MATHEMATICAL MODELLING AND EXPERIMENTAL INVESTIGATION OF TWO-TANK VALVE-LESS IMPEDANCE PUMPING SYSTEM

SHARSAD KARAKUNI1,*, HAIDER ABBAS MOHAMMAD2, YOUSIF ABDALLA ABAKR1

1Department of Mechanical, Materials and Manufacturing Engineering, University of Nottingham, Jalan Broga, Semenyih, Selangor, Malaysia 43500, 2Department of Electrical and Electronic Engineering, University of Nottingham, Jalan Broga, Semenyih, Selangor, Malaysia 43500 *Corresponding Author: sharshad.kk@gmail.com

Abstract

When two tanks open at one end are connected by an elastic tube, a net flow can be generated by periodical excitation at asymmetrical ends of a flexible tube. A simple yet reliable mathematical modelling technique is proposed to represent the essential features of pumping systems based on impedance mismatch in general. Modelling algorithm based on energy balance over small lumps of fluid regions defined for the system is outlined for deriving a set of ordinary differential equations for fluid velocity flux. The developed algorithm is applied for the case of Two-tank model studied in earlier mathematical models and physical experiment studies. Parameters investigated include pinch frequency, pinch location, and physical parameters of a flexible tube. Using the model derived, phase synchronization between external pumping pressure and fluid flux at the junction between the elastic tube and tanks is identified as a key factor in determining the direction of net power. In addition, it has been clear that the magnitude of net power is proportional to the marginal fluid flux amplitudes. With larger elastic tube radius and smaller tube thickness, a substantial increase in net flow generated has been achieved at lower external pumping frequency. The numerical model further validated using experimental simulation is shown to well describe the fundamental flow characteristics of open tank Impedance pumping. Due to its simplicity, modelling technique could be promising to be extended for the system with more than two tanks and even more complicated structures such as in coronary blood flow where branching and varying geometries are present.

Keywords: Energy conservation, Impedance, Multi-stage, Net flow rate, Two-tank, Valve-less, Velocity flux.
1. Introduction

Two-tank impedance pumping refers to a system consisting of two rigid tanks with one open end connected using a valve-less elastic tube. In the case of closed-loop of impedance, pumping, external pinching causes a net flow in the preferential direction, while it can store the potential energy under gravity in an open tank system [1]. Valve-less pumps based on impedance mismatch are of great promise to lab-on-chip, biomedical, microfluidic applications and in understanding the dynamics of cardiovascular blood circulation [2-4]. Previous works show a complex non-linear behaviour of the flow in response to the compression frequency, including distinct resonance peaks and reversals in the flow direction. The valve-less impedance pumping, in general, has been investigated by numerous analytical, numerical and experimental studies since Liebau [5] pioneer work in 1954.

McQueen and Peskin [6] in 2000 developed a formally second-order accurate immersed boundary method in order to numerically simulate blood flow in a three-dimensional model of human heart based on impedance pumping phenomenon. Jung and Peskin [7] first introduced by in 2001, the dominant role of the wave dynamics in the pump using immersed boundary method, which is applicable to problems involving an elastic structure interacting with a viscous incompressible fluid. In 2003, Ottesan [8] modelled the closed-loop system using a one-dimensional theory by averaging and ignoring the higher-order terms in ‘Navier-stokes’ equations. In 2003, Borzi and Propst [9] used a more advanced numerical scheme to investigate the same system of equations. They proposed that the nonlinearity of the model equations is not a necessary condition for valve-less pumping to take place.

In 2004, Auerbach et al. [10] came up with a different analytical approach in such a flat tube system, part of which, is inelastic and part of which, constitutes a periodic wave-like motion based on ‘Euler’s equation’ to derive an analytic solution, which yields a finite pumping effect. Manopoulos et al. [11] presented an improved one-dimensional model in 2006 by taking into account the effects of the hydraulic losses due to the stenosis. Jung [12] in 2007 developed a new lumped model for closed-loop impedance system governed by the ordinary differential equations for pressure and flow, with time-dependent compliance, resistance and inertia. Avrahami and Gharib [13] in 2008 conducted a more advanced numerical FSI (fluid and solid interaction) study, in which, was further revealing that the main driving mechanism of the flow is the reflection of waves at the tube boundary and the wave dynamics in the tube.

Among experimental works, Hickerson et al. [14] contributions are at the forefront since she published her works in 2005. In particular, it was observed that the net flow is highly sensitive to the pinching frequency and the closed-loop is not a requirement to sustain pressure head. They continued to propose a simple 1-D ‘wave-pulse model’ accounting for wave amplitude attenuation and reflection at impedance-mismatched interfaces in an impedance pump. In case of an open valve-less pumping system, which consists of fluid tanks and the connecting elastic tubes or rigid pipes, there are marked results using numerical simulations by Timmermann and Ottesen [15] in 2009. Rosenfeld and Avrahami [16] in 2010 work on the effects of sequential excitations on a single elastic tube in which, showed promising results where an increase in flow rates was observed.
Lee et al. [17] in 2011 published their work on the fluid behaviour for an open-loop multi-stage system and showed that the pressure head of the multi-stage system is double that of the two-tank system using experimental results. Similar observations have been revealed using numerical simulation of our model. In 2013, Lee et al. [18] developed a two-stage impedance system and showed that the flow may significantly improve while maintaining the same energy consumption. In his recent works, Lee et al. [19, 20] and Lee and Chong [21] showed that the middle reservoir between the terminal tanks has been shown to play a major role in the multi-stage system, serving as the driving mechanism inflow enhancement, which agrees with the findings of current work.

