

## **A NOVEL METHOD FOR GLOBAL VOLTAGE SAG COMPENSATION IN IEEE 69 BUS DISTRIBUTION SYSTEM BY DYNAMIC VOLTAGE RESTORERS**

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### **Abstract**

In the paper, a new model for optimizing the placement of Dynamic Voltage Restorers (DVR) for global voltage sag mitigation in a distribution system is introduced. The placement of one or a number of DVRs is optimally selected on the basis of minimizing the system average RMS variation frequency index -  $SARFI_X$  of the system of interest. For calculating  $SARFI_X$  of the system of interest with the presence of a number of DVRs, a new method for modelling different cases of DVR placement for mitigating globally voltage sag due to short-circuits in a distribution system using the Thevenin's superposition principle is proposed. The DVR's performance of global voltage sag mitigation is considered for a given maximum current generated by the DVR. The paper uses the IEEE 69-buses distribution system as the test system for voltage sag simulation and discussion on cases of study for optimal placement of DVRs.

Keywords: Distribution system, Dynamic voltage restorer-DVR,  $SARFI_X$ , Voltage sag.

## 1. Introduction

Voltage sag/dip [1] is one of Power Quality (PQ) issues that occurs rather frequently because its main cause is the faults in power systems. A single voltage sag event may not cause serious problems to a large number of customers, but its high frequency of occurrence still results in costly damages, especially in the distribution system. With the recent development of power electronic applications, the phenomenon can be effectively mitigated by using the Custom Power Device (CPD) [2, 3] under two approaches named “distributed improvement” [4] and “central improvement” [5]. The first is mainly considered for protecting a single sensitive load while the latter is introduced for globally improving PQ in the power system that is mainly interested by utilities.

Among CPD based solutions for voltage sag mitigation, using the Dynamic Voltage Restorer (DVR) have been proved to be effective for “distributed improvement” [6-8] with regard mainly to DVR’s controller designing improvement for mitigating PQ issues at a specific load site. When DVR is used for “central improvement” of PQ in general, the problem of optimizing its placement and size always needs to be solved and [5] generally reviewed various researches for modelling and solving the problem. However, the number of reports for “central improvement” of PQ using CPD, especially DVR is much fewer than that for “distributed improvement” of PQ. The main difficulties for researches on “central improvement” are: (i) To find a suitable steady state or short time modelling of CPD for globally mitigating different PQ issues, (ii) To optimize the use of CPD (sizing and locating). Regarding DVR’s application, the research review can be summarized by remarkable reports as follows: Mohammadi [9] introduced interesting research for optimizing DVR’s location and size, but the objective function implies the improvement of system reliability with regard to the events of supply interruption only. Khanh and Nguyen [10] also considered the optimization of DVR’s location, but it’s used for individual fault events. Ali et al. [11] introduced the solving of the optimization problem for the application of Static Compensator (Statcom) under “central improvement” approach that is probably applicable for other CPD like DVR. This research deals with the mitigation of various PQ issues including voltage sag and uses the multi-objective optimization approach for Statcom locating, but such an optimization problem can rarely get the best performance for voltage sag mitigation only. Chang and Yang [12], Chang and Yu [13] and Zhang and Milanovic [14] deal directly with voltage sag mitigation using FACTS devices, but the modelling of FACTS devices for short-circuit calculation still needs to be further improved.

This paper introduces a novel method for estimating the effectiveness of global voltage sag mitigation in some cases of study for the presence of DVRs in the short-circuit of a distribution system. This method optimizes the placement of DVR basing on minimizing a well-known system voltage sag index -  $SARFI_X$  that allows considering not only a single short-circuit event but also all possible short-circuit events in a system of interest. In solving the problem of optimization, the modelling of DVR compensating globally voltage sag in short-circuit events is introduced and discussed. The research uses the IEEE’s 69-bus distribution system as the test system.

For this purpose, the paper is structured as the following parts: Section 2 introduces the modelling of DVR’s effectiveness for global voltage sag mitigation in the problem of short-circuit calculation for the distribution system. Some cases

of study for DVR application are introduced. Section 3 introduces the problem of optimization where objective function, assumption and constraints are defined and discussed. The modelling of DVR is built in the test system modelling for short-circuit calculation. Finally, the results for different pre-set parameters of DVR are analysed in Section 4.

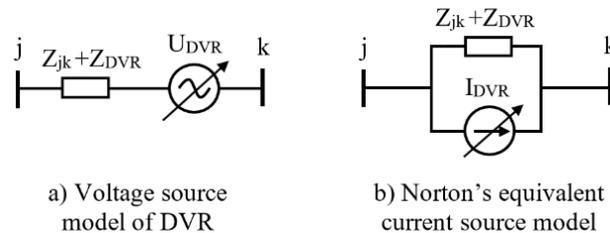
## 2. Modelling of DVR with Limited Current for Short-circuit Calculation in Distribution System

### 2.1. DVR's basic modelling for voltage sag mitigation

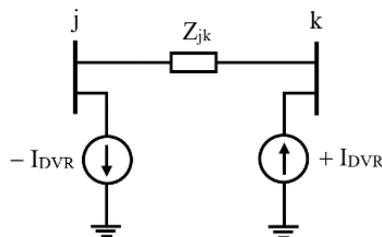
DVR is a FACTS device that is connected in series with the load that needs to be protected or with the source that generates PQ issues to limit its bad influence to the power system operation. The description of the DVR in the steady-state calculation is popularly given as a voltage source [3] connected in series with the impedance of the branch as Fig. 1(a). In modelling, the power system for short-circuit calculation, the method of bus impedance matrix is often used and such DVR's model of a series connected voltage source is difficult to be applied. However, the problem can be eased by replacing the voltage source model with Norton's equivalent current source as shown in Fig. 1(b).

In power system modelling for steady-state calculation, Norton's equivalent current source model of the DVR can be represented as a load current at the output node ( $j$ ) and a current source at the input node ( $k$ ) as shown in Fig. 2 [15].

It is noticed that the node  $k$  is the position where the voltage is compensated by DVR. In the radial distribution system, node  $j$  is the node nearer to the source and node  $k$  is the node farther to the source (i.e., nearer to the load side).



**Fig. 1. Norton's equivalent current source model for DVR.**  
 $U_{DVR}$ : Series voltage source of DVR,  $I_{DVR}$ : Current injected by DVR,  
 $Z_{DVR}$ : Internal reactance of DVR,  $Z_{jk}$ : Impedance of branch  $j-k$ .



