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PARTIALLY PRESTRESSED CONCRETE BEAMS UNDER LIMITED CYCLES OF REPEATED LOADING

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Abstract

This paper deals with the flexural strength as well as serviceability performance of partially prestressed concrete flexural members with bonded and unbonded prestressed strands under limited cycles of repeated loading. The experimental program involves testing twelve beams divided into two groups. The first group, which exposed to monotonic static loading, consists of six beams, three with pretension-bonded strands and the other three with post-tensioned unbonded strands. In all the six specimens seven-wire strands, with 12.7 mm diameter and Grade 270, were used as internal prestressed steel. In addition, different amount of deformed reinforcing steel were used as bonded nonprestressed reinforcement. The second group consists of six beams, identical to the specimens of the first group, subjected to ten cycles of repeated loading. The range of the repeated loading was varied between a lower level and an upper level, equal to 40 and 60 percent, respectively, of the ultimate flexural capacity of the specimen. The effect of repeated loading on first cracking load, cracking behaviour, load carrying capacity, strain in nonprestressed and prestressed steel, strain in concrete and load deflection hysteresis were discussed in detail. It was observed that the limited cycles of repeated loading have a minor effect on the ultimate strength of partially prestressed concrete beams but it has a significant effect on deformability and cracking behaviour. The increase in midspan deflection was between 4.5% and 56%, while the crack width increase was between 14% and 300% for partially prestressed compression-controlled and partially prestressed tension-controlled with unbonded strands beams, respectively.

Keywords: Bonded, Cracking, Deflection, Partial prestressing, Repeated load, Unbonded.

1. Introduction

The structural members should be designed for serviceability requirements in addition to strength. A better understanding of deformation and cracking of concrete beams under different types of loading will improve the serviceability design. Research on the serviceability of structural concrete members has become more important in the last years.

Loading on offshore structure, highways, multi-story car parking, and many other structures are often repeated in nature. Extensive theoretical and experimental studies of both reinforced and prestressed concrete beams over so many years have led to very well established methods for serviceability design under static loads. However, the effects of repeated loading on cracking and deflection of prestressed concrete beams are still rare and scarcely understood.

Concrete structures exposed to repeated loading experience higher deflection comparing with static loading. These deflections are including considerable permanent sets. With the increasing of the number of load cycles, the permanent deflections are also increased. Many researchers [1-7] have noticed this phenomenon; however, relevant experimental information is still poor. It appeared that the deformability and cracking of prestressed concrete beams under repeated loading deserve detailed investigations. This paper project aims to enrich this area of research to extensively study the effect of limited repeated loading on the behaviour of partially prestressed concrete flexural members.

The main objective of this study is to investigate the serviceability (deformability and cracking) of partially prestressed concrete beams under monotonic static and repeated loads. The considered variables included the type of loading (monotonic static and repeated), the partial prestressing ratio, the reinforcing index, and the nature of bond condition between concrete and prestressed reinforcement (bonded and unbounded).

2. Partially Prestressed Concrete

Joint ACE-ASCE Committee 423 [8] reported that a unified definition of the term "partial prestressing" according to ACI 423.5R-99 is "An approach in design and construction in which, prestressed reinforcement or a combination of prestressed and nonprestressed reinforcement is used such that tension and cracking in concrete due to flexure are allowed under service dead and live loads, while serviceability and strength requirements are satisfied". Naaman [9] proposed that the partial prestressing is now used to describe one of three specific features of prestressed concrete members as follows:

- A member, which is design to crack under service load.
- A member in which, the effective prestress in steel is purposely kept lower than its maximum allowable value.
- A concrete member, which reinforced with both prestressed and nonprestressed reinforcement, where the nonprestressed steel may be ordinary mild steel or high strength steel.

