MITIGATION OF SUB-SYNCHRONOUS RESONANCE WITH STATIC VAR COMPENSATOR

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Abstract

The rapid growth of the power sector and emergence leads towards bulk power transfer over long transmission lines. This issue demands series compensation of transmission line. Series compensation of transmission line not only enhances the stability of the power system but also may potentially lead the system towards the problem of Subsynchronous-Resonance (SSR). This paper presents the detailed small signal model of IEEE First Benchmark Model (FBM) for eigenvalue analysis in order to identify responsible modes of oscillations for SSR. To mitigate SSR, the application of Static Var Compensator (SVC) located in the transmission line has been investigated. Two individual supplementary control strategies, namely generator terminal voltage deviation and generator rotor speed deviation, for SVC have been implemented. The coordinated operation of the power system stabilizer in the generator control loop and terminal voltage deviation as a supplementary signal for SVC is tested in this paper. It is found that the combined operation can successfully mitigate SSR and stabilize the system with a high degree of series compensation.

Keywords: Eigenvalues, IEEE FBM, Series compensation, Static VAR compensator, Subsynchronous resonance.
1. Introduction

The different masses of turbine-generators shafts are much susceptible to interaction with the electrical resonances of a transmission network, which is compensated with conventional series capacitors. This can lead to a form of instability known as Subsynchronous Resonance (SSR), which can result in failure of turbine-generator shafts [1, 2]. Extensive research has been started to understanding SSR phenomenon after two successive shaft failures incidences reported at Mohave power plant in USA in 1970 and 1971. There are two main characteristics of SSR phenomenon, namely, (a) self-excitation (also known as steady state SSR) and (b) transient torques (also known as transient SSR) [1-5]. The currents entering generator terminals and oscillating with sub-synchronous frequency produce sub-synchronous frequency voltage components at generator terminal. These voltages can sustain sub-synchronous frequency currents to produce the effect that is termed as self-excitation, which is further categorized into two categories: (i) Induction Generator Effect (IGE) and (ii) Torsional Interaction (TI). In IGE, only rotor electrical dynamics is involved whereas TI deals with both rotors electrical and mechanical dynamics.

The system disturbances such as sudden load changes, faults or tripping of the lines can excite oscillatory torques on the generator rotor. The transient oscillatory electrical torques thus produced may have unidirectional, exponentially decaying as well as oscillatory torques components ranging from sub-synchronous to multiples of network frequency. The sub-synchronous frequency components of torques with large amplitudes just after the disturbance affect the shaft life due to fatigue damage and are analyzed under transient SSR phenomena. Several mitigation techniques for SSR mitigation such as blocking filters using static circuitry, excitation controllers using supplementary signals, torsional relays and many others are reported in the literature [5]. Kumar and Kumar [6] commented, line current and active power-based controller are used for SSR mitigation using SVC. According to Padiyar and Varma [7], damping torque investigation of SMIB system has been presented with SVC located at mid-way of the transmission line. SVCs are basically used for bus voltage regulation. However, SVC output using some supplementary signals can be used to mitigate and damp out power system oscillations [8-12]. Zhijun et al. [13] and Jovicic et al. [14] presented the dynamic phasor and analytical model of SVC. Jusan et al. [15] also described the SSR study using SVC, where the front and rear speeds of HP stage of turbine and exciter, respectively. Wasynczuk [12] earlier addressed this is used as control concept. Zhu et al. [16] explained that the rotor angle deviation signal as a supplementary control signal has been used along with reactive power control loop of unified power flow controller to damp out oscillation caused by subsynchronous resonance. Based on studies by Panda et al. [17], the similar approach is used, but here supplementary signal is used for static synchronous series compensator along with main control loop for effective damping of SSR. Nagarajan and Kumar [18] highlighted that the fuzzy logic control of static VAR system is described; also, the superiority of static VAR system is explained with respect to reactive power generation and absorption. Sreeranganayakulu et al. [19] explained that the effectiveness of SVC is proved using IEEE second benchmark model, but here the linearized model is not tested.

In this paper, systematic mathematical modelling of IEEE FBM model has been formulated with the linearized equation to carry out eigenvalue analysis. The generator model of basic benchmark system has been modified to incorporate
modelling of Automatic Voltage Regulator (AVR) and Power System Stabilizer (PSS). In addition to that, compressive SVC model has been prepared and an attempt is made to mitigate SSR using two different supplementary signals. In all previous work, the modelling of AVR and PSS are not included, also the comprehensive linearized modelling of SVC with its control part are not included.