Motivated by Jung’s compartment model Jung and Kim [1], a system of first-order ordinary differential equations based on energy balance is developed. Even though a similar technique to that of Jung and Kim [1] has been used; through the application of energy equations based on fluid velocity flux; a simpler form of energy model has been achieved. Using ECCM, they were able to show that the direction and magnitude of the impedance pump system depend on the external pumping frequency. Yet, we intend to further investigate the same cause using our model hoping to contribute towards achieving full control of the direction and magnitude of net flow based on external pumping and the nature of such dependency in predicting the direction of net flow has been revealed.

2. Mathematical modelling of two-tank impedance pumping

A mathematical model has been developed for deriving the set of fluid flux equations to describe impedance pumping system. Modelling technique considers a uniform flow region of small lumps of fluid by dividing the pumping system called ‘compartments’ [1]. Principle of energy balance is then applied over the chosen control volume to derive fluid velocity flux equations for each compartment. An algorithm describing this methodology, which could be applied in general for modelling all such impedance-driven system has been illustrated using a flow chart in Fig. 1.

To derive the equations governing the flow in an open valve-less pump system, compartment model as described in Jung and Kim [1] ECCM model is developed as in Fig. 2. Two tanks $C_L$ and $C_R$ with one end open have cross-sectional areas $A_L$ and $A_R$ respectively are connected through a valve-less flexible tube. The tube has a finite wall thickness and preserves its volume under deformation. Both tanks have rigid walls and have fluid levels at height, $H_L(t)$ and $H_R(t)$ at a time ‘$t$’. The fluid contained in the tanks is affected downwards by gravity. An incompressible fluid fills the entire system and the flow of fluid is generated by external pinching at the asymmetrical position of the elastic tube.

Throughout this modelling, $L$ and $R$ as subscripts mean the correspondence to the left and right tank. Flexible tube of length $L$ is divided into ‘$n$’ small compartments, $C_i$ of length $l$. $C_i$ denotes the $i^{th}$ compartment where $i = 0, 1, 2\ldots n - 1$, ‘$n$‘ being any positive integer. Likewise, $P_i$ represents the pressure of fluid in the compartment $C_i$; $v_i$ is the fluid velocity flux between $C_{i,j}$ and $C_{i,j}$. $A_i$, $V_i$, $r_i$, and $h_i$ are area, volume, radius and thickness of corresponding tube compartments respectively. Particularly $v_{i0}$ is flux between $C_L$ and $C_0$; $v_n$ is flux between $C_{n-1}$ and $C_R$. Also, subscript ‘$d$’ represents the value of the corresponding elastic tube parameter at the initial state prior to the flow occurs. The flow between two points occurs as a
result of a change in the total energy of the points. In order to derive the mathematical model describing the system, we have to consider three different tube positions for flux change to occur. Fluid flux ‘\(v_i\)’ inside the flexible tube, flux ‘\(v_0\)’ between left tank and tube outlet \(C_0\) and flux ‘\(v_n\)’ between tube inlet \(C_{n-1}\) and right tank.

**Step 1:** Describe fluid compartments within the system
- Small fluid region defined in space characterized by a specific uniform flux at any given time

**Step 2:** Define control volume for each region of flux change
- Define control volume over a region of flux change to include all forces acting on the fluid

**Step 3:** Identify rate of work done on the fluid by those forces
- For a control volume fluid, find rate of change of pressure, potential, dissipative and kinetic energies

**Step 4:** Apply Energy conservation principle
- Apply energy balance using the identity: rate of change of \((\text{pressure} + \text{potential} - \text{dissipative} = \text{kinetic})\)

**Step 5:** Find the system of ODE’s describing the system
- Solve the energy balance equation for fluid velocity and repeat from step 2 if more regions of flux change are present

Fig. 1. Flow chart illustrating algorithmic steps in numerical modelling.

Fig. 2. Schematic of compartment model for two-tank system.

### 2.1. Deriving the flux equations inside the tube compartments

In this section, we derive an expression to describe the velocity of fluid flow inside the flexible tube. Let define the control volume as shown in Fig. 3. The \(i^{th}\) control volume is defined to occupy the right and left half of the compartments \(C_{i-1}\) and \(C_i\) respectively where ‘\(i\)’ represents any positive integer from 1 to \(n-1\); \(n\) being the total number of compartments considered. Here we define fluid velocity ‘\(v_i\)’ as fluid flux at the junction of compartments \(C_{i-1}\) and \(C_i\).
This allows the freedom to make the assumption that the volume deformation occurs inside the compartments and flow occurs at a point between two compartments, which is not affected by volume change at any time. This assumption simplifies the resulting model without losing the generality. The flow velocity of a fluid is a vector field, which gives the velocity of an element of fluid at any given position and time. Flux is defined as the time derivative of fluid volume through a given surface area or in other words the surface integral of fluid velocity. In this literature, the flow velocity vector of the incompressible fluid in the unit surface area is referred to as fluid flux, velocity flux or simply flux without any distinction.

Fig. 3. Control volume for deriving fluid velocity inside the tube.

In the absence of external pressure, the pressure at any point in a flow-through the elastic tube is the pressure by which, the fluid is exerting on the tube wall. In other words, internal fluid pressure ($P_i$) at any point is the transmural pressure ($P'_i$) at that point when no external pressure is present [22]. The total pressure generated is used to change the tube volume due to the elastic properties of the structure and part of the pressure energy is dissipated in the process of overcoming the resistive characteristics of the fluid flow [23]. The elastic potential energy stored is then converted to kinetic energy, which accelerates the fluid in the direction of flow.

