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INVESTIGATION OF DIFFERENT FAILURE THEORIES FOR A LAMINA OF CARBON FIBER/EPOXY MATRIX COMPOSITE MATERIALS

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Abstract

Unidirectional composite materials are one of the most important materials in many industries and products. It is easy to control their mechanical properties by controlling the orientation of the fibres within the resin. The proper selection of a suitable theory of failure is very important to evaluate the laminated of this type of composite materials. In this paper, some theories of failure were studied for a lamina consists of unidirectional carbon fibres immersed in epoxy. A comparison between theoretical and experimental tensile strength was conducted for many prepared samples in order to select a more accurate theory to be used with this specific type of materials. It was found that the Tsai-Hill theory of failure achieved the best convergence between the theoretical and experimental results compared with other theories of failure. The average deviation between theoretical and experimental results was about (17.9%) in case of using Tsai-Hill theory. This deviation was the minimum compared with other theories of failure. The maximum deviation had occurred when using the Tsai-Wu theory, which was about (69.1%). Therefore, Tsai-Hill theory is more suitable than other failure theories in case of analysis of carbon fibre-epoxy composite materials.

Keywords: Carbon fibres, Composite materials, Lamina, Laminates, Theories of failure.

1. Introduction

Because of their unique features, composite materials play a major role in today's technology and have used extensively over the past four decades [1]. The significant low weight to strength ratio, the good corrosion and fatigue resistance in addition to the simple assembly requirements are among the main advantages that have contributed to the widespread use of composite materials.

Unidirectional fiber-reinforced CM are consisting of fibres embedded in a resin in a way of fitted matrices to obtain high strength and higher elastic moduli. The strength of the resulting CM will be affected by the transiting force from the matrix to the fibres. Many factors may affect this transition force like the relative alignment between fibres orientation and the externally applied load, properties of fibres in addition to the interfacial bonding between the fibres and matrix [2].

Composite materials offer a great weight reduction in products especially when we are talking about the monolithic structural materials. If they are properly designed, CM may represent the ideal engineering material for many industrial tasks and products. Many fibres are use nowadays in producing CM. One of the most popular fibres used is Carbon Fibre (CF). The high performance of polymer composite using carbon fibres as a strengthening material makes them suitable for many industrial products especially in sporting goods, automotive industry and structural components [3]. The higher strength related to a given weight is the goal of using fibre-reinforced CM. Nowadays, a very low-density fibre and matrix materials are used to produce high specific strengths and moduli CM [4].

According to Bere and Krolczyk [5], it is well known that polymeric reinforced CM were successful uses to replace the traditional metallic materials. The multi-axial multi-ply fabric (MMF) composite materials have high-performance comparing with other kinds of CM and therefore, they are becoming increasingly popular as reinforcing materials [6]. The theoretical failure analysis of composite materials of more than one layer involves some complexity. However, it depends primarily on the proper choice of the failure theory appropriate for the specific materials, which the composite made from. From this point of view, this research is concerned with reviewing different theories of failure and comparing the results of these theories with the experimental results of tensile testing of samples made from carbon fibre-epoxy composite with different fibre angles for selecting the appropriate failure theory for this type of material.

2. Reviewing Failure Criteria of Composite Materials

Many failure criteria were developed for unidirectional fibres CM over the last four decades. Micromechanical and macro-mechanical characteristics had been investigated extensively to build a clear view of the failure in CM. Micromechanical analyses deal with the mechanisms of failure and consider the loading type and the properties of the matrix and reinforcements. Although the micromechanical failure criteria are accurate in predictions of failure initiation at critical points, their accuracy will be less regarding the global failure of a lamina. It will be only an approximated prediction when we talk about the failure of laminated CM. For these reasons, and to be more accurate in prediction of the global failure of a lamina, macro-mechanical approach is preferred [7].