**Fig. 2. Model for DVR for steady state analysis.**

## 2.2. Modelling of DVR for global voltage sag mitigation

### 2.2.1. Modelling test system in a short-circuit event

For modelling, the effectiveness of the DVRs for global voltage sag mitigation, the paper introduces the application of the superposition principle according to the Thevenin theorem for the problem of short-circuit calculation in distribution system [10]. It is assumed that the initial state of the test system is the short-circuit without the presence of DVRs. Thus, the system bus voltage can be calculated as follows:

$$[U^0] = [Z_{bus}] \times [I^0] \quad (1)$$

where:

$[U^0]$ : Initial bus voltage matrix (voltage sag at all buses during power system short-circuit).

$[I^0]$ : Initial injected bus current matrix (short-circuit current).

$$[U^0] = \begin{bmatrix} \dot{U}_{sag,1} \\ \vdots \\ \dot{U}_{sag,k} \\ \vdots \\ \dot{U}_{sag,n} \end{bmatrix} \quad (2)$$

$$[I^0] = \begin{bmatrix} \dot{I}_{f1} \\ \vdots \\ \dot{I}_{fk} \\ \vdots \\ \dot{I}_{fn} \end{bmatrix} \quad (3)$$

$[Z_{bus}]$ : System bus impedance matrix calculated from the bus admittance matrix:  $[Z_{bus}] = [Y_{bus}]^{-1}$ . If the short-circuit is assumed to have fault impedance, we can add the fault impedance to  $[Z_{bus}]$ .

With the presence of DVRs, according to Thevenin theorem, the bus voltage equation system should be modified as follows [16]:

$$\begin{aligned} [U] &= [Z_{bus}] \times ([I^0] + [\Delta I]) \\ &= [Z_{bus}] \times [I^0] + [Z_{bus}] \times [\Delta I] = [U^0] + [\Delta U] \end{aligned} \quad (4)$$

where:

$$[\Delta U] = [Z_{bus}] \times [\Delta I] \quad (5)$$

$$\text{or } \begin{bmatrix} \Delta \dot{U}_1 \\ \vdots \\ \Delta \dot{U}_k \\ \vdots \\ \Delta \dot{U}_n \end{bmatrix} = [Z_{bus}] \times \begin{bmatrix} \Delta \dot{I}_1 \\ \vdots \\ \Delta \dot{I}_k \\ \vdots \\ \Delta \dot{I}_n \end{bmatrix} \quad (6)$$

$\Delta U_i$ : Bus  $i$  voltage improvement ( $i = 1 \div n$ ) after adding DVRs in the system.

$\Delta I_i$ : Additional injected current to the bus  $i$  ( $i = 1 \div n$ ) after adding the DVRs in the system.

**2.2.2. Voltage sag mitigation when placing a multiple of DVRs in system of interest**

It assumes that  $M$  is the set of  $m$  branches to connect to  $m$  DVRs (Fig. 3), a DVR on a branch between bus  $j$  (near source side) and bus  $k$  (near load side) is equivalently replaced by a current injected in bus  $k$  and a current going out from bus  $j$ . From (6), the voltage of a bus  $i$ , is calculated as follows

$$\Delta \dot{U}_i = \sum_{j=1}^n Z_{ij} \times \Delta \dot{I}_j \tag{7}$$

where:

$$\Delta \dot{I}_j = \sum_{t \in T_j} k_{t,DVR} \times k_{t,*} \times I_{t,DVR} \tag{8}$$

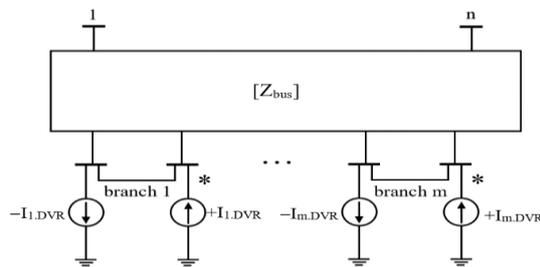
$T_j$ : Set of branches connected to bus  $j$ .

$k_{t,DVR}$ : Factor for placing DVR on the branch  $t$ . If the branch  $t$  is placed with DVR,  $k_{t,DVR} = 1$ . Otherwise,  $k_{t,DVR} = 0$ .

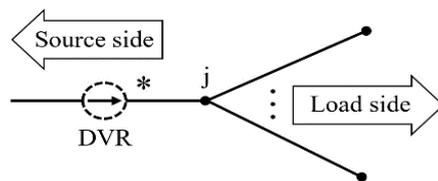
$k_{t,*}$ : Factor for DVR's voltage compensating position. If bus  $j$  is the voltage compensating position by the DVR on the branch  $t$ ,  $k_{t,*} = +1$ . Otherwise, if bus  $j$  is not the voltage compensating position by the DVR on the branch  $t$ ,  $k_{t,*} = -1$ .

$I_{t,DVR}$ : Injected current by DVR on the branch  $t$ .

Note that for the distribution system with typical radial network configuration, each bus is only connected with one branch toward the source. However, it is possibly connected to more than one branch toward the load side (Fig. 4). Therefore, if we define that the voltage compensating bus of the DVR coupled branch is the bus toward the load side, each bus will definitely be the voltage compensating position of not more than one DVR. In other words,  $m$  DVR coupled branches have corresponded to  $m$  different buses of voltage compensation (marked “\*”). For  $m$  buses of voltage compensation by DVR, we have the condition of voltage compensation as follows:



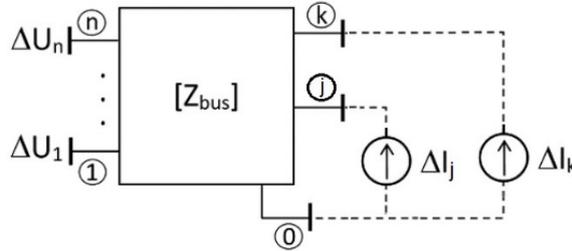
**Fig. 3. System modelling with presence of  $m$  DVRs.**



**Fig. 4. System bus as voltage compensating position.**

**2.2.3. Modelling test system with presence of one DVR**

Assuming a DVR is placed on the branch  $j-k$ . Basing on the DVR modelling in Fig. 2, in the matrix of additional injected bus current Eq. (6), there are only two elements that do not equal to zero (Fig. 5). They are  $\Delta I_k = + I_{DVR}$  and  $\Delta I_j = -I_{DVR}$ . Other elements equal zero ( $\Delta I_i = 0$  for  $i=1 \div n, i \neq j$  and  $i \neq k$ ).