Rate of work done by fluid pressure inside the compartments can be equated to the rate of change of kinetic energy of fluid and the rate of dissipative friction energy. Work is done in a positive direction by $P_i$ on the fluid in control volume considered and in the negative direction by pressure $P_i$. For pipe with the internal cross-sectional area $A_d$, we approximate the viscous resistance parameter due to fluid viscosity $\mu$ inside the compartment of length $l$ using Hagen-Poiseuille laminar flow as $R=8\pi\mu l/A_d^2$ [1, 2]. Also, assuming the fluid to be incompressible, an expression can be derived for modelling the fluid flux as shown below. Here, expression $V'$ denotes the time derivative $dV/dt$ of the variable $V$.

$$\left( P'_iA_{i-1} - P'_iA_i \right) v_i = \rho l v'_i A_d v_i + 8\pi \mu l v_i^2$$

(1)

Cancelling out like terms from the equation, we have:

$$P'_iA_{i-1} - P'_iA_i = \rho l A_d v'_i + 8\pi \mu l v_i$$

(2)

Note that due to the previous assumption in defining control volume, only terms associated with the pressure are affected by volume deformation. Hence, the flux equation for compartments $i=1:n-1$ can be derived as

$$v'_i = \frac{1}{\rho l^2 A_d} (P'_i v_{i-1} - P'_i v_i - 8\pi \mu l^2 v_i)$$

(3)

The negative sign in the equation shows that viscosity opposes the fluid flow.
2.2. Deriving the flux equations at the ends of flexible tube

Consider the flux, \( v_0 \) and \( v_n \), at the bottom outlet of the tanks to the elastic tube. Two rigid tanks are connected to the compartments \( C_0 \) and \( C_{n-1} \) at the end of the tube. Left tank denoted as \( C_L \) and right tank denoted as \( C_R \) have fluid levels \( H_L(t) \) and \( H_R(t) \) at any time \( t \) respectively. As illustrated in Fig. 4, the control volumes for these cases are defined to contain half of the marginal compartments \( C_0 \) and \( C_{n-1} \) along with left and right tanks \( C_L \) and \( C_R \) respectively in each case. The viscous dissipation of fluid inside the rigid tanks is assumed to be negligible compared to the inertial force of fluid and hence, can be ignored.

\[
\frac{1}{2} \rho g H_L + P_{atm} A_L \nu_L - (P_e) A_L \nu_L = \frac{d}{dt} \left( \frac{1}{2} \rho A_L H_L \nu_L^2 \right) + 8 \pi \mu \left( \frac{1}{2} \nu_L^2 \right) + \rho A_L \frac{1}{2} \nu_L \nu_L' 
\]

(4)

It is important to recognize that the positive direction of flow is defined from left to right for the elastic tube and top to bottom inside the tanks. The first term on the left side represents the pressure energy of fluid inside the left tank, which is acting in the positive direction of the fluid flow. The atmospheric pressure, \( P_{atm} \) can be set to zero since it plays no role. The fluid pressure inside the compartment \( C_0 \) acts in the opposite direction of flow towards the left tank. In the right-hand side, the first term denotes the kinetic energy of fluid inside the left tank and the second and the third term accounts for viscous dissipative force and kinetic energy of fluid associated with the left half of the compartment \( C_0 \) respectively. \( A_L, H_L \) and \( \nu_L \) stands for tank cross-sectional area, fluid level and velocity of fluid inside the left tank respectively. Likewise; for the right tank, we write the same equation as

\[
\frac{1}{2} \rho g H_R + P_{atm} A_R \nu_R + (P_e) A_R \nu_R = \frac{d}{dt} \left( \frac{1}{2} \rho A_R H_R \nu_R^2 \right) + 8 \pi \mu \left( \frac{1}{2} \nu_R^2 \right) + \rho A_R \frac{1}{2} \nu_R \nu_R' 
\]

(5)

The pressure energy of fluid inside the right tank is acting in the negative direction of fluid flow. The atmospheric pressure \( P_{atm} \) can be set to zero again. The fluid pressure inside the compartment \( C_{n-1} \) exerts on the direction of positive flow, \( A_R, H_R \) and \( \nu_R \) represents tank cross-sectional area, fluid height level and velocity of fluid inside the right tank respectively.

Fluid velocities, \( \nu_L \) and \( \nu_R \) inside the tanks are related to fluid level by:
\[ v_L = -H_L' \; \text{and} \; v_R = H_R' \]  

(6)

Also, by conservation of mass, we have the relationship

\[ A_d v_0 = A_L v_L \; \text{and} \; A_n v_n = A_R v_R \]  

(7)

Hence, kinetic energy possessed by fluid inside the left tank can be evaluated as follows:

\[
\frac{d}{dt} \left( \frac{1}{2} \rho A_L H_L v_L^2 \right) = \rho A_d v_0 \left[ \frac{A_d}{A_L} H_L v_L' - \frac{1}{2} \left( \frac{A_d}{A_L} v_0 \right)^2 \right]
\]  

(8)

Substituting Eq. (8) in Eq. (4) and setting \( P_{atm} = 0 \) gives

\[
\left( \frac{1}{2} \rho g H_L \right) A_d v_0 - \left( P_i' \right) A_d v_0 = \rho A_d v_0 \left[ \frac{A_d}{A_L} H_L v_L' - \frac{1}{2} \left( \frac{A_d}{A_L} v_0 \right)^2 \right] + 4 \pi l v_0 v_L' + \rho A_d \frac{l}{2} v_0 v_L'
\]  

(9)

Finally, the above equation can be solved for \( v_0 \) as:

\[
v_0 = \frac{1}{\alpha_t} \left[ \frac{g H_L}{2} + \frac{1}{2} \left( \frac{A_d}{A_L} v_0 \right)^2 - \frac{1}{\rho A_d} \left( P_i' v_0 + 4 \pi l v_L' \right) \right]
\]  

\[ \alpha_t = \left( \frac{A_d H_L + l}{A_L} \right) \]  

(10)