Some theories by Daniel and Ishai [8] had been proposed to predict the failure of the composite materials. These proposed theories of failure are classified into three groups; non-interactive or limit theories (maximum stress theory, maximum strain theory); interactive theories (Tsai-Wu theory, Tsai-Hill theory); and failure mode based or partially interactive theories (Hashin-Rotem, Puck). Some on theories of failure, like the maximum stress and strain theories, distinguish between the tensile and compressive strengths in their equations. While other theories, such as Tsi-Hill theory, does not distinguish between the tensile and compressive loading but focused on the interaction between layers of CM. The applicability and validity of any of the given theories depend on the nature of the application and the degree of agreement with experimental results. Unfortunately, there is a dearth of reliable experimental data, which can have considered when examining the suitability of each theory to any selected CM.

Studying failure of CM has two branches, the first one deals with the prediction of the single laminae failure while the other branch deals with prediction of first ply-failure and the progression of damage in case of studying a multi-directional laminate. Some researchers compared theoretical failure predictions and the experimental results despite the lack of research in this field [9]. Usually, theoretical failure predictions of a single lamina are the base for failure predictions for multi-layers or laminated CM. According to that, the proper selection for the theory of failure for a lamina of any CM, which will make failure prediction for the laminated of the same CM be more accurate.

2.1. Maximum stress failure theory:

This theory is similar to the maximum shearing stress theory by Tresca and the maximum normal stress theory by Rankine, which are applied to isotropic materials. Normal and shear stresses in the local axes can be obtained by resolving the stresses acting on a lamina. Using local axes of the lamina, if any of the shear or normal stresses equal to or exceeds the corresponding ultimate strengths related to the unidirectional lamina, failure will be predicted for that lamina. If the stresses or strains in the global axes of the lamina are given, it can be predicted that the lamina will be failed if one of the following criteria is violated [10]:

$-(\sigma_1^C)_{ult} < \sigma_1 < (\sigma_1^T)_{ult}$, or

$-(\sigma_2^C)_{ult} < \sigma_2 < (\sigma_2^T)_{ult}$, or

$$-(\tau_{12})_{ult} < \tau_{12} < (\tau_{12})_{ult} \tag{1}$$

2.2. Maximum strain failure theory

This theory is based on the Tresca maximum shear stress theory and the maximum normal strain theory by Venant for isotropic materials. The global strains applied to a lamina must be resolved towards local axes. In case of equity between the local shearing or normal strains and the corresponding unidirectional laminates ultimate strains, it can be predicted that failure will occur in a lamina. The strains in the local axes can found when the strains/stresses in an angle lamina are given. A lamina considered to fail if one of the following criteria is violated [10].

$-(\varepsilon_1^C)_{ult} < \varepsilon_1 < (\varepsilon_1^T)_{ult}$, *or*

$-(\varepsilon_2^c)_{ult} < \varepsilon_2 < (\varepsilon_2^T)_{ult}$, *or*

$$-(\Upsilon_{12})_{ult} < Y_{12} < (\Upsilon_{12})_{ult} \quad (2)$$

2.3. Tsai–Hill failure theory

This theory based on the distortional energy yield criterion of Von-Mises for isotropic materials. Generally, strain energy consists of two parts; one called dilation energy, which related to the change in volume and the other, related to the change in shape, which called distortion energy. Despite that the distortion energy is a part of the strain energy, however, prediction of failure can base on. It can be said that failure will occur if the distortion energy will be greater than the specific materials failure distortion energy.

Von-Mises distortional energy criterion adopted by Hill to applied with anisotropic materials and then Tsai adapted this idea to a unidirectional lamina. He proposed that a lamina would be considered failed if the following equation is adopted [10]:

$$\left[\frac{\sigma_1}{(\sigma_1^T)ult}\right]^2 - \left[\frac{\sigma_1\sigma_2}{(\sigma_1^T)^2 ult}\right] + \left[\frac{\sigma_2}{(\sigma_2^T)ult}\right]^2 + \left[\frac{\tau_{12}}{(\tau_{12})}\right]^2 = 1 \quad (3)$$