**Fig. 5. Test system modelling using  $[Z_{bus}]$  with presence of one DVR.**

Replace the above-assumed values of  $\Delta I_i$  in Eq. (6), we have

$$\Delta \dot{U}_k = Z_{kk} \times \Delta \dot{I}_k + Z_{kj} \times \Delta \dot{I}_j = (Z_{kk} - Z_{kj}) \times \dot{I}_{DVR} \tag{7}$$

According to the DVR modelling in Fig. 2, the voltage of bus  $k$  is compensated up to the desired value. Khanh and Nguyen [10] proposed the desired value is 1 p.u. It means the bus  $k$  voltage is boosted by DVR from  $U_k^0 = U_{sag,k}$  up to  $U_k = 1$  p.u. Therefore,

$$\Delta \dot{U}_k = 1 - \dot{U}_{sag,k} \tag{8}$$

Replace Eqs. (8) into (7), we get  $I_{k,DVR}^*$ :

$$I_{k,DVR}^* = \Delta \dot{I}_k = \frac{\Delta \dot{U}_k}{Z_{kk} - Z_{kj}} = \frac{1 - \dot{U}_{sag,k}}{Z_{kk} - Z_{kj}} \tag{9}$$

Then the comparison of  $I_{DVR}$  calculated in Eq. (9) and a given  $I_{DVRmax}$  is as follows:

- If  $I_{k,DVR}^*$  calculated by Eq. (9) is not greater than a given  $I_{DVRmax}$ , the voltage of bus  $k$  is boosted up to 1 p.u. And the upgraded voltage for other bus  $i$  ( $i = 1 \div n; i \neq k$ ) in the test system can be calculated as follows:

$$\Delta \dot{U}_i = Z_{ik} \times \Delta \dot{I}_k + Z_{ij} \times \Delta \dot{I}_j = (Z_{ik} - Z_{ij}) \times I_{k,DVR}^* \tag{10}$$

- If  $I_{k,DVR}^*$  calculated by Eq. (9) is greater than  $I_{DVRmax}$ , the voltage of bus  $k$  is calculated as follows:

$$\dot{U}_k = (Z_{kk} - Z_{kj}) \times I_{DVRmax} + \dot{U}_{sag,k} < 1 \text{ p.u.} \tag{11}$$

The upgraded voltage for other bus  $i$  ( $i = 1 \div n; i \neq k$ ) in the test system can be calculated as follows:

$$\Delta \dot{U}_i = (Z_{ik} - Z_{ij}) \times I_{DVRmax} \tag{12}$$

Finally, system bus voltages with the presence of DVR.

$$\dot{U}_i = \Delta \dot{U}_i + \dot{U}_{sag,i} \tag{13}$$

### 2.2.4. Modelling test system with presence of two DVRs

In the case of a number of DVRs placed in the test system, for simply demonstrating the algorithm, we consider the placement of two DVRs. Assuming DVR1 and DVR2 are coupled on the branch  $j-k$  and branch  $e-f$  where bus  $k$  and bus  $f$  are the voltage compensating positions respectively. The Norton's equivalent circuits using injected currents of DVRs are shown in Fig. 6.

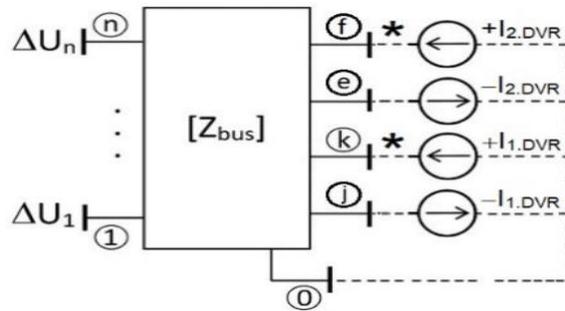


Fig. 6. Test system modelling using  $[Z_{bus}]$  with presence of two DVRs.

Therefore, Eq. (6) is written for this case of study as follows for the voltage compensating buses  $k$  and  $f$ :

$$\begin{cases} \Delta \dot{U}_k = (Z_{kk} - Z_{kj}) \times \dot{I}_{1,DVR}^* + (Z_{kf} - Z_{ke}) \times \dot{I}_{2,DVR}^* \\ \Delta \dot{U}_f = (Z_{fk} - Z_{fj}) \times \dot{I}_{1,DVR}^* + (Z_{ff} - Z_{fe}) \times \dot{I}_{2,DVR}^* \end{cases} \quad (14)$$

Note that because of typically radial network configuration as said in 2.2.2., bus  $e$  can be identical to bus  $j$  (buses toward the source side), however, bus  $f$  is never identical to bus  $k$  (buses toward to load side).

Applying the voltage compensating condition to buses  $k$  and  $f$ , we have:

$$\begin{cases} \Delta \dot{U}_f = 1 - \dot{U}_{sag,f} \\ \Delta \dot{U}_k = 1 - \dot{U}_{sag,k} \end{cases} \quad (15)$$

Replace Eqs. (15) to (14) and solve the system of two equations, we have the required DVR's currents  $\dot{I}_{1,DVR}^*$  and  $\dot{I}_{2,DVR}^*$ .

We verify the DVR's maximum current and select  $I_{1,DVR}$  and  $I_{2,DVR}$  as step 2 in 2.2.2. Next, we calculate the bus  $i$  voltage increases as follows:

$$\Delta \dot{U}_i = (Z_{ik} - Z_{ij}) \times \dot{I}_{1,DVR}^* + (Z_{if} - Z_{ie}) \times \dot{I}_{2,DVR}^* \quad (16)$$

For  $\forall i=1 \div n, i \neq j, k, e, f$ .

And finally, we calculated all system bus voltages with the presence of two DVRs as (10).

## 3. Problem Definition

### 3.1. Test system

For simplifying the introduction of the new method in the paper, the IEEE 69-bus distribution feeder (Fig. 7) is used as the test system because it just features a balanced

three-phase distribution system, with three-phase loads and three-phase lines. This system is large enough for testing the placement of one or a number of DVRs.

This research assumes base power to be 100 MVA. The base voltage is 11 kV. The system voltage is 1 p.u. System impedance is assumed to be 0.1 p.u.