In a similar fashion, the equation for velocity flux \( v_n \) can be derived as:

\[
v_n = \frac{1}{\alpha_n} \left[ -\frac{g H_R}{2} + \frac{1}{2} \left( \frac{A_d}{A_R} v_n \right)^2 - \frac{1}{\rho A_d} \left( P_i' v_n + 4 \pi l v_n' \right) \right]
\]  

\[ \alpha_n = \left( \frac{A_d H_R + l}{A_R} \right) \]  

(11)

### 2.3. Equations describing two-tank model

In the compartment model of open-tank valve-less pumping, the kinetic energy is apparently affected by the elastic force from the tube, which is applied perpendicularly to the inside fluid in the control volume [1]. Since only the flow in the axial direction is considered in this model, the fluid pressure must be modelled to include the tension force induced by the elastic force in a reasonable manner. Fluid transmural pressure \( P_i' \) inside the tube has been modelled under the assumption that elastic tube in \( C_i \) maintains its cylindrical shape and constant volume with no change in the length \( l \) of compartment \( C_i \) during the deformation of the elastic tube [1]. Under such assumptions, the relationship between the fluid pressure and the elastic pressure induced by the tension of the elastic tube can be derived. The unidirectional pressure along the compartment is given as follows [1].

\[
P_i' = \frac{1}{2l} \left( \frac{E h_i}{r_i + h_i} - P_i' \right) \left( v_i - v_{i+1} \right)
\]  

(12)

The thickness of the pipe is quite small compared to pipe radius and assuming negligible tube mass inside \( C_i \) leads to the identity describing the state equation of
Mathematical Modelling and Experimental Investigation of Two-Tank...

\[ h_i(t) = \sqrt{D_0^2 + r_i^2(t)} - r_i(t) \]  

(13)

Since the fluid is considered incompressible, the volume conservation of fluid can be written as

\[ r_i' = \frac{r_i(v_i - v_{i+1})}{2l} \]  

(14)

A system of ordinary differential equations describing the compartment model for the two-tank system can be addressed with the solutions \( P_i(t), v_i(t), r_i(t), H_L(t) \) and \( H_R(t) \). When external pressure \( P_{ext} \) is applied on the compartment \( C_i \), in the internal fluid pressure inside the compartment is given as the sum of transmural pressure and the external pressure as \( P_i = P_i' + P_{ext} \). Integrating external pressure, the system of ordinary differential equations describing the Two-tank impedance pumping system can be summarized as follows

- The general equation representing flux equations inside the flexible tube is for \( i = 1, 2, \ldots, n-1 \).

\[ v_i' = \frac{1}{\rho A_h} \left[ (P_i' + P_{ext}) v_{i-1} - (P_i' + P_{ext}) v_i \right] - \frac{8\pi \rho v_i}{\rho A_h} \]  

(15)

- Velocity flux equation for terminal tanks can be written as:

\[ v_0' = \frac{1}{2\alpha_L} \left[ gH_L + \left( \frac{A_{dL}}{A_L} v_0 \right)^2 \right] - \frac{1}{2\alpha_L} \left[ (P_0' + P_{ext}) v_0 + 4\pi \alpha_L v_0 \right] \]  

(16)

\[ v_n' = -\frac{1}{2\alpha_R} \left[ gH_R + \left( \frac{A_{dR}}{A_R} v_n \right)^2 \right] + \frac{1}{2\alpha_R} \left[ (P_n' + P_{ext}) v_{n-1} - 4\pi \alpha_R v_n \right] \]  

(17)

\[ H_L' = -\frac{A_d}{A_L} v_0; \quad H_R' = \frac{A_d}{A_R} v_n \]  

(18)

- For \( i = 0, 1, \ldots, n-1 \):

\[ P_i' = \frac{1}{2l} \left( \frac{Eh_i}{r_i + h_i} - P_i' \right) (v_i - v_{i+1}) \]  

(19)

\[ r_i' = \frac{r_i(v_i - v_{i+1})}{2l} \]  

(20)
\[
\begin{align*}
 h_i(t) &= \sqrt{D_i^2 + r_i^2(t) - r_i(t)} \\
 D_i^2 &= \left( r^2_{d} + h^2_{d} \right) - r^2_{d} \\
 V'_i &= \frac{V_i}{l} (v_i - v_{i+1})
\end{align*}
\] (21)

where \( r_{d} \) and \( h_{d} \) denote the original radius and thickness of the pipe respectively.

3. Numerical results on two-tank valve-less pumping

Impedance pumping mechanism generates a directional net flow when an external periodic excitation is applied on the flexible tube. In general, valve-less pumping based on impedance could be addressed using three key flow parameters. One being the pumping frequency of input external pressure applied and the other two variables are direction and magnitude of the resultant net flow [23]. Benefits of valve-less pumping could only be fully exploited once the relation between input pumping frequency and direction and magnitude of flow output is fully understood. The mathematical model derived in the previous section has been able to come up with a less complicated set of ordinary differential equations. Reliability and validity of the model developed will be investigated analytically and physically in the coming sections. Before investigating the effect of pumping frequency on direction and magnitude of net power generated, the model is verified using constant parameters such as pinching location and tube specifications in order to establish previously known results.

Due to the oscillatory flow caused by the external pinching, the level difference \( (H_R(t)-H_L(t)) \) inside tanks fluctuates for a sufficiently long time in both directions. Hence, let us define \( 'hR-hL' \) as the average value of \( (H_R(t)-H_L(t)) \) as a variable for measuring net power to provide a reliable measurement of net energy storage occurs at a specific excitation frequency. Note that, pinching compartment denoted as \( C[1,2] \) represents that external pumping is being applied on compartments \( C_1 \) and \( C_2 \) simultaneously.

Since only cases of equal tank cross-sections are considered, subscript term \( \tau \) is used in conjunction with variables to denote for common tank parameters such as in \( \delta \), where it represents diameter for both tanks. Simulation parameters used in the numerical experiments are listed in Table 1. Initially, when \( P^{ext} \) is zero, the fluid level inside the tanks are set to be equal, therefore, the system is in equilibrium under gravity.