2. Power System Modelling for SSR Studies

Figure 1 shows IEEE First Benchmark Model (FBM) for the analysis of SSR [2]. The test system data utilized in this paper is described in Appendix A. Before representing the modelling of SVC to mitigate SSR, the turbine-generator models, excitation system and electrical network interfacing with series compensation are modelled to carry out SSR without considering SVC. The formulation of state equations first starts with individual component modelling, which subsequently integrated to form overall combined state space systems for the computation of eigenvalues of the entire network to represent different modes of oscillations.

![Fig. 1. Turbine-Generator shaft representation along with network as per IEEE FBM.](image)

2.1. Modelling of synchronous generator

Type 2.2 model of the synchronous machine is used in this work. This model of synchronous machine has one field winding \( f_d \) and one damper windings \( k_d \) on \( d \)-axis of rotor whereas two damper windings \( k_{q1} \) and \( k_{q2} \) on \( q \)-axis of the rotor [20]. The state space equations for the synchronous generator is given as per Eq. (1) using flux linkage dynamics of stator and rotor windings as state variables.

\[
\frac{d}{dt} \Delta x = \begin{bmatrix} A_x \end{bmatrix} \Delta x + \begin{bmatrix} B_{xq} \end{bmatrix} \begin{bmatrix} \Delta V_{eq} \\ \Delta V_{ed} \end{bmatrix} + \begin{bmatrix} B_{x\omega} \end{bmatrix} \Delta \omega + \begin{bmatrix} B_{xT} \end{bmatrix} \begin{bmatrix} \Delta V_{eq} \\ \Delta V_{ed} \end{bmatrix}
\]

where
\[
\Delta x = \begin{bmatrix}
\Delta \psi_{qr} & \Delta \psi_{d} & \Delta \psi_{q1} & \Delta \psi_{q2} & \Delta \psi_{j1} & \Delta \psi_{j2}
\end{bmatrix}
\]
and \( \Delta \) represents a small change in quantities.

Other matrices are listed in Appendix B.

2.2. Modelling of turbine-generator unit-mechanical system

The turbine–generator unit considered as lumped masses forms the mechanical system and it consists of six masses of different pressure stages as shown in Fig. 1 [2]. The synchronous generator and an exciter are also coupled on the same shaft. The state equation for a mechanical system is given in per unit as per Eq. (2). In the presented mechanical system, there are total twelve state variables of mechanical systems including six of speed deviation of each masses and another six are angular position of each masses.

\[
[2H]p\omega+[D]\omega+[K]+T=0
\]

(2)

where the shaft stiffness matrix is denoted by \([K]\). The \([H]\) and \([D]\) matrices are diagonal matrix of inertias and damping constants, respectively. \(T\) gives torque vector acting on the different pressure stages of shaft and \(\omega\) is the speed vector. The inertia constants and shaft stiffness of different stages are given in Appendix A. Damping is ignored in presented work.

2.3. Electrical network interfacing

The electrical equivalent circuit for the IEEE first benchmark model to study SSR is shown in Fig. 2. The generator with constant voltage source \(E_g\) is connected to the an infinite bus through series compensated transmission line. The terminal voltage of the generator is \(V_t\).

\[
\begin{align*}
V_{ld} &= R_i l_d + \frac{X_l}{\omega_b} p_i_d - \frac{\omega}{\omega_b} X_{l} j_q + v_{ld} + V_e \sin \delta \\
V_{dq} &= R_i l_q + \frac{X_l}{\omega_b} p_i_q + \frac{\omega}{\omega_b} X_{l} j_d + v_{dq} + V_e \cos \delta
\end{align*}
\]

(3)

Fig. 2. Network model of FBM.
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\[
\begin{align*}
    pV_{cd} &= \omega v_{cq} + \omega_0 X_{c_d} i_d \\
    pV_{cq} &= -\omega v_{cd} + \omega_0 X_{c_q} i_q
\end{align*}
\]

(4)

2.4. Integration of system components

The set of state space Eqs. (1) to (4) of individual components are integrated to formulate the state equations of the overall system in order to capture the different dynamic events related to SSR. The dimensions of A matrix of the integrated system consist of 20×20.

3. Network Model with SVC

The main component of the SVC is the parallel combination of Fixed Capacitor (FC) and Thyristor-Controlled Reactor (TCR). Figure 3 shows the presence of SVC at the predetermined location of the line. The location of SVC is ascertained by considering equivalent values of line impedance \( R_1 + j\omega L_1 \) and \( R_2 + j\omega L_2 \). The state space equations after incorporating SVC are formulated as per Eqs. (5) to (10).