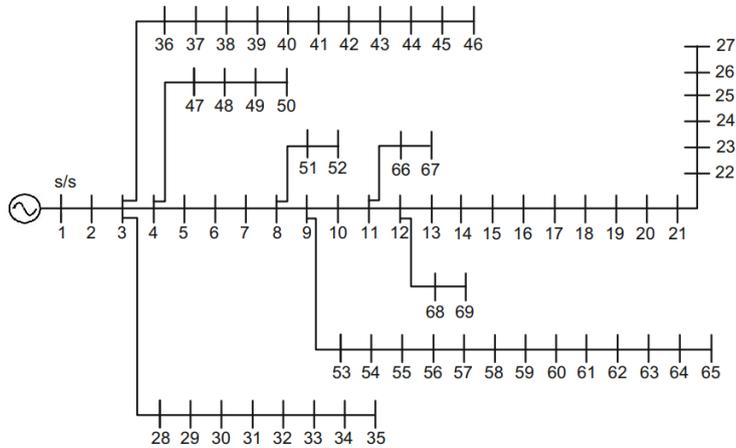


Fig. 7. IEEE 69-bus distribution system.

### 3.2. Short-circuit calculation

The paper only considers voltage sags caused by faults. Because the method introduced in this paper considers  $SARFI_X$ , we have to consider all possible fault positions in the test system. However, to simplify the introduction of the new method, we can consider only three-phase short-circuits. Other short-circuit types can be taken similarly in the model if detailed calculation is needed.

Three-phase short-circuit calculation is performed using the method of bus impedance matrix. The resulting bus voltage sags with and without the presence of DVRs can be calculated for different cases of given influential parameters as analysed in Section 4.

### 3.3. Problem of optimization

#### 3.3.1. Objective function, assumptions and constraints

Reviewed publications in Part I suggests that for an application of power quality solution, a cost-effect model can be introduced when the cost of power quality solution (investment) and benefits from the resulting power quality mitigation are addressed. However, for such a systematic solution as global voltage sag mitigation by DVRs, its benefit is hard to quantify. Therefore, in this research, the application of DVRs for global voltage sag mitigation is performed that based on the problem of optimizing the location of one or a multiple of DVRs in the test system where the objective function is only to minimize the System Average RMS Variation Frequency Index -  $SARFI_X$  where  $X$  is a given RMS voltage threshold [17].

$$SARFI_X = \frac{\sum_{i=1}^n n_{i,X}}{n} \Rightarrow \text{Min} \quad (17)$$

where:

$n_{i,X}$ : The number of voltage sags lower than  $X\%$  of the load  $i$  in the test system.

$n$ : The number of loads (assuming all buses in the system).

Therefore, this research accepts an important assumption that the problem of optimization does not take account of any costs for power quality investment as well as resulting benefices. In this problem, for an in-advance given a number of DVRs with a given limited current, we need to find the optimal scenarios of DVR placement in order to achieve the best performance of global voltage sag mitigation. That is why  $SARFI_X$  is used as an objective function.

For a given fault performance (fault rate distribution  $r_f$ ) of a given system and a given threshold  $X$ ,  $SARFI_X$  calculation is described as the block diagram in Fig. 8 [18]. For  $SARFI_X$  calculation in Fig. 8, one important thing is the calculating all bus voltages of the system of interest without or with the presence of a number of DVRs in a certain scenario of placement (blocks (\*) and (\*\*)). These calculations are newly introduced in subsection 2.2.2.

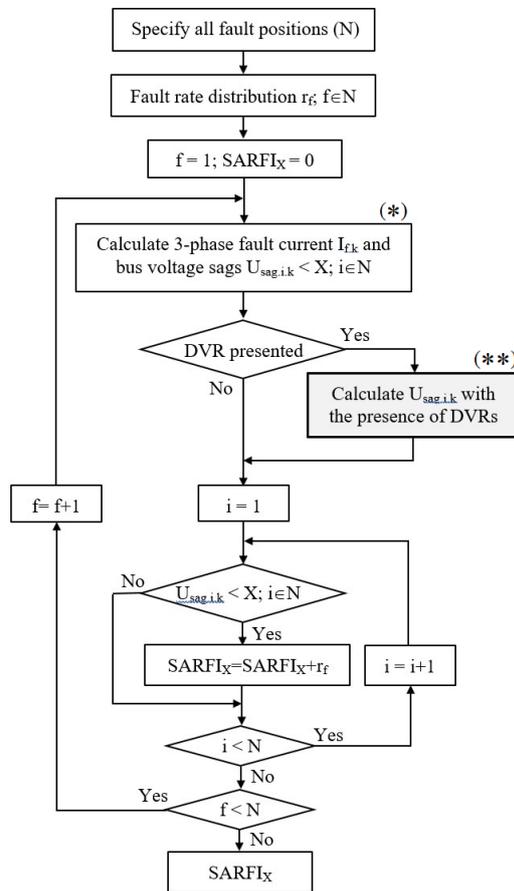


Fig. 8. Block-diagram of  $SARFI_X$  calculation.

For this problem of optimization, the main variable is the scenario of positions (branches) where DVRs are placed. Because of radial network topology, a distribution system with  $n$  buses has  $n-1$  branches. For the 69-bus test system, we have 68 branches. If  $m$  DVRs are considered, the total scenarios of DVR placement to be tested are as follows:

$$T_m = C_{n-1}^m = \frac{(n-1)!}{m!(n-1-m)!} = \frac{68!}{m!(68-m)!} \quad (18)$$

For example, if we consider the placement of one DVR in the test system, we have  $m = 1$  and total scenarios for placing this DVR is  $T_1 = C_{68}^1 = \frac{68!}{1!(68-1)!} = 68$ .

Each candidate scenario to be tested is the bus number  $k$ , ( $k = 1 \div 68$ ).

If we consider the placement of two DVRs in the test system, we have  $m = 2$  and the total scenarios for placing these two DVRs is  $T_2 = C_{68}^2 = \frac{68!}{2!(68-2)!} = 2278$ .

The problem of optimization has no constraint, but there're two important assumptions: Firstly, a DVR's parameter, which is the limited current of DVR is in-advance given. The modelling about how DVR with a limited current can compensate globally voltage sag is introduced in subsection 2.2.2. Secondly, the DVR's operation is assumed [6-8] that DVR only works if it is placed on the branch that is not a part of fault current carrying path (from the source to the fault position). In this case, the bypass switch is actually closed to disable DVR's operation.

### 3.3.2. Problem-solving

For such a problem of optimization, with pre-set parameters ( $X\%$ , and DVR's limited current), the objective function -  $SARFI_X$  is always determined for any candidate scenarios of DVR's placement in  $T_m$ . Therefore, we use the method of direct search and test all scenarios of DVR positions. The block-diagram of solving this problem in Matlab is given in Fig. 9. In this block-diagram, firstly, the set  $M$  of  $T_m$  candidate scenarios of placement of  $m$  DVRs Eq. (18) is listed. Then, according to the said method of direct search, for each candidate scenario  $k$  ( $k \in M$ ), the corresponding  $SARFI_X$  is calculated.