Under equilibrium condition, \( v_i(0) \) is zero and hence, the internal fluid pressure inside the tube caused by the kinetic energy of the fluid is also zero. Since no volume deformation occurs at his stage, initial radius and thickness of the flexible tube is the original radius and thickness of the tube before deformation. External pressure creating from periodic pinching on the flexible tube is integrated in to simulation by employing the following time-dependent non-negative function \[1\].

\[
P^e_{ext}(t) = \begin{cases} 
\frac{P^{ext}}{2}(1 - \cos(2\pi nt)), & \text{for all } k \in \{0,1,...,n-1\}, \\
0, & \text{otherwise}
\end{cases}
\] (23)
Table 1. Values of parameters and initial conditions for simulation.

<table>
<thead>
<tr>
<th>Initial conditions</th>
<th>Values</th>
</tr>
</thead>
<tbody>
<tr>
<td>n (no. of compartments)</td>
<td>20</td>
</tr>
<tr>
<td>(r_0) (initial radius of fluid in (C_1)) (cm)</td>
<td>0.5</td>
</tr>
<tr>
<td>(h_0) (initial thickness of elastic tube in (C_1)) (cm)</td>
<td>0.1</td>
</tr>
<tr>
<td>(P_i(0)) (initial pressure of fluid in (C_1)) (dyne/cm(^2))</td>
<td>0</td>
</tr>
<tr>
<td>(v(0)) (initial flux between fluids in tube) (cm/s)</td>
<td>0</td>
</tr>
<tr>
<td>(E) (young’s modulus) (dyne/cm(^2))</td>
<td>(10^7)</td>
</tr>
<tr>
<td>(g) (gravitational acceleration) (g cm/s(^2))</td>
<td>980</td>
</tr>
<tr>
<td>(\rho) (fluid density) (g/cm(^3))</td>
<td>1.0</td>
</tr>
<tr>
<td>(\mu) (fluid viscosity) (gcm(^{-1})/s)</td>
<td>0.01</td>
</tr>
<tr>
<td>(P_{max}) (Maximum external pressure) (dyne/cm(^2))</td>
<td>1.01325*10^5</td>
</tr>
<tr>
<td>(H_L(0)=H_R(0)) (cm)</td>
<td>20</td>
</tr>
<tr>
<td>(d_L=d_R=d_T) (Tank diameters) (cm)</td>
<td>5</td>
</tr>
<tr>
<td>Tank cross-section type</td>
<td>Circular</td>
</tr>
<tr>
<td>(L) (Length of elastic tube) (cm)</td>
<td>40</td>
</tr>
</tbody>
</table>

3.1. Effect of elastic tube properties on net flow

At tube radius of 0.5 cm, maximum net power is generated at pinching frequency of 10.7 Hz. With 20 percent increase in tube radius at 0.6 cm, the net flow rate has been increased to a value of 1.75 cm with peak frequency shifting towards the left at 8.6 Hz. With tube radius of 0.7 cm, net flow skyrocketed to a maximum value at 4 cm with actuation frequency decreasing to a value of 7.2 Hz. Bar plot in Fig. 5 illustrates that tube radius has been inversely related to peak excitation frequency while generating greater net flow with greater elastic tube size. Figure 6 shows that when tube thickness is decreased from 0.125 cm to 0.10 cm and further to 0.075 cm, the external pumping frequency required to produce maximum net power has been reduced from 11.7 Hz to 10.7 Hz and then 9.5 Hz respectively. At the same time, net flow developed has decreased from 1.3 cm at 0.075 cm thickness to 0.5 cm for a tube thickness of 0.125 cm. Under gravity, assuming no frictional force, similar results have been mentioned in prior literatures for tube physical parameters and resulting net flow by Jung and Kim [1] and Jung [12].

With a larger tube radius, the duty cycle of excitation pressure applied increases as the contact time of pincher with tube increases. Secondly, large tube radius also means volume displaced per external excitation is larger than with smaller tube radius thus, resulting in higher net flow rate. The total volume of the tube may change due to the capacitance effect. Since the length of the tube is preserved at all times in our model, the volume change results as a change in the circumferential radius of the compartment sections. Capacitance effect can be modelled in the case of fluid flow in an elastic tube using the relation where \(E\) is young’s modulus and \(h\) is the thickness of the elastic pipe respectively [2, 4].

\[
\frac{dP}{dt} = \frac{E h V'}{2\pi r^3} \tag{24}
\]

According to the above Eq. (24), increasing elastic tube radius and reducing tube thickness would considerably decrease transmural pressure on the tube wall acting against external pumping force. As a result, more of the external pressure applied can transfer to the fluid internal pressure. It is noticeable from Fig. 5 that the level change at tube radius of 0.7 cm is far greater than the other two cases
investigated. This can be also explained using the above equation where the net flow changes non-linearly with the change in tube radius. Transmural pressure on the flexible tube wall is inversely proportional to the cube of tube internal radius, which could be attributed to the significant rise in flow rate with a small change in radius.

**Fig. 5.** Excitation frequency and net flow analysis for varying elastic tube radii, \( r_d = 0.5, 0.6, 0.7 \) cm, \( d_t = 5 \) cm, \( L=40 \) cm and \( h_d = 0.1 \) cm are set to be constant.

**Fig. 6.** Excitation frequency and net flow analysis for varying elastic tube thickness, \( h_d = 0.075, 0.1, 0.125 \) cm \( d_t = 5 \) cm, \( L=40 \) cm and \( r_d = 0.5 \) cm are set to be constant.
3.2. Effect of pumping locations on net flow

Examining the peak value analysis plot (Fig. 7), for excitation compartments such as C[2,3] and C[6,7], external pinching results in net flow characteristics of the same magnitude but opposite in direction. Similar characteristics have been observed for C[1,2] and C[7,8]. We may refer such pairs of pinching compartments as ‘symmetrical pairs’ since such pairs have been shown to exhibit symmetry about the centre of the flexible tube.

