometric parameters based on constraints imposed by the unperturbed relative motion, and just as [31], provide a compelling argument for the simplicity and geometric insight provided by the new relative motion description through straightforward impulsive manoeuvres to reconfigure the orbital parameters.

In response to the lack of perturbations considered in the HCW solution, [32, 33] derive extensions of the model for the incorporation of the first-order effects of the Earth oblateness $J_2$ perturbation. Developments in correct for first-order $J_2$ effects on the reference orbital period nodal drift and present numerical results indicating position modelling errors at the meter level when compared with a numerical integration of the absolute $J_2$ perturbed equations of motion. Although Stringer et al. [34] did not initially consider the effect of $J_2$ on the chief orbit, the authors have included first-order perturbing accelerations due to atmospheric drag in the extension of the HCW model.

The ensuing analysis uses the Floquet theory to predict a slowly manifesting relative motion instability arising as a result of the perturbation considered, consistent with the presented simulation results. Perez et al. [35] later expanded the research to consider the perturbation of the reference frame angular velocity but did not arrive at closed-form solutions. Early work by Leonard examines the use of the HCW dynamics model in conjunction with differential drag acting as a constant acceleration in the anti-flight direction. Imposition of a simple coordinate transformation to describe the relative motion with respect to an average position of the unperturbed equations has seen the perturbed equations resulting in two uncoupled second-order differential equations, for which the authors demonstrate closed-form control laws for formation-keeping.
Yamanaka and Ankersen [36] developed extensions of the HCW analytical solution to incorporate the effect of atmospheric drag on two spacecraft flying in formation. Humi and Thomas are considered a linear drag model where the variation in drag force varies is proportional to velocity. Have improved this formulation by considering a quadratic drag model, where force varies with the square of spacecraft velocity. It is important to note that, for simplifying the solution authors have assumed an atmospheric density profile that varies inversely with altitude and that both spacecraft have identical ballistic coefficients.

The result is a modified version of the HCW STM that includes the effect of atmospheric drag on the relative motion in the radial direction. In a more recent work, Bevilacqua and Romano have developed a method for using atmospheric drag to conduct rendezvous manoeuvres with multiple spacecraft in near-circular orbits by adopting the original approach of Leonard. The analytical control design uses a J₂ perturbed dynamic model built from the aforementioned Schweighart-Sedwick solution that is improved to enable incorporation of the drag-based manoeuvres.

With respect to the aforementioned work of Lovell and Tragesser, research by Bevilacqua and Lovell presents a new analytical solution for the evolution of the relative state subject to continuous thrust. The authors establish the practical merit of this new formulation by incorporating orbital re-phasing and rendezvous manoeuvres designed using the input shaping theory. Bennett and Schaub have developed a set of variational equations for the relative state developed by Lovell and Tragesser using a Lagrange bracket formulation that captures the evolution of the state parameters arising from the perturbing accelerations expressed in the chief RTN frame.

The resulting model finds use in designing closed-form control policies subject to this perturbation. Finally, it is important to mention the work of Colombo 1989, which provides an analytical solution to the HCW model while includes constant and once-per-orbit harmonic empirical accelerations in the radial, along-track, and cross-track components. The framework developed in that work can be used for modelling a multitude of perturbations that manifest as a superposition of constant and harmonic accelerations occurring at the orbital angular rate.

4.2. Nonlinear models

For relative motion considerations where the previous linearization assumption breaks down, several authors have directly considered the simplified autonomous nonlinear equations of relative motion under the circular chief orbit assumption (Stringer et al. 2012, Newman et al, 2014, Perez et al.2015). In this context, the assumption of small inter spacecraft separation is relaxed for obtaining new closed-form solutions to the equations of relative motion. The double transformation solution uses the nonlinear mapping between cylindrical (or spherical) coordinates and rectilinear coordinates to rewrite the cylindrical (or spherical) HCW solution in terms of the nonlinear combination of rectilinear initial conditions.

The approximated double transformation methods retain nonlinear terms only up-to the second order from the associated double transformation results. This is how the double transformation and approximated double transformation models build a solution from the linear HCW model that is nonlinear in the initial conditions. The authors highlight the double transformation and multiple-scales models as subject to persistent but slowly drifting error due to retaining the linear
HCW along-track drift term alone, whereas the quadratic Volterra predicts additional drifting motion in the other directions. It is important to note that, although these nonlinear models extend the inter-satellite range of applicability by capturing higher-order separation effects, they neglect the potentially larger effects of chief orbit eccentricity entirely.