### Fig. 3. Network model with SVC.

The voltages in d-q frame at the location of SVC in d-q frame are given by Eqs. (5) and (6) and voltages across series capacitor are represented by Eqs. (7) and (8).

\[
\begin{align*}
    V_{dq} &= R_2 i_{dq} + \frac{X_{L2}}{\omega_0} p i_{dq} + \frac{\omega}{\omega_0} X_{L2} i_{dq} + V_{eq} + V_a \cos \delta \\
    V_{ld} &= R_2 i_{ld} + \frac{X_{L2}}{\omega_0} p i_{ld} - \frac{\omega}{\omega_0} X_{L2} i_{ld} + V_{eq} + V_a \sin \delta
\end{align*}
\]

(5)

\[
\begin{align*}
    pV_{eq} &= -\omega V_{eq} + \omega_0 X_{c} i_{eq} \\
    pV_{eq} &= \omega V_{eq} + \omega_0 X_{c} i_{eq}
\end{align*}
\]

(6)

The voltages \( V_i \) at the location of SVC in d-q frame are represented below.

\[
\begin{align*}
    C_a pV_{iq} + \omega C_a V_{id} &= I_{iq} - I_{3q} - I_{2q} \\
    C_a pV_{id} - \omega C_a V_{iq} &= I_{id} - I_{3d} - I_{2d}
\end{align*}
\]

(9)

(10)
The set of Eqs. (5) to (10) in linearized form are obtained to form the state equation in terms of the voltage across the series capacitor and the current flowing through it as well as the voltage at SVC terminal.

\[
\Delta V_{sd} = \begin{bmatrix}
\frac{a_b}{X_{l2}} - \frac{a_b}{X_{l2}} & 0 & \frac{a_b}{X_{l2}} & 0 \\
\frac{a_b}{X_{l2}} & 0 & \frac{a_b}{X_{l2}} & 0 \\
a_bX_{s} & 0 & a_bX_{s} & 0 \\
-\epsilon_bX_{s} & 0 & 0 & a_b \\
-\epsilon_bX_{s} & 0 & 0 & a_b \\
\end{bmatrix} \begin{bmatrix}
\Delta i_{d} \\
\Delta i_{q} \\
\Delta V_{q} \\
\Delta V_{d} \\
\end{bmatrix}
\]

(11)

Similarly, voltage differences between generator terminal and SVC location are given in Eqs. (12) and (13).

\[
V_{q} - V_{iq} = R_{i} i_{q} + \frac{X_{l1}}{a_{0}} p_{i_{q}} + \frac{\omega}{a_{b}} X_{l} i_{d}
\]

(12)

\[
V_{d} - V_{id} = R_{i} i_{d} + \frac{X_{l1}}{a_{0}} p_{i_{d}} - \frac{\omega}{a_{b}} X_{l} i_{q}
\]

(13)

Using the linearized value of above mentioned \(V_{iq}\) and \(V_{id}\) into Eq. (1), the linearized state equations of stator and rotor flux linkages can be formed.

The current \(i_{d}\) through reactor of SVC in Fig. 3 can be modeled by considering only the reactance \(L_{s}\) as below after neglecting its resistance \(R_{s}\).

\[
V_{iq} = L_{s} i_{q} + \omega L_{q} i_{sd} = \frac{X_{l5}}{a_{b}} p_{i_{q}} + \frac{\alpha X_{l5}}{a_{b}} i_{sd}
\]

(14)

\[
V_{id} = L_{s} i_{sd} - \omega L_{q} i_{iq} = \frac{X_{l5}}{a_{b}} p_{i_{sd}} - \frac{\alpha X_{l5}}{a_{b}} i_{q}
\]

(15)

Linearizing and rearranging, we have:

\[
p \Delta i_{iq} = \alpha_{0} \Delta V_{iq} \Delta B + \alpha_{0} R_{l} \Delta V_{iq} - \alpha_{0} \Delta i_{iq} - i_{id} \Delta \omega
\]

(16)

\[
p \Delta i_{id} = \alpha_{0} \Delta V_{id} \Delta B + \alpha_{0} R_{l} \Delta V_{id} + \alpha_{0} \Delta i_{id} + i_{iq} \Delta \omega
\]

(17)

Control of SVC

The small signal model for SVC control to be used for SSR mitigation is shown in Fig. 4. The perturbation in terminal voltage at SVC location and current through
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TCR in Fig. 3 are fed to the reference point. The measurement time constant for both voltage and current is assumed to be equal and represented by $T_m$. PI control block acts as a voltage regulator. Firing delay time and average dead time are given by $T_D$ and $T_S$, respectively. These parameters values are listed in Appendix A. The feedback signal of voltage perturbation changes the susceptance. $\Delta V_f$ may be used as a supplementary feedback signal, it is ignored at present.