Tube centre is characterized by having no net flow rate. In other words, impedance pumping exhibits symmetry behaviour about the origin, which corresponds to middle excitation position C[4,5] in this case. This property could be easily explained by virtue of symmetry in position as such compartment pairs are located at the same distance and opposite direction from the centre of the elastic tube as in C[2,3] and C[6,7].

![Flow rate at various pinching locations](image1)

![Peak net flow at various pinching locations](image2)

**Fig. 7.** Net flow behaviour of two-tank system at various pumping locations for excitation frequency range of 0-15 Hz.

3.3. Effect of pinching frequencies on net flow behaviour

Three pinching positions at compartments C[1,2], C[2,3] and C[6,7] have been chosen to investigate the underlying reason for that specific direction to be preferred for net energy storage at the applied pinching frequency. In this context; for sake of convenience, we redefine positive and negative direction with respect to pinching location on the elastic tube.

For the pumping locations on the right half of the tube, directions are reversed from that of left half side making the right tank to left tank as a positive direction.
of net flow and vice versa. External pumping periodic function is given in Eq. (23) is applied at those compartments for pinching frequencies ranging from 0.1 Hz to 15 Hz with an increment of 0.1 Hz to find the average fluid level difference in tanks for each frequency.

From Fig. 8, it can be seen that within these range of frequencies, there happened to have a peak positive net flow, a peak negative net flow and frequencies of no significant net flow in all selected pump locations. At compartment position C[1,2], maximum net flow in positive direction occurs at 10.9 Hz and peak negative value happens at 10.7 Hz denoting power storage happens in the reverse direction in this case. At pinching position C[2,3], positive peak frequency is 10.7 Hz and a negative maximum net flow occurs at 10.6 Hz.

Third pumping location chosen at right half of tube, C[6,7] being ‘symmetrical pair’ of compartment C[2,3] have same positive and negative peak frequencies and net flow values when directions are defined as above. Pinching frequency of 6 Hz is chosen for the case with no significant net flow rate for all three-tube positions.

Phase synchronization between two waves occurs when the positive half cycle of the first wave coincides with the positive half cycle of the second wave and negative half cycle with the negative and likewise skew-synchronization means the positive half cycle of one wave synchronizing with the negative half of the other and vice versa. It has been revealed that maximum positive net flow occurs when external pumping synchronizes with the wave profile of left marginal fluid flux \(v_0\). On the contrary, if the skew-synchronization occurs between external pumping and marginal flux at the left tank, then the net power becomes negative.

Similarly, in terms of right marginal flux \(v_n\), peak positive flow occurs for skew-synchronization between external pressures applied and wave profile of flux \(v_n\) and if the external pumping synchronizes with velocity flux \(v_n\), negative power storage takes place. These conditions have been met at frequencies 10.9 Hz for pinching location C[1,2] and at 10.7 Hz for compartments C[2,3] and C[6,7] in generating positive net flow as illustrated in Fig. 9(a). Also, Fig. 9(b) shows that conditions for negative directional net flow occurs at an excitation frequency of 10.7 Hz for C[1,2] and at 10.6 Hz for pumping locations C[2,3] and C[6,7].

In general, when one complete cycle of external pressure starts and ends within one complete cycle of oscillatory velocity flux at a tube-tank junction, peak net flow in a preferential direction occurs. Such relation between peak net flow and excitation frequency could be explained using wave physics as for the case of synchronization, the amplitudes of flux wave profile at a tube-tank junction and external pressure wave are being added up for positive half cycle while excitation pressure attenuates negative wave profile of the oscillating flux resulting peak net flow. Hence, in the case of positive peaks, synchronization of waves of external pressure applied and fluid velocity flux results rise in flux amplitude towards a positive direction.

On the other hand, amplitudes of both waves cancel each other in the case of skew-synchronization. In the absence of these conditions, no net flow in any of the directions is observed as shown in Fig. 9(c). With these findings, the direction of net power storage could be predicted if marginal velocity flux profile could be identified.
Also; from the figures, it has been clear that magnitude of net power is proportional to the fluid flux amplitudes since marginal flux amplitude of compartment C[1,2] has found to be smaller than that of C[2,3] and C[6,7]. In addition, having the same net peak flow rate, same amplitudes are noticed in the case of marginal flux waves of symmetrical pairs. An accurate deduction of magnitude has been difficult due to the fact that average flow rate has been calculated for net power stored since level differences in tanks fluctuate for a sufficiently long time in both directions.

Fig. 8. Level differences in tanks for pinching compartments C[1,2], C[2,3] and C[6,7].

(a) Flow behaviour at positive peak.
4. Experimental Validation

Numerical studies using mathematical model proved to be in good agreement with the flow behaviour observed in previous works of impedance pumping based on the pump’s physical parameters and excitation locations. For same pumping position, because of changing pumping frequencies, the direction of net power could be reversed along with nonlinear varying in average flow rate [13-15]. Lee et al. [24] in 2017 conducted an experimental study of such impedance-driven flow with emphasis on the dynamical study of the reverse flow in the open-loop environment. Study conducted shows that the reverse peak flow is rather significant.
with an estimate of 23 percent lower than the forward peak flow. The flow dynamics, on the other hand, has shown to exhibit different characteristics as per the forward peak flow. Hence, to study open-tank impedance pumping with complex non-linear flow dynamics comprising of distinct resonance peaks and reversal in the flow direction, to experimental simulation has become a necessity rather than a luxury.