4.3. Dynamics models using an orbital element state representation

The proposed CW model has the capability of close proximity operation. Equation (1) helps obtaining a solution of closed form, assuming both the spacecraft are in near circular orbit and the external forces considered as zero continuously i.e., \( \vec{F} = 0 \). The sensible assumption of impulse thrust \( \Delta V \) which is exerted on the spacecraft in a real-time application is taken into consideration.

\[
\ddot{x} - 2\omega \dot{y} - 3\omega^2 x = f_x ; \quad \ddot{y} - 2\omega \dot{x} = f_y ; \quad \ddot{z} + \omega^2 z = f_z \tag{1}
\]

Consideration of external force \( \vec{F} = 0 \), and Eq. (1) is possible by taking a one derivative order of X component in Eq. (1), we have,

\[
\ddot{x} = 2\omega \dot{y} + 3\omega^2 x ; \quad \dddot{x} = 2\omega \dot{y} + 3\omega^2 \dot{x} \tag{2}
\]

Equation (3) is obtained by substituting the value \( \ddot{y} = 2\omega \dot{x} \) into the above equation

\[
\dddot{x} = -\omega^2 \dot{x} \tag{3}
\]

Then, enhancing the Laplace transform of Eq. (3), we get:

\[
s^3 X(s) - s^2 x_0 - sx_0 - sX(s)\omega^2 - x_0\omega^2 = 0 \quad ; \quad X(s) = \frac{x_0}{s} + \frac{x_0}{(s^2+\omega^2)} + \frac{s\dot{x}_0}{s(s^2+\omega^2)} \tag{4}
\]

where \( x_0, \dot{x}_0, \ddot{x}_0 \) are representing in x-component initial relative position, velocity and acceleration respectively. To resolve the equation in the Laplace transform, let

\[
\frac{x_0}{s(s^2+\omega^2)} = A + \frac{Bx + C}{(s^2+\omega^2)} \tag{5}
\]

Multiply both sides on the numerator of A for finding a coefficient value of A. Set \( S = 0 \) Thus,

\[
A = \frac{\dddot{x}_0}{\omega^2} \tag{6}
\]

Apply the same concept to obtain the value of B and C that are

\[
B = -\frac{\ddot{x}_0}{\omega^2} and \quad C = 0 \tag{7}
\]

Substitute the coefficient of A, B and C in the Eqns. (6) and (7) into (5), now Eqn. (4) becomes,

\[
X(s) = \frac{x_0}{s} + \frac{x_0}{(s^2+\omega^2)} + \frac{x_0}{s^2+\omega^2} - \frac{s\dot{x}_0}{s(s^2+\omega^2)} \tag{8}
\]

Then, the inverse Laplace transform become in this form,

\[
x(t) = x_0 + \frac{x_0}{\omega^2} + \frac{\dot{x}_0}{\omega} \sin(\omega t) - \frac{\ddot{x}_0}{\omega^3} \cos(\omega t) \tag{9}
\]

Equation (19) represents the finest closed form solution of CW equation, which shows the relative position of the x component. The expression of \( \dddot{x} \) is considered
in Eq. (12) and the expression of $\ddot{x}$ in Eq. (2) is considered, allowing after $\ddot{x}_0 = 2\omega \dot{y}_0 + 3\omega^2 x_0$, Eq. (9) becomes,

$$x(t) = 4x_0 + \frac{2y_0}{\omega} + \frac{x_0}{\omega} \sin (\omega t) - \left(3x_0 + \frac{2y_0}{\omega}\right) \cos (\omega t) \tag{10}$$

The relative velocity can be obtained through the order of the derivative. Hence consideration of the initial condition mentioned in Tables 1 and 2 (position, velocity) are constant, having Eq. (11),

$$\dot{x}(t) = \dot{x}_0 \cos(\omega t) + \left(3x_0 + \frac{2y_0}{\omega}\right) \omega \sin (\omega t) \tag{11}$$

The value of $\dot{y}$ substitute in Eq. (10)

$$\ddot{y}(t) = -2\omega \dot{x}_0 \cos (\omega t) - 2\omega (3\omega x_0 + 2\dot{y}_0) \sin (\omega t) \tag{12}$$

The expression of $\dot{y}$ and $y$ can be obtained through twice integration in Eq. (1).

$$\dot{y}(t) = -2\omega \dot{x}_0 \sin (\omega t) + 2(3\omega x_0 + 2y_0) \cos (\omega t) \tag{13}$$

The constant of $C$ and $D$ can be determined by considering of time, $t=0$, where $y'(0) = \dot{y}_0$ and $y(0) = y_0$. That is,

$$C = -6\omega x_0 - 3\dot{y}_0; D = -\frac{2y_0}{\omega} + y_0 \tag{14}$$

Consequently,

$$y(t) = \frac{2x_0}{\omega} \cos (\omega t) + \frac{2}{\omega} (3\omega x_0 + 2\dot{y}_0) \sin (\omega t) - (6\omega x_0 +) t - \frac{2\dot{x}_0}{\omega} + y_0 \tag{15}$$