The dynamic equations of SVC compensated network using the notations used in Fig. 4 are as follows.

$$p\Delta Z_1 = \Delta V_{ref} - \Delta Z_2 + \Delta V_f$$ (18)

where $V_f$ is for supplementary feedback signal, which has been ignored.

$$p\Delta Z_2 = \frac{1}{T_m} (\Delta V_1 - K_p \Delta i_t) - \frac{1}{T_m} \Delta Z_2$$ (19)

$$\Delta V_f = \frac{V_{ref}}{V_{ref}} \Delta V_{id} + \frac{V_{ref}}{V_{ref}} \Delta V_{iq}$$ (20)

Similarly,

$$\Delta i = \frac{i_{d0}}{i_{d0}} \Delta i_{id} + \frac{i_{q0}}{i_{q0}} \Delta i_{iq}$$ (21)

From Eqs. (19), (20) and (21), the following equation is obtained,

$$p\Delta Z_2 = \frac{1}{T_m} \left[ \frac{V_{ref}}{V_{ref}} \Delta V_{id} + \frac{V_{ref}}{V_{ref}} \Delta V_{iq} - K_D \left( \frac{i_{d0}}{i_{d0}} \Delta i_{id} + \frac{i_{q0}}{i_{q0}} \Delta i_{iq} \right) \right] - \frac{1}{T_m} \Delta Z_2$$ (22)

Other state variable linearized equations are,

$$p\Delta Z_3 = -\frac{K_f}{T_s} \Delta Z_1 + \frac{K_p}{T_s} \Delta Z_2 - \frac{1}{T_s} \Delta Z_3 - \frac{K_p}{T_s} \Delta V_{ref} - \frac{K_p}{T_s} \Delta V_f$$ (23)

$$p\Delta B = \frac{\Delta Z_3}{T_D} - \frac{\Delta B}{T_D}$$ (24)
The matrix for control diagram equations is obtained using Eqs. (16) to (18) and (22) to (24). The total state variable of the system with the SVC and its control will be thirty. Hence, the A matrix will be of the order of 30x30.

4. Modelling of AVR and PSS

The use of AVR is needed for the synchronous generator to restore its terminal voltage automatically in the event of load changes or fault condition. To make a response of AVR faster, the gain $K_A$ of PSS is set to a higher value, which in turn reduces the damping torque of the system.

When the system is working at higher loading conditions and the synchronous generator is connected to load through a larger reactance, the use of AVR can result in negative damping torque in the system and the system may become oscillatory unstable. To avoid oscillatory instability and to compensate for negative damping torque effect of AVR, PSS as shown in Fig. 5 is used. PSS can provide the necessary phase shift through its lead-lag blocks depending on the requirement and can successfully make the system stable.

Total five state variables will be added in the system with the addition of AVR and PSS. The state equation of AVR and PSS are not listed here, it can be easily derived from its block diagram figure. After addition of AVR and PSS into the test system, the total state variable will be thirty-five.

![Fig. 5. Block diagram of AVR and PSS.](image)

5. Use of Supplementary Signal for Mitigation of SSR

5.1. Rotor speed deviation as a supplementary signal

Here rotor speed deviation is taken as a supplementary signal to mitigate SSR. The block diagram of the transfer function is given in Fig. 6.
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\[
\Delta V_F = -\frac{\Delta V_L}{T_4} + C_i \left[ \frac{\Delta V_q}{\Delta V_d} \right] + C_2 \frac{d}{dt} \left[ \frac{\Delta V_q}{\Delta V_d} \right]
\]

5.2. Generator terminal voltage deviation as a supplementary signal

Here generator terminal voltage deviation is taken as a supplementary signal to mitigate SSR. The block diagram of the transfer function is given in Fig. 7.

\[
pZ_c = -\frac{1}{T_4} Z_c + \left[ \frac{K_\beta}{T_4} \left( \frac{1 - T_3}{T_4} \right) \right] \Delta \omega
\]

\[
V_r = K_\beta \left( \frac{T_3}{T_4} \right) \Delta \omega + Z_c
\]
where,
\[ C_1 = K_B \left[ V_{q0} \ V_{d0} \ V_{q0} \ V_{d0} \ V_{q0} \ V_{d0} \right] \]
\[ C_2 = K_B \left( \frac{T_2}{T_1} \right) \left[ V_{q0} \ V_{d0} \ V_{q0} \ V_{d0} \ V_{q0} \ V_{d0} \right] \]

Incorporating above equations in the system discussed in section IV, we have now total thirty-six state variables, hence the A matrix of state space equation will be 36x36.