Pump’s behaviour could be explicitly measured using an experimental approach. Experimental studies were conducted to investigate the pump’s behaviour under various pinching frequencies and pinching positions on the elastic tube in order to determine their effects those that are vital in the pump’s performance. Two tanks are connected in parallel using an elastic tube of silicon type in between. Water with standard properties is used as the working incompressible fluid. Solenoid actuators are used to pinch the valveless impedance tube at desired frequencies and level sensors measure the water level in each tank. Figure 10 shows the schematic of the experimental set-up for this project. dSPACE RT1104 Controller board with Connector Panel CP1104 is used to control and data acquisition of sensors and solenoids.

To begin the experiment, all physical connections are made between sensors, solenoids, dSPACE Connector Panel ADC and DAC channels. Pull type linear solenoids with a box frame are used for periodic excitation of the elastic tube by pinching the tube at various frequencies. In pull-type solenoids, the plunger is pulled into the solenoid coil when the coil is energized. This makes them ideally suited for applications asking for low power consumption or low heat dissipation with high life expectancy. A continuous level sensor, Milone eTape liquid level sensor is used in this project for measuring the water level inside the tanks. The eTape liquid level sensor is an innovative solid-state sensor with a resistive output that makes use of printed electronics instead of moving mechanical floats. Sensor outputs resistance that varies with the fluid height.

Different flow rates are obtained by controlling the pinching frequency of solenoid on the elastic pump. Solenoid actuators are used to pinch the valve-less tube at desired frequencies. For elastic tube length of 70 cm divided into 20 small compartments, pinching positions on the tube are compartments; \( C = 4, C = 6, C = 9, C = 13 \) and \( C = 15 \) in the increasing order in positive x-direction of the flexible tube as illustrated in Fig. 11. At each position, experiments were carried out from 6 Hz to 8 Hz pinching frequency with an increment of 0.1 Hz.
Flow rates inside tanks are calculated from the difference in water level between the tanks measured using level sensors. The average height difference is then plotted against frequencies and repeated for various pinching positions and results were compared with a numerical model for analysis. Table 2 lists the material specifications used in the experiment. Experiment conditions are listed in Table 3. In Fig. 12, water level difference between tanks has been plotted against various pumping frequencies for five different excitation positions on the tube and compared against the analytical plot. In both studies, at $C=9$, pinching location being the centre of the tube, zero net flow has been noted. If middle location $C=9$ is considered as the reference, flow profile for compartments, $C=15$ and $C=13$ lies above and flow behaviour for pinching locations $C = 4$ and $C = 6$ lies below the reference line. Noting that compartments $C=13$ and $C=15$ are at the right side of the tube with the positive direction of resultant net flow at resonant frequencies while compartments $C = 4$ and $C = 6$ at the left side of the tube causes follow the negative direction for the maximum net flow.

Within the range of excitation frequencies investigated, all chosen pinching positions are found to exhibit a peak flow rate around 7.2-7.3 Hz in Fig. 13. Maximum net flow rate from experiments achieved tank level differences of 1.10 cm and -1.15 cm for positions $C = 15$ and $C = 4$ respectively while height difference of 1.07 cm achieved in both directions for same pumping positions using numerical methods.

Table 2. Material specifications for experiment setup.

<table>
<thead>
<tr>
<th>Specifications</th>
<th>Values</th>
</tr>
</thead>
<tbody>
<tr>
<td>Young’s modulus(E)(dyne/cm$^2$)</td>
<td>$10^7$</td>
</tr>
<tr>
<td>Fluid viscosity,µ (kg m$^{-1}$ s$^{-1}$)</td>
<td>0.001</td>
</tr>
<tr>
<td>Fluid density, ρ (kg m$^{-3}$)</td>
<td>998</td>
</tr>
<tr>
<td>Tube diameter, d (mm)</td>
<td>11</td>
</tr>
<tr>
<td>Tube wall thickness, h (mm)</td>
<td>1.5</td>
</tr>
<tr>
<td>Tube length, (cm)</td>
<td>70</td>
</tr>
<tr>
<td>Tank cross-sectional area (cm$^2$)</td>
<td>$10^4$10</td>
</tr>
</tbody>
</table>

Table 3. Simulation parameters for experiment.

<table>
<thead>
<tr>
<th>Simulation parameters</th>
<th>Values</th>
</tr>
</thead>
<tbody>
<tr>
<td>Initial tank level ($H_0$) (cm)</td>
<td>15</td>
</tr>
<tr>
<td>Excitation frequency, (Hz)</td>
<td>6-0.1-8</td>
</tr>
<tr>
<td>Frequency waveform</td>
<td>Sinusoidal</td>
</tr>
<tr>
<td>No. of compartments</td>
<td>20</td>
</tr>
<tr>
<td>Compartment length (cm)</td>
<td>3.5</td>
</tr>
</tbody>
</table>
Experimental data has been limited to excitation frequencies below 10 Hz. Discrepancies in net flow rate values are likely to occur mainly for the following reason. The difference in the original and chosen values for constant parameters such as Young’s modulus and maximum value of external pressure applied. Value of $P_{\text{max}}$ for use in the analytical simulation could be found through iteration to match with average flow rate values from experiments. Another main issue faced was vibrations caused by pinching of the solenoid, which would send shock waves in water level in tanks resulting in inconsistent data. Since we are only interested in net flow caused by peak frequencies, irregular data from vibrations and other noises were filtered considering only significant readings. Within the luxury of above said constrains, experimental work validates that numerical model derived for Two-tank system in Section 2 could very well describe the net flow direction caused by a
particular frequency for a given excitation position, the resonant frequency value, flow behaviour caused by pinching location and hence, impedance pumping mechanism consisting two open tanks in general.