2.4. Tsai-Wu failure theory

The total strain energy failure theory of Beltrami is the base of the Tsai-Wu failure theory since Tsai-Wu applied this failure theory to a lamina under the action of plane stress. This theory takes into accounts if the strength of the lamina is tension or compression and can be considered more general than the Tsai-Hill failure theory. According to this theory by Li et al. [11], a lamina considered to fail if the following criterion is violated:

$$H_1\,\sigma_1 + H_2\,\sigma_2 + H_6\,\tau_{12} + H_{11}\,\sigma_1^2 + H_{22}\,\sigma_2^2 + H_{66}\,\tau_2^2 + 2\,H_{12}\,\sigma_1\,\sigma_2 < 1 \quad (4)$$

The parameters H_1, H_2, H_6, H_{11}, H_{22}, and H_{66} mentioned in the previous equation can found by using unidirectional laminae's strengths parameters were [10]:

$$H_1 = \frac{1}{(\sigma_1^T)ult} - \frac{1}{(\sigma_1^c)ult}, \qquad H_{11} = \frac{1}{(\sigma_1^T)ult\,(\sigma_1^c)ult},$$

$$H_2 = \frac{1}{(\sigma_2^T)ult} - \frac{1}{(\sigma_2^c)ult}, \qquad H_{22} = \frac{1}{(\sigma_2^T)ult\,(\sigma_2^c)ult},$$

$$H_6 = 0,$$

$$H_{66} = \frac{1}{(\tau_{12})^2 ult} \quad (5)$$

On the other hand, the parameter H_{12} can be estimated using some empirical suggestions as follows [10]:

$$H_{12} = -\frac{1}{2(\sigma_1^T)^2 ult}, \text{ per Tsai-Hill failure theory} \quad (6)$$

$$H_{12} = -\frac{1}{2(\sigma_1^T)ult\,(\sigma_1^c)ult}, \text{ per Hoffman criterion} \quad (7)$$

$$H_{12} = -\frac{1}{2}\sqrt{\frac{1}{(\sigma_1^T)ult\,(\sigma_1^c)ult\,(\sigma_2^T)ult\,(\sigma_2^c)ult}}, \text{ per Mises-Henchys criterion} \quad (8)$$

3. Experimental procedures

3.1. Materials

The goal of experiments in this study is to determine the tensile strength characteristics of carbon fibre-epoxy CM. Epoxy resin of type (quick mast 105) was used as a matrix during preparing samples with a mixing weight ratio of (30%), which were recommended according to manufacturer instructions. Unidirectional carbon fibres, which are fabricated by different angles (0^o - 90^o) were uses as a reinforcement material. Many samples were fabricated and prepared for tensile testing and each sample was prepared with a specific fibre angle.

3.2. Samples preparing technique

To obtain the required composite samples, vacuum bag technology used as shown in Fig. 1. Initially, a good impregnation of the reinforced material inside the resin should be satisfied using wet technology and hand lay-up process.

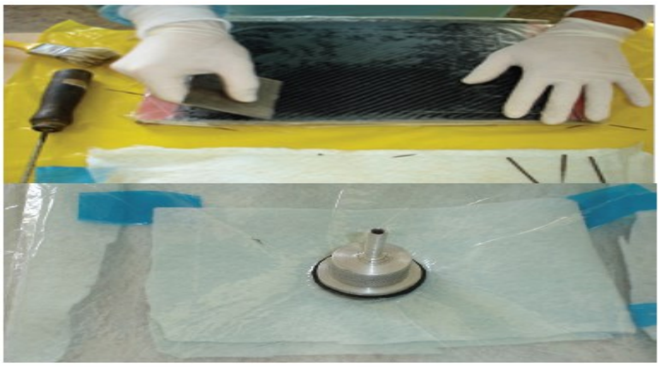
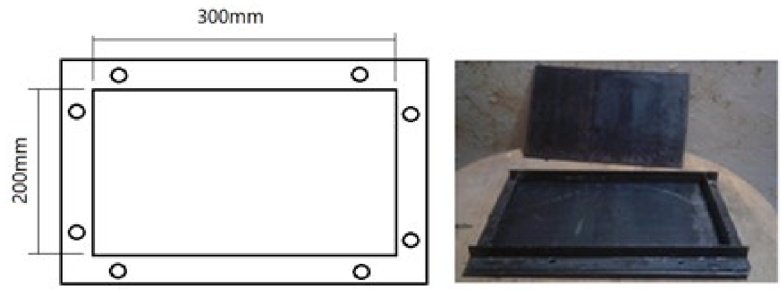


Fig. 1. Composite plates manufacturing process.