6. Results and Discussion

In this paper, the various state space equations formulation is categorized in different cases as below.

Case I: The overall base system explained in Section II.

Case II: Inclusion of SVC in case I system.

Case III: Inclusion of AVR and PSS in case II system.

Case IV:
(a) Addition of rotor speed deviation as a supplementary signal in case III system.
(b) Addition of generator terminal voltage deviation as a supplementary signal in case III system.

The data used for SSR analysis with IEEE FBM shown in Fig. 2 are given in Appendix A. The series compensation level considered in this work is equal to 50%, i.e., the total inductive reactance (including transformer and transmission line) is compensated by 50% by incorporating capacitive compensation. The objectives of the work are to carry out eigenvalue analysis for all above-listed cases and to investigate the effect of series compensation on SSR.

The eigenvalue analyses for first three cases have been reported in Table 1 to identify the torsional modes, which have the frequency of oscillations in a sub-synchronous frequency range. Some torsional modes are highlighted, which have the positive real part, which is responsible to create subsynchronous resonance in the system. The real part of eigenvalues must be negative for stable operation. In Table 1, some eigenvalues of torsional modes have positive real values; hence this case situation leads towards the subsynchronous resonance. The use of SVC can improve one of the torsional modes as compared to the base system but not fully succeed to mitigate SSR. The addition of AVR and PSS in the generator control loop can improve the torsional mode but left the system with insufficient damping, which can be noticed through larger real part of eigenvalues related to modes of rotor windings.

Hence, it is observed that the system with SVC in transmission network and utilizing AVR and PSS in generator control loop are unable to mitigate the torsional oscillations when the system is compensated with a series capacitor. In order to mitigate the impact of SSR and to stabilize the system with series compensation, the application of two different supplementary signals has been explored. Table 2 shows the performance of the system after using two
supplementary signals. The use of rotor speed deviation is still not capable to mitigate SSR because it has some eigenvalues with positive real parts, but it can be noticed from Table 2 that the use of generator terminal voltage deviation as a supplementary signal can successfully mitigate all torsional modes and make the system stable. All eigenvalues show the negative real parts and result in higher damping even with 50% compensation level.

Time domain simulations are also carried out for case III and case IV(b) in MATLAB-Simulink using derived equations. Figures 8 and 9 show the time domain response of oscillations of masses and the shaft torque between LPA and LPB turbine for the disturbance applied at t = 6 seconds for case III and case IV(b) respectively. The results of time domain simulations validate the results obtained from the eigenvalue analysis.

Table 1. Results of eigenvalue analysis for case I to III.