In this study, partial prestressing reveals to the combination of prestressed and nonprestressed reinforcement, where both types sharing the resistance of the members to the externally applied load. Partial prestressing covers the whole range of reinforcing and/or prestressing schemes. Accordingly, in an attempt to provide a general terminology, the term "partial prestressing" becomes the synonym of the term "structural concrete". They both occupy the whole spectrum of reinforcing range between fully reinforced and fully prestressed concrete beams.

To quantify the size of prestressing in partially prestressed concrete members, several indices have been generally introduced such "Degree of Prestress" [10], "Prestressing Index" [11] or "Partial Prestressing Ratio" [12]. The partial prestressing ratio (PPR) allows a unified treatment of the “ultimate flexural strength” limit state for reinforced, prestressed and partially prestressed concrete. It is defined as follows:

$$PPR = \frac{M_{np}}{M_n} = \frac{A_{ps}\, f_{ps}\,(d_p - a/2)}{A_{ps}\, f_{ps}\,(d_p - a/2) + A_s\, f_y\,(d_s - a/2)} \qquad (1)$$

where

M_{np} = nominal moment capacity provided by prestressed steel, kN.m;

M_n = total nominal moment capacity, kN.m;

A_{ps} = area of prestressed steel in tension zone, mm^2;

A_s = area of nonprestressed steel in tension zone, mm^2;

f_{ps} = stress in prestressed steel at nominal flexural strength, MPa;

f_y = specified yield strength of nonprestressing tensile steel, MPa;

d_p = distance from the extreme compressive fiber of concrete to the centroid of prestressed steel in tension zone, mm;

d_s = distance from the extreme compression fiber of concrete to the centroid of nonprestressed steel in tension, mm, and

a = depth of equivalent compressive stress block of concrete, mm

Naaman [13] explained that the characterizing the total quantity of reinforcement in members is also important. The reinforcing index ($\overline{\omega}$**)** is used for that as follows:

$$\overline{\omega} = \begin{cases} \omega_p + \omega - \omega' \\ \frac{A_{ps}\, f_{ps}}{b\, d_p\, f_c'} + \frac{A_s\, f_y}{b\, d_s\, f_c'} - \frac{A_S'\, f_y'}{b\, d_s\, f_c'} \\ \rho_p\, \frac{f_{ps}}{f_c'} + \rho\, \frac{f_y}{f_c'} - \rho'\, \frac{f_y'}{f_c'} \end{cases} \qquad (2)$$

where

ω_p = prestressed steel reinforcing index;

ω = nonprestressed tensile steel reinforcing index;

ω' = compression steel reinforcing index;

f_c' = specified compressive strength of concrete at 28 days age, MPa;

b = width of compression face of the member, mm

A'_S = area of nonprestressed steel in compression zone, mm^2;

f'_y = specified yield strength of nonprestressing compression steel, MPa;

ρ = nonprestressed tensile steel ratio;

ρ_p = prestressed tensile steel ratio, and

ρ' = nonprestressed compression steel ratio.

It should be noted that these parameters are used as design parameters in addition to their main function for describing the extent of partial prestressing. Each parameter has two limits, one related to fully reinforced concrete and the other to fully prestressed concrete. These limits ranged between zero and one. Almost one parameter is sufficient to characterize the extent of partial prestressing in the flexural design of structural members, while an individual group of researchers preferred to tie their methodologies of flexural analysis and design of structural concrete members with more than one parameter of the above described. In this study both PPR and $\bar{\omega}$ are used to describe the extent of partial prestressing.

3. Experimental Program

The experimental program was undertaken to study the behaviour of partially prestressed concrete beams under monotonic static and limited repeated loading. The load carrying capacity, serviceability limit states in terms of crack, crack width and spacing between cracks, the deflection prior to and after cracking was investigated. The modes of failure were also examined.

This research included testing twelve rectangular-section simply supported concrete beams that have the same geometric layout, same shear reinforcement, and variable flexural reinforcement as shown in Table 1. All beams were with 200 mm width×300 mm height rectangular cross section simply supported on a 3000 mm span. The span to depth ratio was 10. All beams were loaded in four-point loading using two symmetrical monotonic concentrated static loads applied at one-third of the span length as shown in Fig. 1. Beams were designed according to ACI-318-2014 [14].