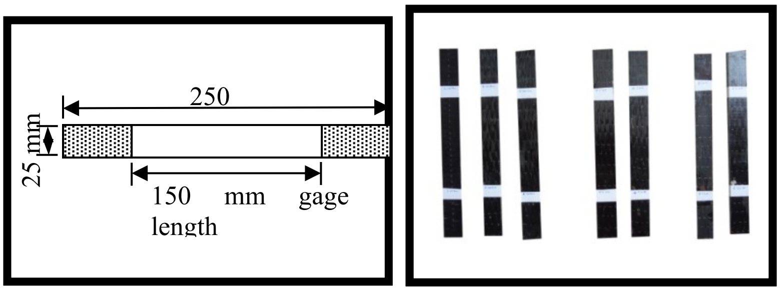
About thirty tensile testing samples were prepared; every three samples have the same angle of fibres orientation. The first three samples have a zero as an angle of fibres orientation, the next three groups have an angle of 10^o and so on till the last group, which have been prepared with a fibre orientation angle of 90^o. The fibres impregnation process is done by using a metal mould shown in Fig. 2. To prevent any bonding between the moulded materials and mould surface, the mould moisturized by a slight amount of grease as a moisture agent. After achieving good impregnation of the carbon fibre with the resin inside the metal mould, a vacuum bag foil was used to cover all prepared layers and subjected to a vacuum pressure of (-0.9 Bar). Vacuum environment is necessary to eliminate the air bubbles from composite materials and will be useful to eliminate any excessive resin. In addition, the vacuum can be very useful to create very thin and uniformed composite plates. The polymerization process is also required and done using an oven at (80 ^{o}C) for 8 hours.

Thirty samples were cut from the prepared plates for tensile testing and every three samples were cut with a specific unidirectional fibres angle of orientation. The dimensions of the cut samples shown in Fig. 3, which were select according to (ASTM) standard. During the tensile testing, all samples were loaded until failure with constant loading speed of (3 mm/min) using a WDW-200 E III hydraulic tensile test machine (100 KN) testing machine. All tests conducted at room temperature (20 °C) and humidity in the range of 45-50%.



(a) Schematic diagram. **(b) Real image of mould.**

Fig. 2. The metal mould used for fibres impregnation process.



(a) Schematic specimen. **(b) Samples of CM specimens.**

Fig. 3. Sample dimension according to the ASTM D3039 standard.

4. Results and Discussion

For many selected angles of laminae, a comparison study between the output of the various failure theory and some experimental results has done by Kolios and Proia [12]. Those comparisons conducted by considering an angle lamina loaded by a uniaxial load in the *x*-direction, σ_x, as shown in Fig. 4.

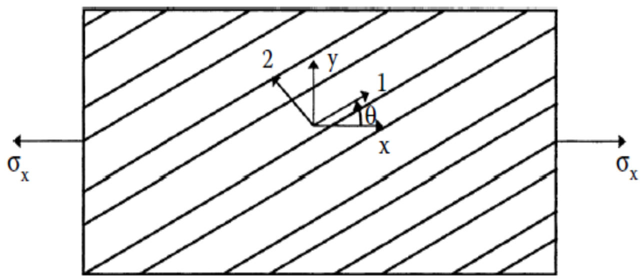


Fig. 4. Unidirectional loading in *x*-direction [10].