<table>
<thead>
<tr>
<th>Modes of oscillations</th>
<th>IEEE FBM with 50% series compensation (Case I)</th>
<th>IEEE FBM 50% series compensation + SVC installed on transmission line (Case II)</th>
<th>IEEE FBM SVC+AVR-PSS (Case III)</th>
</tr>
</thead>
<tbody>
<tr>
<td>Supersynchronous mode</td>
<td>-4.716 ± 623.63i</td>
<td>-1.298 ± 879.60i</td>
<td>-2.636 ± 130.35i</td>
</tr>
<tr>
<td>Torsional mode</td>
<td>± 297.97i</td>
<td>± 202.84i</td>
<td>± 160.50i</td>
</tr>
<tr>
<td>Torsional mode</td>
<td>0.0004 ± 126.94i</td>
<td>0.001 ± 98.70i</td>
<td>0.001 ± 59.90i</td>
</tr>
<tr>
<td>Torsional mode</td>
<td>0.001 ± 98.70i</td>
<td>0.001 ± 98.70i</td>
<td>0.998 ± 11.870i</td>
</tr>
<tr>
<td>Torsional mode</td>
<td>-2.636 ± 130.35i</td>
<td>-1.987 ± 49.45i</td>
<td>-5.88 ± 376.84i</td>
</tr>
<tr>
<td>Subsynchronous mode</td>
<td>0.001 ± 1357.1638373</td>
<td>-1.868 ± 125.255i</td>
<td>-101.12 ± 3.78i</td>
</tr>
<tr>
<td>Swing mode</td>
<td>0.0001 ± 1357.1638373</td>
<td>-3.498 ± 70.93i</td>
<td>-4.79 ± 1.58i</td>
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<tr>
<td>Modes related to</td>
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<td>-1.20 ± 202.64i</td>
</tr>
<tr>
<td>Modes related to</td>
<td>0.0001 ± 1357.1638373</td>
<td>-4.074 ± 879.60i</td>
<td>-1.00 ± 160.48i</td>
</tr>
<tr>
<td>Damper winding, field</td>
<td>0.0001 ± 1357.1638373</td>
<td>-4.074 ± 879.60i</td>
<td>-1.20 ± 202.64i</td>
</tr>
<tr>
<td>winding and SVC</td>
<td>0.0001 ± 1357.1638373</td>
<td>-4.074 ± 879.60i</td>
<td>-1.00 ± 160.48i</td>
</tr>
<tr>
<td>Modes related to</td>
<td>0.0001 ± 1357.1638373</td>
<td>-4.074 ± 879.60i</td>
<td>-1.20 ± 202.64i</td>
</tr>
<tr>
<td>Supersynchronous mode</td>
<td>-4.074 ± 879.60i</td>
<td>-2.454 ± 704.320i</td>
<td>-0.79 ± 0.462</td>
</tr>
<tr>
<td>Torsional mechanical</td>
<td>0.0001 ± 1357.1638373</td>
<td>-4.074 ± 879.60i</td>
<td>-1.309</td>
</tr>
<tr>
<td>mode</td>
<td>-4.074 ± 879.60i</td>
<td>-2.454 ± 704.320i</td>
<td>-0.019 ± 202.83i</td>
</tr>
<tr>
<td>Subsynchronous mode</td>
<td>0.0001 ± 1357.1638373</td>
<td>-4.074 ± 879.60i</td>
<td>-1.20 ± 202.64i</td>
</tr>
<tr>
<td>Swing mode</td>
<td>0.0001 ± 1357.1638373</td>
<td>-4.074 ± 879.60i</td>
<td>-1.00 ± 160.48i</td>
</tr>
<tr>
<td>Modes related to</td>
<td>0.0001 ± 1357.1638373</td>
<td>-4.074 ± 879.60i</td>
<td>-1.20 ± 202.64i</td>
</tr>
</tbody>
</table>
-0.47 ± 126.69i  
-18.255 ± 101.48i  
-26.60 ± 33.21i  
3.72 ± 83.46i  
-5.00 ± 0.46i  
-1.081 ± 0.147i  
-0.001, -0.0886  
2.443, 16.051  
-20.498  
-54.271  
-94.63, -108.99  
-112.82, -1000.43

<table>
<thead>
<tr>
<th>Mode Type</th>
<th>Frequency</th>
<th>Real Part</th>
<th>Imaginary Part</th>
</tr>
</thead>
<tbody>
<tr>
<td>Subsynchronous mode</td>
<td>5.00 ± 0.846i</td>
<td>-0.041 ± 14.327i</td>
<td></td>
</tr>
<tr>
<td>Swing mode</td>
<td>1.081 ± 0.147i</td>
<td>0.001, 0.0886</td>
<td></td>
</tr>
<tr>
<td>Torsional mechanical mode</td>
<td>-0.006 ± 126.94i</td>
<td>-4.732 ± 17.873i</td>
<td></td>
</tr>
<tr>
<td>Torsional mechanical mode</td>
<td>-0.11 ± 98.547i</td>
<td>-5.738 ± 31.325i</td>
<td></td>
</tr>
</tbody>
</table>

Modes related to damper winding, field winding and SVC.

Fig. 8. Time domain simulation results.

(a) Oscillations of masses.

(b) LPA-LPB shaft torque for case III.
The paper presents the state space modelling of different electrical components to study SSR of IEEE FBM. The series compensation of transmission network with a certain degree of compensation may result in SSR. In this paper, an attempt is made to present a systematic approach to form the state space equation of IEEE FBM with different component modelling. For mitigating SSR, the modelling of the system with SSR is formulated and use of two supplementary feedback signals to operate SVC has been presented. Only SVC can’t mitigate the subsynchronous resonance. As per case III, the use of AVR and PSS helps to improve the torsional modes of oscillations, but they resulted in lack of damping. The application of SVC with generator terminal voltage deviation as supplementary feedback signal has proved to be superior for mitigating SSR. The obtained results are strongly supported by the time domain simulation waveforms. In the presented work, the location of the series capacitor is at the far end of the transmission line but the interested reader can carry out further work by choosing any location of the series capacitor on the transmission line.