In the lumped model analysis, an attempt is made to infer the dynamic properties of fluid flow in a tube in terms of only input and output. The focus of the lumped models has been on a time-dependent relation between pressure and flow. Pressure and flow waves are considered as a function of time only [2-4]. Lumped model of impedance pumping using electrical, mechanical, hydraulic and wave analogies and modelling using immersed boundary method have been widely discussed over the literatures [6-14].

However, to understand the dynamic properties of the flow in an elastic tube in terms of internal properties and structure of the system, analysis using un-lumped model is necessary. In the un-lumped model of such a system, pressure and flow waves are considered as not only a function of time but also a function in space.

The basic approach is to start from the level of a small section of tube and move up to the entire length of the tube [4]. Jung introduced compartment model of the flow driven by pumping without valves in an open-tank system consisting of two tanks [1]. Her works concluded that the valve-less pumping effect that the energy stored in the valve-less pumping system under gravity is attributed to the directional net power caused by the simple external pumping on the elastic tube.

However, Jung’s model showed higher complexities with the introduction of the compounded compartment and the artificial compounded area and length of fluid associated with it. In this study, we define control volume in such way that the volume deformation occurs inside the compartments and flow occurs at a point between two compartments, which is not affected by volume change at any time. This assumption simplifies the resulting model without losing the generality.

Few previous works have been documented on analytical modelling of impedance system consisting of more than two tanks. The pressure at the external pinching is the only driving force in a flow-through an elastic tube connected to tanks. However, the relationship between this driving force and the ultimate flow at the end of the tubes is everything but simple [2, 3]. The only prospect for a good understanding of the flow is by modelling the system. Modelling is thus a necessity rather than a luxury in the study of pulsatile flow in an elastic tube.

The main objective of the current literature is to come up with a much simpler compartment model to be extended to a more sophisticated open-tank system with interim tanks. Such a model could also be promising towards controlling the power of the valve-less pumping. Extension of the current model to a multi-tank valve-less pumping with interim reservoirs has been achieved and is a subject of different literature written as a follow-up to the current work.

With such an objective in mind, experimental studies are conducted to validate the numerical model, which is based on assumptions. The standard deviation measures how accurately the sample represents the actual value from, which the data has been taken. Experimental points with error bar have been plotted to show better the good agreement between the model and the experiments. Error bars are plotted in Fig. 14 and compared to numerical results to show the agreement between the model and the experimental data.
Fig. 14. Standard deviation error analyses for two-tank system.

5. Conclusions

A simple model based on energy balance has been developed and shown to describe the essential features of impedance pumping observed for Two-tank model in earlier mathematical models and experimental studies. Among the physical parameters, radius and thickness of the elastic tube have been revealed to have much impact on net flow characteristics under external pumping. While inspecting the factor related to the direction of net power using numerical experiments, maximum positive flow occurs when external pumping synchronizes with left marginal fluid flux \( V_0 \). On the contrary, if skew-synchronization occurs between external pumping and marginal flux at the left tank, then the net power becomes negative. In addition, it has been clear that the magnitude of net power is proportional to the marginal fluid flux amplitudes. In the absence of these conditions, no net flow in any of the directions has been observed. The derived mathematical model has been compared with experimental simulations and numerical results obtained analytically have been found to be in good agreement with the experiment data.

Using the algorithm developed, similar model as in two-tank could be extended for Multi-stage impedance pumping system aiming to generate augmented flow. Having full control achieved over the pumping frequency and resulting directional flow and its magnitude would be the primary objective of future research in this field.

<table>
<thead>
<tr>
<th>Nomenclatures</th>
</tr>
</thead>
<tbody>
<tr>
<td>( A )</td>
</tr>
<tr>
<td>( C_i )</td>
</tr>
<tr>
<td>( H(t) )</td>
</tr>
<tr>
<td>( h )</td>
</tr>
<tr>
<td>( hL )</td>
</tr>
<tr>
<td>( hR )</td>
</tr>
</tbody>
</table>

\[ \text{Pinching compartments} \]

\[ \text{Experimental vs Simulated} \]
<table>
<thead>
<tr>
<th>Symbol</th>
<th>Description</th>
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</thead>
<tbody>
<tr>
<td>$L$</td>
<td>Length of the flexible tube, cm</td>
</tr>
<tr>
<td>$P$</td>
<td>Fluid pressure inside tube, dyne/cm$^2$</td>
</tr>
<tr>
<td>$P_{ext}$</td>
<td>External fluid pressure, dyne/cm$^2$</td>
</tr>
<tr>
<td>$P_t$</td>
<td>Fluid transmural pressure inside tube, dyne/cm$^2$</td>
</tr>
<tr>
<td>$r$</td>
<td>Radius of the tube, cm</td>
</tr>
<tr>
<td>$V$</td>
<td>Volume of the flexible tube, cm$^3$</td>
</tr>
<tr>
<td>$v$</td>
<td>Velocity flux of the fluid inside tube, cm/s</td>
</tr>
</tbody>
</table>

**Greek Symbols**

<table>
<thead>
<tr>
<th>Symbol</th>
<th>Description</th>
</tr>
</thead>
<tbody>
<tr>
<td>$\mu$</td>
<td>Fluid viscosity, gcm$^{-1}$s$^{-1}$</td>
</tr>
<tr>
<td>$\rho$</td>
<td>Fluid density, g/cm$^3$</td>
</tr>
<tr>
<td>$\omega$</td>
<td>Frequency of the excitation signal, Hz</td>
</tr>
</tbody>
</table>

**Abbreviations**

<table>
<thead>
<tr>
<th>Abbreviation</th>
<th>Description</th>
</tr>
</thead>
<tbody>
<tr>
<td>ECCM</td>
<td>Energy Conserving Compartment Model</td>
</tr>
<tr>
<td>FSI</td>
<td>Fluid and Solid Interaction</td>
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</tbody>
</table>

**References**