Calculating  $SARFI_X$  of the test system with and without the presence of DVRs is performed as Figs. 8 and 10. With the presence of  $m$  DVRs coupled on  $m$  branches, as discussed in subsection 2.2.2., we have  $m^*$  buses of voltage compensation by DVR. Figure 8 shows the algorithm starting with verifying of branch  $t$ .

The ( $t = 1 \div m$ ) connected with DVR is part of a fault current carrying path or not. If it's the case, DVR on this branch is disabled by the bypass switch and we have  $I_{t,DVR} = 0$ . After that, the condition of voltage compensation by DVR is applied for above said  $m^*$  buses to calculate the required  $I_{t,DVR}^*$ . Then, DVR's maximum current is checked to finally calculate the system bus voltage as (10).

In the block-diagram, input data that can be seen as the above said pre-set parameters. The "postop" is the intermediate variable that fixes the scenario of DVR's location corresponding to the minimum  $SARFI_X$ . The initial solution of objective function Min equals  $B$ , which is a big enough value (e.g.,  $B = 69$ ) for starting the search process. The scenarios for different parameters of fault events are considered.

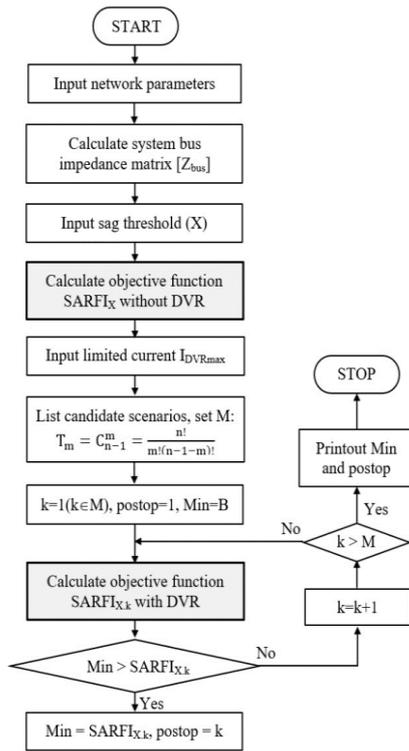


Fig. 9. Block-diagram of solving problem of optimization.

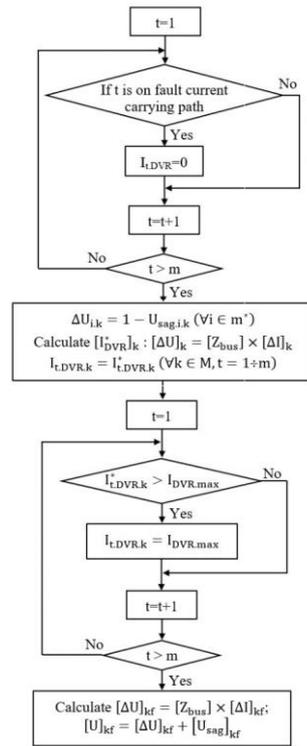


Fig. 10. Block-diagram of calculating system voltage with presence of DVRs.

## 4. Result Analysis

### 4.1. Preset parameters

The research considers the following pre-set parameters:

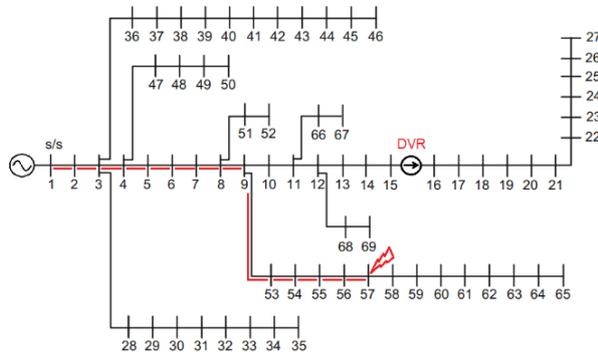
- For calculating  $SARFI_x$ , the fault performance, which is fault rate distribution, is assigned to all fault positions. According to Khanh et al. [18], just for introducing the method, the paper assumingly uses the uniform fault distribution (simplest fault distribution modelling) and fault rate equals 1 time per period of time for each fault position (assumed at each bus).
- For RMS voltage threshold, the paper only considers voltage sags, so  $X$  is given as 90, 80, 70, 50% of  $U_n$ .
- For DVR's limited current, the paper considers  $I_{DVRmax} = 0.1, 0.5, 1$  and  $1.5$  p.u.

### 4.2. First case of study: Placing one DVR in test system

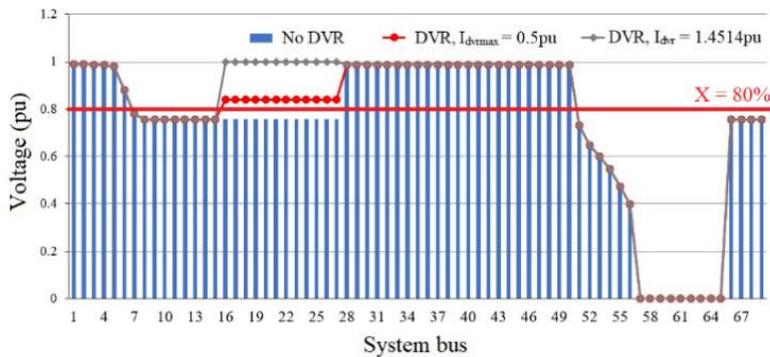
The simplest case is that with one DVR placed in the test system. In solving the problem of optimization considering the above said pre-set parameters, results are step-by-step introduced for better analysis and discussion. For a case of pre-set parameters, we initially consider sag  $X = 80\%$ ,  $I_{DVRmax} = 0.5$  p.u. For calculating  $SARFI_x$  of the test system with the presence of one DVR at a certain location, we

have to collect the sag frequency for all load buses (69 buses) caused by all possible fault positions (69 buses). For each fault position, firstly, the algorithm will check to see whether the DVR is on the fault current carrying path or not. For example, if the fault occurs at the bus 57, branches on the path from bus 1 to bus 57 (red marked in Fig. 11) are the locations where DVR is disabled if it is placed on these branches.