This comparison conducted after calculating σ_x in terms of the local stresses using the following Eq. [10]:

$$\sigma_1 = \sigma_x \cos 2\Theta$$
$$\sigma_2 = \sigma_x \sin 2\Theta$$
$$\tau_{12} = -\sigma_x \sin\Theta \cos \Theta \tag{9}$$

The corresponding local strains are as follows [10]:

$$\varepsilon_1 = (1/E_1)\,(\cos 2\Theta - \upsilon_{12} \sin 2\Theta)\,\sigma_x,$$
$$\varepsilon_2 = (1/E_2)\,(\sin 2\Theta - \upsilon_{21} \cos 2\Theta)\,\sigma_x$$
$$\gamma_{12} = -(1/G_{12})\,(\sin\Theta \cos\Theta)\,\sigma_x \tag{10}$$

Using local stresses and strains given by Eqs. (1) to (8), the ultimate allowable load σ_x can found as a function of fibres orientation. The micromechanical analysis used to find the required stiffness's and strengths of the unidirectional lamina to use during applying the main theories of failure and they found to be as following:

E_1 = 72.266 GPa, E_2 = 6.61 GPa v_{12} = 0.325, G_{12} = 2.451 GPa,

$(\sigma_1^T)_{ult}$ = 401.3 MPa , $(\sigma_1^c)_{ult}$ = 63.4 MPa, $(\sigma_2^t)_{ult}$ = 11.72 MPa,

$(\sigma_2^c)_{ult}$ = 21.8 MPa, $(\tau_{12})_{ult}$ = 31.72 MPa

All of these characteristics calculated using weight fraction ratio of 30%, which had been adopted during the experimental work.

Theoretical results, which was obtained using the Matlab package, show that according to Tsai-Hill and Tsai-Wu failure theories, the tensile strength will smoothly change as the angle of fibres inclination is increased. On the other hand, this changes in the tensile strength will contain two regions separated by a sharp region if the maximum stress and maximum strain failure theories are adopted as can be noticed through Fig. 5. This phenomenon is due to the change in the failures modes in case of using the maximum stress and the maximum strain theories of failure. With using small fibres orientation angle, failure will be occurring due to fibre failure. As such angle increase, the shear criterion will be the main mode of failure. Finally, transverse (either matrix or interface) failure mode will be noted with large fibre orientation angles as they approach 90°. Despite this disadvantage, the more frequently used failure criteria are the maximum strain and maximum stress failure criteria. This is because they are easy to use and simple to understand [13].

It is also clear from the figure that there is a great convergence of results between Tsai-Hill and maximum stress theories of failure. This is because the specific failure modes predicted by each of those theories are similar to each other. Both theories are based on comparing the individual laminae stresses with the corresponding strength of them [14]. The average experimental results for the tensile test for every angle of fibres orientation listed in Table 1. Each result printed in this table represents an average of three experimental readings. As it is expected, the tensile strength for any unidirectional fibres CM will increase when the direction of the externally applied load approaches to the direction of the fibres within CM.

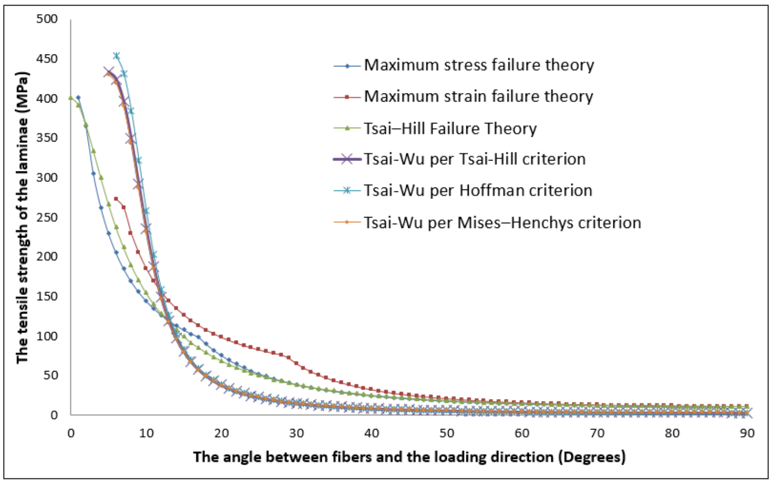


Fig. 5. Theoretical schematic diagrams for the main theories of failure for carbon fibre-epoxy CM.

Table 1. Experimental results for the tensile strength of the tested samples.