Comparison of the $x$ and $y$ component from the solution of $z$-component because it is uncoupled in Eq. (1) Equation (20) becomes closed form solution by recalling the equation of motion in $z$ component is,

$$\ddot{z} = -\omega^2 Z \tag{16}$$

By taking into the consideration of Laplace transform of the equation is obtained Eqn. (20) as:

$$s^2 Z(s) - sz_0 - \dot{z}_0 = -\omega^2 Z(s) \tag{17}$$

Similarly, the same condition can be followed with respective time derivative and the inverse Laplace transforms’ solution for the $z$-component.

$$z(t) = \frac{\dot{z}_0}{\omega} \sin (\omega t) + z_0 \cos (\omega t)$$

$$\dot{z}(t) = \dot{z}_0 \cos (\omega t) - z_0 \omega \sin (\omega t) \tag{18}$$

The closed form solution stated in Eqs. (10), (11), (15) and (20) shows a linear function with respect to their position and velocities. Thus, it can be expressed in matrices form that are,

$$\vec{r} = \Phi \vec{r}_0 + \Phi \dot{r}_0$$

$$\ddot{\vec{r}} = \Phi \ddot{r}_0 + \Phi \dot{r}_0 \dot{r}_0 \tag{19}$$

where,

$$\Phi = \begin{pmatrix} \Phi_{rr} & \Phi_{rv} \\ \Phi_{vr} & \Phi_{vv} \end{pmatrix}$$


\[
\Phi_r = \begin{bmatrix}
4 - 3 \cos(\omega t) & 0 & 0 \\
6(\sin(\omega t) - \omega t) & 1 & 0 \\
0 & 0 & \cos(\omega t)
\end{bmatrix},
\]

\[
\Phi_{\tau} = \begin{bmatrix}
\frac{1}{\omega} \sin(\omega t) & 2 \frac{1}{\omega} (1 - \cos(\omega t)) & 0 \\
2 \frac{\cos(\omega t) - 1}{\omega} & 1 \frac{4 \sin(\omega t) - 3 \omega t}{\omega} & 0 \\
0 & 0 & \frac{1}{\omega} \sin(\omega t)
\end{bmatrix},
\]

\[
\Phi_{\nu} = \begin{bmatrix}
3 \omega \sin(\omega t) & 0 & 0 \\
6 \omega (\cos(\omega t) - 1) & 0 & 0 \\
0 & 0 & -\omega \sin(\omega t)
\end{bmatrix},
\]

\[
\Phi_{\psi} = \begin{bmatrix}
\cos(\omega t) & 2 \sin(\omega t) & 0 \\
-2 \sin(\omega t) & 4 \cos(\omega t) - 3 & 0 \\
0 & 0 & \cos(\omega t)
\end{bmatrix}
\]

5. Results and Discussion

Figures 3-7 display two test cases with and without the consideration of an atmospheric drag. The simulation results show an improvement in a closed form solution for non-linear solution, and it is remarkable, is relating to counterparts of linear form while doing a separation of spacecraft. The results obtained have been attained through the numerical integration equation (19). In the event of a change the initial condition of eccentricity and inclination in the closed form solution, the linearized solution struggles to replicate the real motion. The results plotted are show in-track and cross track separation in a stipulated time interval. Also, the relative error separation vs. actual separation and angular rate of the target frame has been observed. It is a remarkable solution.

The improvement in the proposed non-linear solution is remarkable. Particular attention is paid to the dynamical state representation, the immediate assumptions on the model, the types of perturbations included, and the metrics on accuracy and computational complexity provided. In the development of these mappings, Koenig et al clarifies that the HCW solution parameterized in terms of the ROE state demonstrates improved accuracy over the translational state formulation considering the relative eccentricity vector retains second-order terms in eccentricity normally dropped in the linearization of the translational state equations of motion. Simulation results presented by Yamanaka and Ankersen [36] for reference orbit with a 500-km perigee altitude and eccentricity values of 0.1 and 0.7 show a strong agreement of their model with the numerical integration, whereas the HCW model completely breaks down, as expected. The following architecture is structured to provide a consistent, repeatable, and rigorous assessment via high-fidelity simulation and for providing a meaningful comparison of several surveyed dynamic models.