If DVR is not on the fault current carrying path, for example, DVR is on branch 15 (between bus 15 and bus 16), the bus voltage improvement is shown on Fig. 12 to illustrate the performance of DVR's model as introduced in subsections 2.1 and 2.2. With the DVR placed on branch 15, the voltage at bus 16 is boosted to 1 p.u. and the required injected current from DVR is 1.4514 p.u., which is quite large. The buses from bus 16 to the end of this lateral tap (bus 27) are all compensated to 1 p.u. Other bus voltages remain unchanged. However, with regard to the DVR's limited current, for example, we assumed  $I_{DVRmax} = 0.5$  p.u., the voltages from bus 16 to bus 27 are just upgraded as the red line (0.8397 p.u.) in Fig. 12. For the  $X = 80\%$ , 40 buses experiencing voltage sag are counted. However, with the presence of DVR, only 28 buses having the voltage lower than 80% are counted.



**Fig. 11. Checking locations where DVR is disabled for a give fault position.**



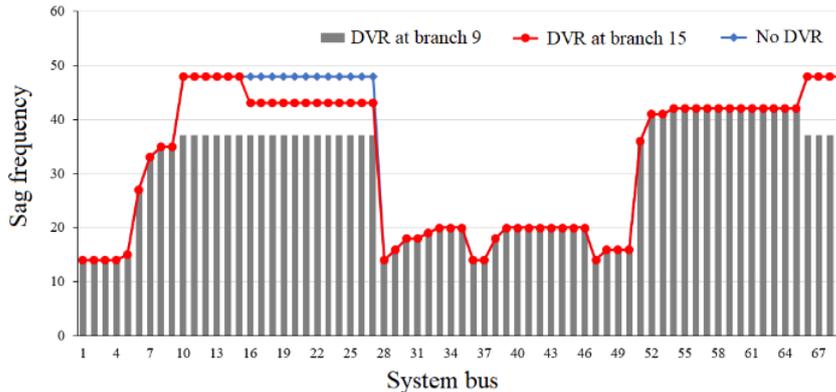
**Fig. 12. Bus voltage without and with DVR placed on branch 15 (15-16) for short-circuit at bus 57.**

Similarly, the algorithm (as shown in Fig. 8) calculates the frequency of voltage sag for the magnitude  $X$  (resulted by all possible fault positions) at all buses and

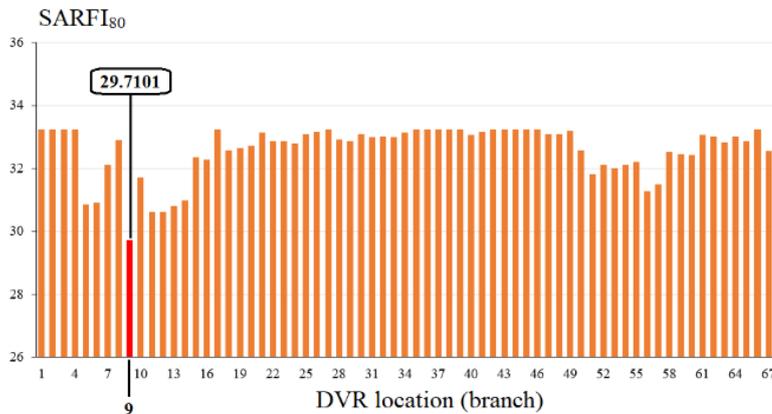
finally, the  $SARFI_X$  is obtained. Figure 13 shows sag frequency at all system buses for the cases without DVR (blue line) and with DVR having  $I_{DVRmax} = 0.5$  p.u. (red line) placed on the branch 15. The sag frequency is improved from buses 16 to 28. The resulting  $SARFI_{80}$  is 32.3478.

For other DVR's locations, the corresponding  $SARFI_{80}$  is similarly calculated. Values of  $SARFI_{80}$  for all scenarios of DVR placement are depicted in Fig. 14 for comparison.

DVR F location resulting in the minimum  $SARFI_X = 29.7101$  for the above-said case of pre-set parameters is at branch 9 (between bus 9 and bus 10). Sag frequency at all buses with DVR (having  $I_{DVRmax} = 0.5$  p.u.) placed at the branch 9 is also plotted (grey bars) in Fig. 13.



**Fig. 13. Sag frequency for  $X = 80\%$  at all buses without and with a DVR placed on branch 9 (optimal) and branch 15,  $I_{DVRmax} = 0.5$  p.u.**



**Fig. 14.  $SARFI_{80}$  for all scenarios of DVR placement and  $I_{DVRmax} = 0.5$  p.u.**

For analysing the influence of DVR's limited current on  $SARFI_X$ , we consider other cases of  $I_{DVRmax} = 0.1$  p.u., 0.5 p.u., 1 p.u. and 1.5 p.u. with  $X = 80\%$  in the same way, the  $SARFI_{80}$  for DVR's placement and different values of  $I_{DVRmax}$  are integrated in the same chart as shown in Fig. 15. The "0" means the  $SARFI_{80}$  in the

case without DVR. Obviously, a higher limited current produces a better (smaller)  $SARFI_X$  improvement.

For considering the improvement of  $SARFI_X$  for different levels of voltage sag magnitude  $X$ , the results of  $SARFI_X$  for  $X = 50\%$ ,  $70\%$ ,  $80\%$  and  $90\%$  with  $I_{DVRmax} = 0.5$  p.u. are shown in the Fig. 16. The “0” means the  $SARFI_X$  without DVR. Finally, remarkable results for all pre-set parameters are summarized in Table 1. We can see that the  $SARFI_X$  improvement is generally not big for DVR because DVR can only compensate for the voltage of the buses from the DVR’s location towards to load side.

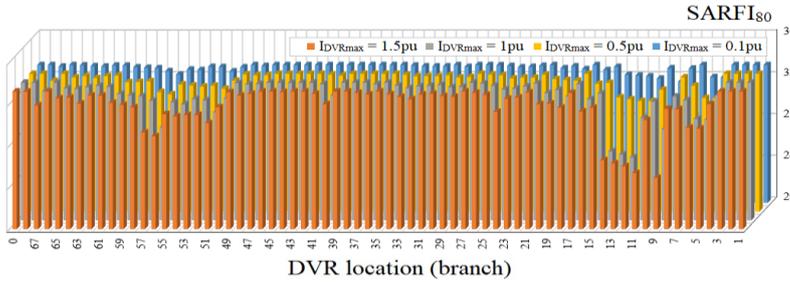


Fig. 15.  $SARFI_{X=80\%}$  for all scenarios of DVR placement,  $I_{DVRmax} = 0.1, 0.2, 0.3, 0.5$  p.u.