No. of sample	Inclination of fibres (degrees)	Tensile strength (MPa)
1	0	410.1
2	10	108.1
3	20	79.4
4	30	36.4
5	40	29.4
6	50	25
7	60	20.8
8	70	17.3
9	80	12.8
10	90	5.1

Figure 6 presents the stress-strain diagrams for the first five tested samples. The orientation of fibres within each sample printed as shown. It can be noted that the behaviour of the first two tested samples, which were the strongest among other samples showed an elastic behaviour.

The remaining three tested samples did not show clearly this elastic behaviour, and this was probably caused due to a failure occurred in the bonding between fibres and the resin. Therefore, it can be said that this elastic behaviour is typical for the fibres, which is the reinforcing material. This phenomenon will always be very clear whenever there is converges between the direction of the externally applied load and the direction of the fibres during the test.

Figure 7 shows the stress-strain diagrams for the remaining five tested samples, which their fibres were oriented by (50, 60, 70, 80 and 90) degrees. Again, elastic behaviour appeared, however, when fibres directions diverged with the direction of the applied load. This elastic behaviour can be expected for the matrix, which was epoxy when a small fraction of the load will be carried by fibres.

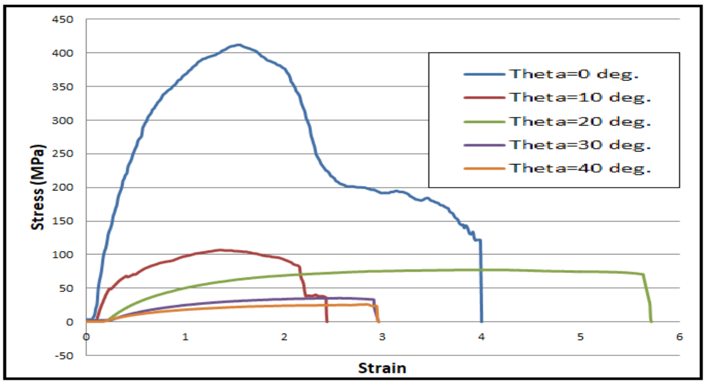


Fig. 6. Stress-strain diagrams for the first tested group of carbon fibres-epoxy cm.

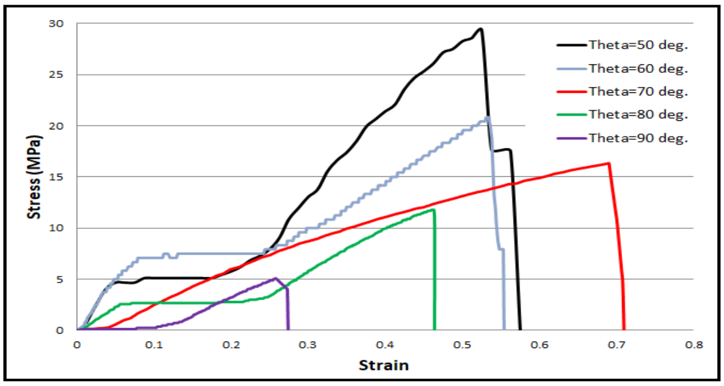


Fig. 7. Stress-strain diagrams for the second tested group of carbon fibres-epoxy CM.

The experimental results listed in Table 1 were compared with the theoretical values of the strengths corresponding to the same angles of inclination of fibres within the composite material. The average deviations of the output are calculated between the theoretical and experimental results of each theory of failure. It found that the Tsai-Hill theory of failure achieved the best matching between the theoretical and experimental results of Fig. 8 followed by the maximum stress theory and then the maximum strain theory. Figure 9 clearly shows how well the theoretical and experimental results are matching each other using Tsai-Hill theory of failure. All stress components are included in one expression. Therefore, the predicted failure will represent the overall failure in case of using Tsai-Hill theory rather than the particular failure mode in case of using the limit or non-interactive theories like maximum stress and maximum strain failure theories [14].

On the other hand, Tsai-Wu theory of failure failed in achieving acceptable conformity between theoretical and experimental results for the carbon fibre-epoxy composite material. It is recommended to study this failure theory using other different composite materials and compare its experimental results with the theoretical outputs of this theory. It is recommended also to study the partially interactive theories like Hashin's failure criteria and puck theory.