A designated Matlab/Simulink library has been developed for implementing the dynamics models in conjunction with a high-fidelity numerical orbit propagator serving as the simulated truth. The models are tested using consistent initial conditions on the reference orbit and the relative motion for a variety of test scenarios designed to highlight the relative strengths and weaknesses of the chosen dynamic models.
Finally, a detailed error analysis has been carried out that provides a one-to-one comparison of the modelling performance against the numerically propagated truth simulation. In addition, it is extended to compare the values with existing dynamic model such as curvilinear HCW model, the quadratic Volterra model [37], the Schweighart-Sedwick model, the Yamanaka-Ankersen STM in [36], the Broucke STM [38], the Gim-Alfriend STM [39, 40], the GAM STM in 2015, the KGD STM [41], the Yan-Alfriend nonlinear theory [42], and the Biria-Russell Vinti method [43]. Noteworthy, after validating the results with all existing model author concluding that, extension of classical linear CW equation works effectively without loss of accuracy up to the range of eccentricity is 0.0001 to 0.7 and changes of few degrees in inclination when to incorporate non-linear equation.

Fig. 3. Linear and non-linear radial separation vs. time with/without drag.

(a) Radial separation vs. time without drag.

(b) Radial separation vs. time without drag.
Fig. 4. Linear and non-linear in-track separation vs. time with/without drag.

Fig. 5. Linear and non-linear cross-track separation with/without drag.
Fig. 6. Linear and non-linear relative error vs. actual separation in proximity operation.

(a) Relative error in separation vs. actual separation.

(b) Relative error in separation vs. actual separation.

Fig. 7. Linear and non-linear angular rate of the target frame with/without drag.

(a) Angular rate of the target frame without drag.

(b) Angular rate of the target frame with drag.

6. Conclusion
A new approach method has been studied using a small eccentricity for the relative motion in a two-body orbit, usually of RSW coordinates with a Taylor expansion. Significantly, identifying the relative motion in the presence of linear and non-linear counterparts can be employed with 0.00158 of the eccentricity and inclination 97.5 degrees of the follower orbit. In addition, the solution provides a compact form between the linear co-ordinates and the follower orbital elements. The extension of classical linear CW equation works effectively without loss of accuracy up-to the range of eccentricity is 0.0001 to 0.7 and changes by a few degrees in inclination when to incorporate non-linear equation.

**Nomenclatures**

<table>
<thead>
<tr>
<th>Symbol</th>
<th>Description</th>
</tr>
</thead>
<tbody>
<tr>
<td>$A$</td>
<td>Satellite effective area, m$^2$</td>
</tr>
<tr>
<td>$a$</td>
<td>Semi major axis</td>
</tr>
<tr>
<td>$C_d$</td>
<td>Drag coefficient</td>
</tr>
<tr>
<td>$F$</td>
<td>External Force, N</td>
</tr>
<tr>
<td>$i$</td>
<td>Inclination</td>
</tr>
<tr>
<td>$J_2$</td>
<td>Gravitational potential co-efficient</td>
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<tr>
<td>$M_s$</td>
<td>Satellite mass, kg</td>
</tr>
<tr>
<td>$M$</td>
<td>Mean anomaly</td>
</tr>
<tr>
<td>$q_1, q_2$</td>
<td>Eccentricity vector</td>
</tr>
<tr>
<td>$R_e$</td>
<td>Radius of Earth (6.3781×10$^6$ m)</td>
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<tr>
<td>$t$</td>
<td>Time, s</td>
</tr>
<tr>
<td>$\Delta V$</td>
<td>Impulsive thrust, Ns</td>
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<td>$x_0$</td>
<td>x component initial relative position, m</td>
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<tr>
<td>$\dot{x}$</td>
<td>x component initial relative velocity, m/s</td>
</tr>
<tr>
<td>$\ddot{x}$</td>
<td>x component initial acceleration, m/s$^2$</td>
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**Greek Symbols**

<table>
<thead>
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<th>Symbol</th>
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<tbody>
<tr>
<td>$\theta$</td>
<td>Argument of latitude.</td>
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<tr>
<td>$\omega$</td>
<td>Argument of perigee</td>
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<tr>
<td>$\mu$</td>
<td>Gravity constant</td>
</tr>
<tr>
<td>$\rho$</td>
<td>Atmospheric density</td>
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**Abbreviations**

<table>
<thead>
<tr>
<th>Abbreviation</th>
<th>Description</th>
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<tr>
<td>CW</td>
<td>Clohessy- Wiltshire</td>
</tr>
<tr>
<td>HCW</td>
<td>Hill’s Clohessy- Wiltshire</td>
</tr>
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<td>ISS</td>
<td>International Space Station</td>
</tr>
<tr>
<td>LEO</td>
<td>Low Earth Orbit</td>
</tr>
<tr>
<td>RPO</td>
<td>Rendezvous and Proximity Operation</td>
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<td>ROEs</td>
<td>Relative Orbital Elements</td>
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**References**