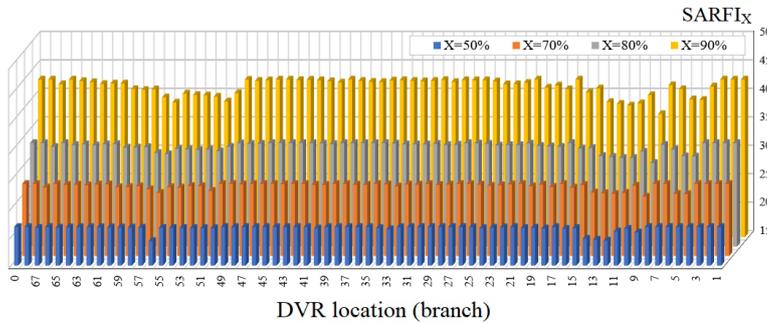


Fig. 16.  $SARFI_{X=80\%}$  for all scenarios of DVR placement for different voltage sag magnitude (50%, 70%, 80% and 90%),  $I_{DVRmax} = 0.5$  p.u.

Table 1. Results for using one DVR placement.

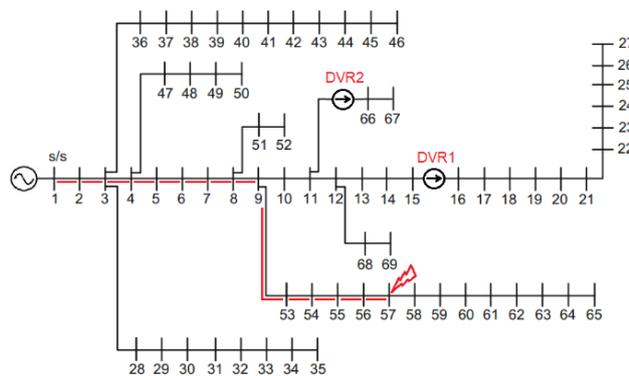
Results	$I_{DVRmax}$ (p.u.)				
	<i>c</i>	0.1	0.5	1	1.5
<b>X = 50%</b>					
<i>minSARFI<sub>X</sub></i>	21.7971	21.4783	19.3188	16.5797	16.5797
<b>DVR branch</b>		9	56	11	6
<b>X = 70%</b>					
<i>minSARFI<sub>X</sub></i>	27.6957	27.1739	25.4638	21	21
<b>DVR branch</b>		56	9	9	9
<b>X = 80%</b>					
<i>minSARFI<sub>X</sub></i>	33.2174	31.6232	29.7101	26.0435	24.9275
<b>DVR branch</b>		9	9	12	9
<b>X = 90%</b>					
<i>minSARFI<sub>X</sub></i>	42.7101	9	36.6522	33.7826	31.2319
<b>DVR branch</b>		22	9	13	9

### 4.3. Second case of study: Placing two (multiple) DVRs in test system

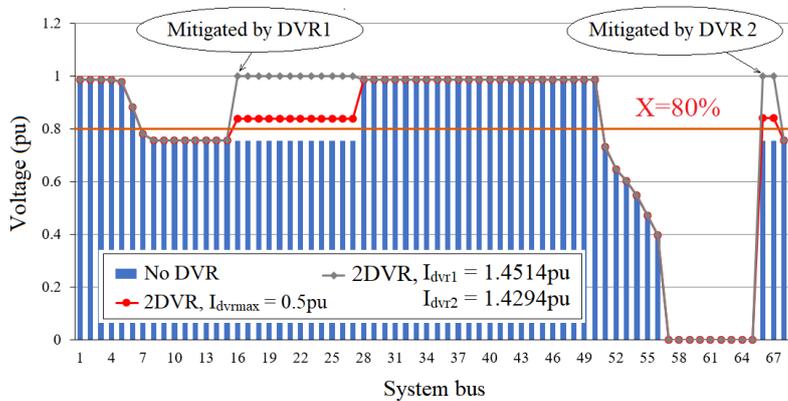
To illustrate the algorithm's performance for the case of a number of DVRs placed in the test system, we consider to optimally select the locations of two DVRs. In solving the problem of optimization considering pre-set parameters as Section 4.1, results are also step-by-step introduced for better analysis and discussion.

Firstly, we also start to consider sag  $X = 80\%$ . We consider a candidate scenario of locating two DVRs on branch 15 and branch 65 (Fig. 17). If short-circuit position is still bus 57, the required currents for DVRs for boosting the voltage at DVR's location to 1 p.u. are  $I_{DVR1} = 1.1451$  p.u. and  $I_{DVR2} = 1.4294$  p.u. respectively, which are quite high. If we consider the DVR's limited current  $I_{DVRmax} = 0.5$  p.u., the bus voltages are mitigated as shown in Fig. 18.

With regard to sag voltage level  $X = 80\%$ , without DVR, there are 40 buses having the voltage lower than  $X$ . This figure reduces to 26 buses for placing DVRs on branches 15 and 65.



**Fig. 17. Checking locations where DVRs disabled for a give fault position.**

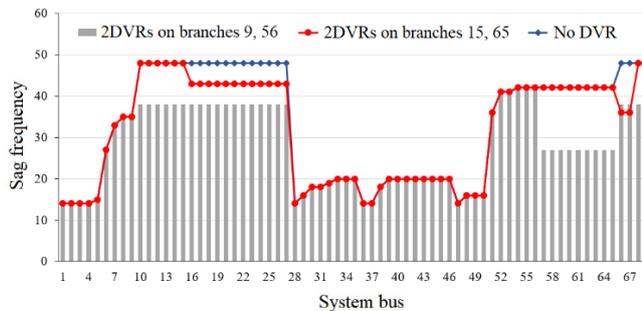


**Fig. 18. System bus voltage without and with 2 DVRs placed on branches 15 (15-16) and 65 (11-66) for short-circuit at bus 57.**

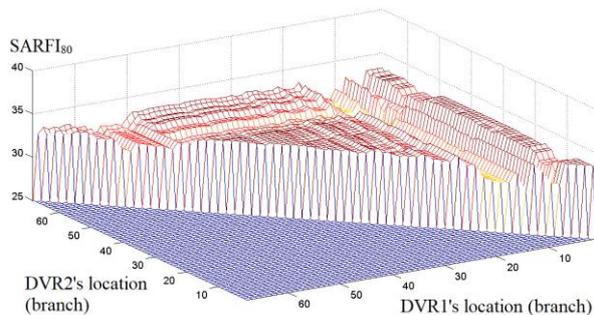
Considering other short-circuit positions (other 68 buses), the resulting sag frequency ( $X = 80\%$ ) of all buses is plotted on Fig. 19. Finally, we calculate the  $SARFI_{80} = 32$ . We similarly calculate  $SARFI_X$  for other candidate scenarios of the location of two DVRs ( $T_2 = 2278$  scenarios).