### Nomenclatures

<table>
<thead>
<tr>
<th>Symbol</th>
<th>Description</th>
</tr>
</thead>
<tbody>
<tr>
<td>$C_n$</td>
<td>Capacitance of capacitor of SVC, farad (in per unit)</td>
</tr>
<tr>
<td>$I_{2d}$, $I_{2q}$</td>
<td>Current through series capacitor in d-q reference frame, pu amp</td>
</tr>
<tr>
<td>$I_{3d}$, $I_{3q}$</td>
<td>Current through TCR in d-q reference frame, pu amp</td>
</tr>
<tr>
<td>$i_{sd}$, $i_{sq}$</td>
<td>Generator stator current in d-q reference frame, pu amp</td>
</tr>
<tr>
<td>$K_p$, $K_i$</td>
<td>Proportional and integral control gain respectively</td>
</tr>
<tr>
<td>$R$</td>
<td>Resistance of transmission line, pu ohm</td>
</tr>
<tr>
<td>$r_{fd}$</td>
<td>Resistance of field winding, pu ohm</td>
</tr>
<tr>
<td>$r_{kd}$</td>
<td>Resistance of damper winding on d axis, pu ohm</td>
</tr>
<tr>
<td>$r_{kq1}$, $r_{kq2}$</td>
<td>Resistance of damper windings on q axis, pu ohm</td>
</tr>
<tr>
<td>$r_s$</td>
<td>Resistance of stator winding, pu ohm</td>
</tr>
<tr>
<td>$T_1$, $T_2$</td>
<td>Time delay for lag-lead network of PSS, second</td>
</tr>
<tr>
<td>$T_p$, $T_d$</td>
<td>Time delay for supplementary signal, second</td>
</tr>
<tr>
<td>$T_D$</td>
<td>Firing delay time, second</td>
</tr>
<tr>
<td>$T_m$</td>
<td>Measuring delay time, second</td>
</tr>
<tr>
<td>$T_S$</td>
<td>Average dead time, second</td>
</tr>
</tbody>
</table>
\( V_e \)  
Infinite bus voltage, pu volt

\( V_{1d}, V_{1q} \)  
Voltage in d-q reference frame at SVC terminal, pu volt

\( V_{cd}, V_{cq} \)  
Voltage across series capacitor in d-q reference frame, pu volt

\( V_{td}, V_{tq} \)  
Generator terminal voltages in d-q reference frame, pu volt

\( x_{ad} \)  
Direct axis reactance of synchronous machine, pu ohm

\( x_{aq} \)  
Quadrature axis reactance of synchronous machine, pu ohm

\( X_C \)  
Series compensation of transmission line, pu ohm

\( X_L \)  
Inductive reactance of transmission line, pu ohm

\( x_{cd}, x_{cq} \)  
Voltage across series capacitor in d-q reference frame, pu volt

\( x_{ad} \)  
Leakage reactance of field winding of synchronous machine, pu ohm

\( x_{al}, x_{al2} \)  
Leakage reactance of q axis rotor winding of synchronous machine, pu ohm

\( x_{ls} \)  
Leakage reactance of stator winding of synchronous machine, pu ohm

\( x_{md} \)  
Direct axis mutual reactance of synchronous machine, pu ohm

**Greek Symbols**

\( \delta \)  
Angular displacement of rotating mass (Fig. 1), radian

\( \psi \)  
Flux linkages, weber

\( \psi_{ds}, \psi_{qs} \)  
Stator flux linkages in d-q axis, weber

\( \psi_{ds0}, \psi_{qs0} \)  
Initial values of stator flux linkages in d-q axis, weber

\( \psi_{fd} \)  
Flux linkages of field winding, weber

\( \psi_{ld} \)  
Flux linkages of damper winding on d axis, weber

\( \psi_{lq1}, \psi_{lq2} \)  
Flux linkages of damper windings on q axis, weber

\( \omega \)  
Angular speed of rotating mass, rad/s.

\( \omega_b \)  
Base speed, rad/s.

**Abbreviations**

AVR  
Automatic Voltage Regulator

FBM  
First Benchmark Model

HP  
High Pressure

IGE  
Induction Generator Effect

PSS  
Power System Stabilizer

SSR  
Subsynchronous Resonance

SVC  
Static Var Compensator

TCR  
Thyristor Control Reactor

TI  
Torsional Interaction

**References**


Mitigation of Sub-synchronous Resonance with Static Var Compensator


### Appendix A

**Synchronous machine parameters for IEEE-FBM: Values are in pu**

\[ X_{T}=0.14 \text{ pu}, R_{L}=0.02, X_{L}=0.50, X_{SYS}=0.06, \\
X_{C}=0.371, P_{e}=0.9, V_{T}=1. \]