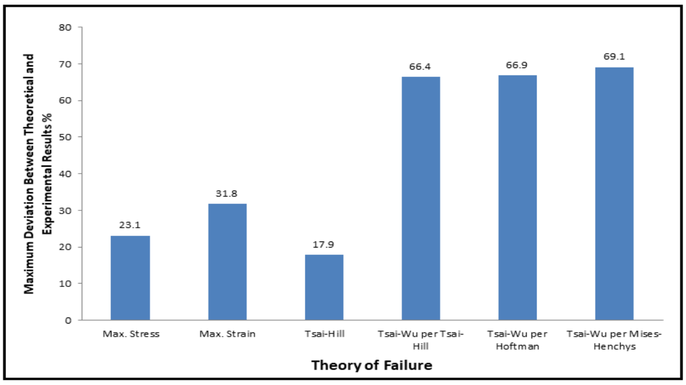


Fig. 8. Percentage deviation between theoretical and experimental result.

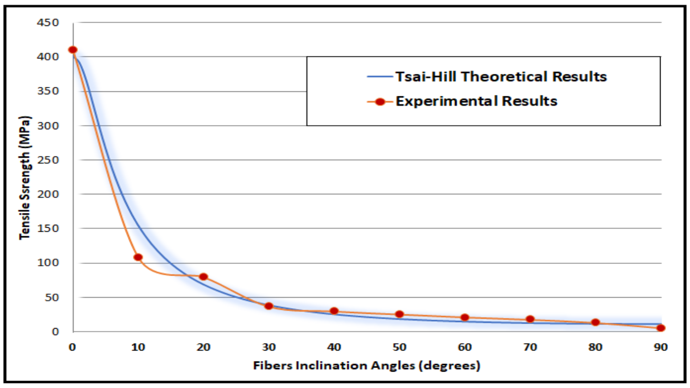


Fig. 9. Convergence between theoretical strengths and the outputs of Tsai-Hill failure theory.

5. Conclusions

Some concluding observations from the investigation are given below.

- The differences between the theoretical results of Tsai-Wu failure theory and the experimental results for carbon fibre-epoxy CM quite pronounced even when using different criteria for calculating some parameters of this theory. It was recommended to exclude this theory when dealing with these specific materials.
- Tsai–Hill and the maximum stress failure theories' outputs are in good agreement with the obtained experimental results regarding carbon fibre-epoxy CM.
- The relationship between strength and the orientations of fibres within the resin found to be more smooth with using both Tsai-Hill and Tsai-Wu theories while it was less smooth in case of maximum stress theory and the worst case was noted when using the maximum strain theory.

Nomenclatures

(Y_{12})	In-plane shear strain (in-plane 1-2)

$(\gamma_{12})_{ult}$	Ultimate in-plane shear strain (in-plane 1–2)
Greek Symbols	
ε_1	Local longitudinal strain (in direction 1)
ε_2	Local transverse strain (in direction 2)
$(\varepsilon_1^c)_{ult}$	Ultimate longitudinal compressive strain (in direction 1)
$(\varepsilon_2^c)_{ult}$	Ultimate transverse compressive strain (in direction 2)
$(\varepsilon_1^T)_{ult}$	Ultimate longitudinal tensile strain (in direction 1)
$(\varepsilon_2^T)_{ult}$	Ultimate transverse tensile strain (in direction 2)
σ_1	Local stresses (in direction 1), MPa
σ_2	Local stresses (in direction 2), MPa
$(\sigma_1^c)_{ult}$	Ultimate longitudinal compressive strength (in direction 1), MPa
$(\sigma_1^T)_{ult}$	Ultimate longitudinal tensile strength (in direction 1), MPa
$(\sigma_2^c)_{ult}$	Ultimate transverse compressive strength (in direction 2), MPa
$(\sigma_2^T)_{ult}$	Ultimate transverse tensile strength (in direction 2), MPa
τ_{12}	In-plane shear stress, MPa
$(\tau_{12})_{ult}$	In-plane ultimate shear strength, MPa
Abbreviations	
CF	Carbon Fibre
CM	Composite Material

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