These beams were divided into two identical groups. The first group was tested under monotonic static loading (S) and regarded as control beams. The second group was exposed to ten cycles of repeated loading (R), which ranged between 40 and 60 percent of ultimate flexural capacity produced from the static test. According to Harajli and Naaman [3], the load levels P_{min} and P_{max} were selected to simulate self-weight and self-weight plus superimposed dead load and service overload respectively. Each group consisted of six beams as follows:

- Three Partially Prestressed concrete (PP) beams with bonded tendons (B). Each beam was reinforced with 2Ø12.7 mm low-relaxation strands and with different amount of nonprestressed reinforcement to get different values of PPR. The first beam was Tension-Controlled (TC), the second was Transition-Controlled (TRC) and the third is Compression-Controlled (CC) [14].
- Three partially prestressed concrete beams with unbonded tendons (U). Each beam was also reinforced with 2Ø12.7 mm low-relaxation strand. As in the

first group, the first beam was tension-controlled, the second was transition-controlled and the third was compression-controlled.

To avoid brittle failure immediately after cracking, the ACI-318 code [14] recommends that the section nominal resisting moment be at least 20 percent higher than a cracking moment. Based on that, the corresponding minimum reinforcement ratio ($\rho_{p,min}$) was considered equal to 0.00154.

On the other hand, the reinforcement area was chosen for tension-controlled beams so that the expression c/d_t should be less than 0.375, while for compression-controlled beams it should be greater than 0.60 [15], where c is the depth of neutral axis and d_t is the distance from the concrete extreme compression fiber to the extreme layer of tension steel.

Deformed steel bars with a yield strength of 570 MPa and ultimate strength of 650 MPa were used as nonprestressed steel in this study. Two bars of 10 mm were used as compression reinforcement for all the beams, as shown in Fig. 2. For tensile reinforcement, a different number of bars of diameter 10 and 12 mm were used depending on whether the beam is tension-controlled, transition or compression-controlled. For shear reinforcement, bars of diameter 10 mm were used as closed stirrups.

Seven-wire low relaxation strand of 12.7 mm nominal diameter conforming to ASTM A416/A416M-05 [16] was used as prestressed steel, as shown in Fig. 2. The strands in all beams were stressed to 0.7 of ultimate tensile strength. All the properties of the prestressed steel strand were described in Table 2. The reference concrete mixture was designed to achieve a target compressive cylinder strength of 40 MPa at 28 days. Table 3 shows the properties of hardened concrete at the date of the test for each tested beam.

Table 1. Reinforcement details of the tested beams.

Beam designation	ρ_p	ρ	ρ'	ω_p	ω	ω'	$\bar{\omega}$	*PPR*
PP-B-TC-S **PP-B-TC-R**	0.00429	0.00302	0.00302	0.185	0.043	0.043	0.185	0.771
PP-B-TRC-S **PP-B-TRC-R**	0.00429	0.00869	0.00302	0.178	0.124	0.043	0.259	0.529
PP-B-CC-S **PP-B-CC-R**	0.00429	0.01818	0.00316	0.168	0.259	0.043	0.384	0.358
PP-U-TC-S **PP-U-TC-R**	0.00429	0.00302	0.00302	0.152	0.043	0.043	0.152	0.734
PP-U-TRC-S **PP-U-TRC-R**	0.00429	0.01207	0.00311	0.143	0.172	0.044	0.271	0.409
PP-U-CC-S **PP-U-CC-R**	0.00429	0.01573	0.00311	0.138	0.224	0.044	0.318	0.339

Table 2. Properties of prestressed steel.