<table>
<thead>
<tr>
<th>Reactance</th>
<th>Value (pu)</th>
<th>Time constant</th>
<th>Value (sec)</th>
</tr>
</thead>
<tbody>
<tr>
<td>X(_d)</td>
<td>1.79</td>
<td>T(_d)(_0)</td>
<td>4.3</td>
</tr>
<tr>
<td>X(_d^*)</td>
<td>0.169</td>
<td>T(_d^*)(_0)</td>
<td>0.032</td>
</tr>
<tr>
<td>X(_q^*)</td>
<td>0.135</td>
<td>T(_q^*)(_0)</td>
<td>0.85</td>
</tr>
<tr>
<td>X(_q)</td>
<td>1.71</td>
<td>T(_q)(_0)</td>
<td>0.05</td>
</tr>
<tr>
<td>X(_q^*)</td>
<td>0.228</td>
<td></td>
<td></td>
</tr>
<tr>
<td>X(_ad)</td>
<td>0.13</td>
<td></td>
<td></td>
</tr>
</tbody>
</table>

#### Mechanical masses value IEEE-FBM.

<table>
<thead>
<tr>
<th>Inertia constant (H)</th>
<th>Shaft section</th>
<th>Spring constant (K) in pu torque/rad</th>
</tr>
</thead>
<tbody>
<tr>
<td>HP turbine</td>
<td>0.092897</td>
<td>HP-IP</td>
</tr>
<tr>
<td>IP turbine</td>
<td>0.155589</td>
<td>IP-LPA</td>
</tr>
<tr>
<td>LPA turbine</td>
<td>0.85867</td>
<td>LPA-LPB</td>
</tr>
<tr>
<td>LPB turbine</td>
<td>0.884215</td>
<td>LPB-GEN</td>
</tr>
<tr>
<td>Generator</td>
<td>0.868495</td>
<td>GEN-EXC</td>
</tr>
<tr>
<td>Exciter</td>
<td>0.034217</td>
<td></td>
</tr>
</tbody>
</table>

#### Parameters of different control circuit.

<table>
<thead>
<tr>
<th>Parameter</th>
<th>K(_P)</th>
<th>K(_I)</th>
<th>K(_D)</th>
<th>T(_S)</th>
<th>T(_D)</th>
<th>T(_M)</th>
<th>B</th>
</tr>
</thead>
<tbody>
<tr>
<td>Value</td>
<td>0.3</td>
<td>30</td>
<td>0.05</td>
<td>0.02066</td>
<td>0.008</td>
<td>0.001</td>
<td>1 pu</td>
</tr>
</tbody>
</table>

<table>
<thead>
<tr>
<th>Parameter</th>
<th>T(_R)</th>
<th>K(_F)</th>
<th>K(_A)</th>
<th>T(_F)</th>
<th>T(_A)</th>
<th>K(_R)</th>
<th>X(_C_R)</th>
</tr>
</thead>
<tbody>
<tr>
<td>Value</td>
<td>0.01</td>
<td>0.058</td>
<td>30</td>
<td>0.62</td>
<td>0.04</td>
<td>30</td>
<td>1 pu</td>
</tr>
</tbody>
</table>

<table>
<thead>
<tr>
<th>Parameter</th>
<th>T(_1)</th>
<th>T(_2)</th>
<th>T(_3)</th>
<th>K(_B)</th>
<th>K(_STAB)</th>
<th>T(_W)</th>
</tr>
</thead>
<tbody>
<tr>
<td>Value</td>
<td>0.154</td>
<td>0.033</td>
<td>0.001</td>
<td>0.03</td>
<td>0.065</td>
<td>10</td>
</tr>
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</table>
Appendix B

\[
[A] = \begin{bmatrix}
\frac{r_e}{x_s} & \frac{r_e}{x_s} & \frac{r_e}{x_{sd}} & 0 & 0 \\
\frac{r_e}{x_s} & \frac{r_e}{x_s} & \frac{r_e}{x_{sd}} & 0 & 0 \\
\frac{r_e}{x_{sd}} & \frac{r_e}{x_{sd}} & \frac{r_e}{x_{sd}} & 0 & 0 \\
0 & \frac{r_e}{x_{sd}} & \frac{r_e}{x_{sd}} & 0 & 0 \\
0 & \frac{r_e}{x_{sd}} & \frac{r_e}{x_{sd}} & 0 & 0 \\
\end{bmatrix}
\]

\[
B_{e} = \begin{bmatrix}
\omega_b & 0 & 0 & 0 & 0 \\
0 & \omega_b & 0 & 0 & 0
\end{bmatrix}
\]

\[
B_{eM} = \begin{bmatrix}
-\psi_{eM0} & \psi_{eM0} & 0 & 0 & 0
\end{bmatrix}
\]

\[
B_{r} = \begin{bmatrix}
0 & 0 & \omega_b & 0 & 0 \\
0 & 0 & \omega_b & 0 & 0 \\
0 & 0 & 0 & \omega_b r_{fd} & 0 \\
0 & 0 & 0 & 0 & \omega_b \\
\end{bmatrix}
\]