Figure 20 shows the  $SARFI_X$  for all candidate scenarios for the case of  $X = 80\%$ ,  $I_{DVRmax} = 0.5$  p.u. Each candidate scenario is a point (in the matrix of scenarios of DVR placement) with its corresponding  $SARFI_X$ . Because we do not consider the permutation of the locations between two DVRs (e.g., 1-2 is the same as 2-1), we only plot  $SARFI_X$  of the scenarios corresponding to points on the triangle from the main diagonal of the matrix of scenarios of placement of 2 DVRs. Other points are not considered and thus, its  $SARFI_X$  is given a certain value (say 25) for distinguishing with the points considered as scenarios of DVR placement. The optimal scenario is DVR1 on branch 9 (between bus 9 and bus 10), DVR2 on the branch 56 (between bus 55 and bus 56) and the minimum  $SARFI_X$  is 27.7536. The resulting voltage sag frequency of all buses in the test system with the optimal placement of two DVRs is also plotted in Fig. 19 for comparison.

Similarly, we consider other cases of pre-set parameters - threshold  $X = 50\%$ ,  $70\%$ ,  $80\%$  and  $90\%$  and DVR's limited current  $I_{DVRmax} = 0.1$  p.u.,  $0.5$  p.u.,  $1$  p.u. and  $1.5$  p.u. Each case of study generates a similar  $SARFI_X$  distribution against two DVRs placement as Fig. 20 and finally the optimal DVR's placement is achieved.



**Fig. 19. Sag frequency of all system buses for  $X = 80\%$  without and with 2 DVRs placed on branches 15 and 65 and 2 DVRs optimally placed on branches 9 (9-10) and 56 (55-56).**



**Fig. 20.  $SARFI_X$  against scenarios of two DVR placement for  $X = 80\%$  and  $I_{DVRmax} = 0.5$  p.u.**

Remarkable results are summarised in Table 2. We can see again, the larger  $I_{DVRmax}$  results in better improvement of  $SARFI_X$ . For many scenarios of pre-set parameters, the optimal placement takes branch 9 and branch 56 that is in the middle of long feeders. Higher threshold  $X$  results in higher  $SARFI_X$  but the improvement of  $SARFI_X$  with different DVR's limited current is the same.

**Table 2. Remarkable results for two DVR placement.**

Results		$I_{DVRmax}$ (p.u.)				
		No DVR	0.1	0.5	1	1.5
$X = 50\%$						
$minSARFI_X$		21.7971	21.3478	16.9275	14.1014	14.1014
DVR placement	DVR1		9	12	11	11
(branch)	DVR2		56	56	56	56
$X = 70\%$						
$minSARFI_X$		27.6957	26.8551	23.8986	17.6087	17.6087
DVR placement	DVR1		9	9	9	9
(branch)	DVR2		56	56	56	56
$X = 80\%$						
$minSARFI_X$		33.2174	30.7101	27.7536	21.7391	20.6232
DVR placement	DVR1		9	9	12	9
(branch)	DVR2		56	56	56	56
$X = 90\%$						
$minSARFI_X$		42.7101	37.8116	32.6087	27.3913	24.8406
DVR placement	DVR1		9	9	13	9
(branch)	DVR2		56	56	56	56

### 5. Conclusion

This paper introduces a new method for global voltage sag mitigation by using a number of DVRs in the distribution system where the effectiveness of global voltage sag mitigation by DVRs for the case of limited maximum current is modelled using Thevenin's superposition theorem in short-circuit calculation of power distribution systems. This method allows us to consider the DVR's effectiveness of voltage sag mitigation not only for event index but also for site and system indices. As a result, the optimal scenario of DVR placement is obtained by minimizing the resulting  $SARFI_X$  with regard to pre-set parameters including the voltage threshold  $X$  and the DVR's maximum injected current. The paper also considers the case of using a number of DVRs for global voltage sag mitigation that is applicable for large size distribution systems.

For the purpose of introducing the method, some assumptions are accompanied by the type of short-circuiting and the fault rate distribution. For real applications, the method can easily include the real fault rate distribution as well as all types of short-circuiting. DVR's effectiveness of global voltage sag mitigation is relatively limited as DVR can only compensate the voltage of buses from the DVR's location toward load side and it is also disabled if it is coupled on the fault current carrying path.

The method of optimizing the DVR placement only considers how to get the best outcome of an in-advance given a number of DVRs without considering DVR investment. That is because the cost-effective model cannot be built for such a systematic solution as global voltage sag mitigation. Further research should address this limitation by trying to quantify the benefice of global voltage sag mitigation.

**Nomenclatures**

$I_{DVR,max}$	DVR's limited current, p.u.
$I^0$	Initial injected bus current matrix (Short-circuit current), p.u.
$I_{fk}$	Fault current at bus $k$ , p.u.
$I_{k,DVR}^*$	Injected current by DVR on the branch $k$ , ( $k \in M$ ) that boosts the voltage of bus $k$ to 1 p.u.
$I_{t,DVR}$	Injected current by DVR on the branch $t$ , p.u.
$k_{t,*}$	Factor for DVR's voltage compensating position
$k_{t,DVR}$	Factor for placing DVR on branch $t$
$M$	Set of $m$ branches to connect to $m$ DVRs
$m^*$	Buses of voltage compensation by DVR
$SARFI_X$	System average RMS variation frequency index
$T_j$	Set of branches connected to bus $j$
$T_m$	Total scenarios of DVR placement to be tested
$U$	Bus voltage matrix after adding DVRs in system, p.u.
$U^0$	Initial bus voltage matrix (Voltage sag at all buses during power system short-circuit), p.u.
$U_{sag,i}$	Voltage sag at bus $i$ during power system short-circuit
$X$	RMS voltage threshold, %
$Z_{bus}$	System bus impedance matrix calculated from bus admittance matrix, p.u.

**Greek Symbols**

$\Delta I$	Additional injected current matrix by DVR, p.u.
$\Delta I_i$	Additional injected current to the bus $i$ after adding DVRs in system, p.u.
$\Delta U$	Bus voltage improvement matrix after adding the DVRs in system, p.u.
$\Delta U_i$	Bus $i$ voltage improvement after adding DVRs in system, p.u.

**Abbreviations**

DVR	Dynamic Voltage Restorer
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