Mechanical property	Value
Nominal area	99.7 mm^2
Ultimate tensile strength	1860 MPa
Breaking strength	187.8 kN
Minimum load of 1% elongation	171.8 kN
Modulus of elasticity	197500 MPa

Table 3. Concrete compressive strength for tested beams.

Beam designation	@ 28 days age, (MPa)	@ Date of testing age, (MPa)
PP-B-TC-S and PP-B-TC-R	39.7	40.5
PP-B-TRC-S and PP-B-TRC-R	39.7	40.5
PP-B-CC-S and PP-B-CC-R	41.4	42.2
PP-U-TC-S and PP-U-TC-R	39.2	41.1
PP-U-TRC-S and PP-U-TRC-R	39.2	41.1
PP-U-CC-S and PP-U-CC-R	41.2	42.0

Fig. 1. General view of testing arrangement.

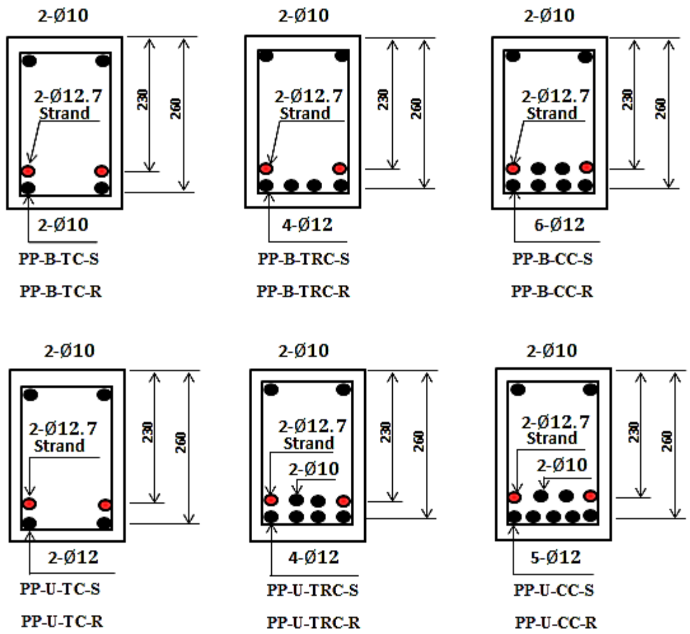

Fig. 2. Steel arrangement for tested beams under static and under repeated loading.

4. Behaviour of Experimental Beams under Monotonic Static Loading

Six beams have been tested under monotonic static load (group I). The flexural behaviours of these beams are as follows.

4.1. Load-deflection response

The load-deflection curves for the six tested beams are shown in Fig. 3. Careful inspection of these diagrams, the following observations can be drawn:

- For partially prestressed concrete beams with tension-controlled and transition-controlled, three distinctive points could be observed. These points are characterized by cracking, yielding and ultimate loads.
- Partially prestressed concrete compression-controlled beams are characterized by two points, which are cracking and ultimate loads. Thus, the clear yielding point cannot be distinguished.
- Prior to cracking all beams behaved in an elastic manner. After cracking, beams with low partially prestressed ratio PPR and high reinforcing index $\bar{\omega}$ developed approximately a linear load-deflection response.
- All beams experienced decreasing of stiffness after cracking, depending on the level of PPR and $\bar{\omega}$. Increasing the area of tension reinforcement produced a stiffer beam, which led to a low rate of deflection versus applied load.
- Beams with bonded strands were stiffer than the counterpart with the unbonded strand. This is obvious for bonded strand beam (PP-B-TC-S) and unbonded strand beam (PP-U-TC-S) taking into consideration that both beams have exactly the same amount of reinforcement (prestressed and nonprestressed).
- The nonprestressed steel yielded prior to prestressed steel for all tension-controlled beams. This is because of the physical location of reinforcing steel relative to concrete bottom fibre.
- Beams with a low amount of nonprestressed steel (i.e., low $\bar{\omega}$) exhibited behaviour that is more ductile by their flatter and longer load deflection curve.

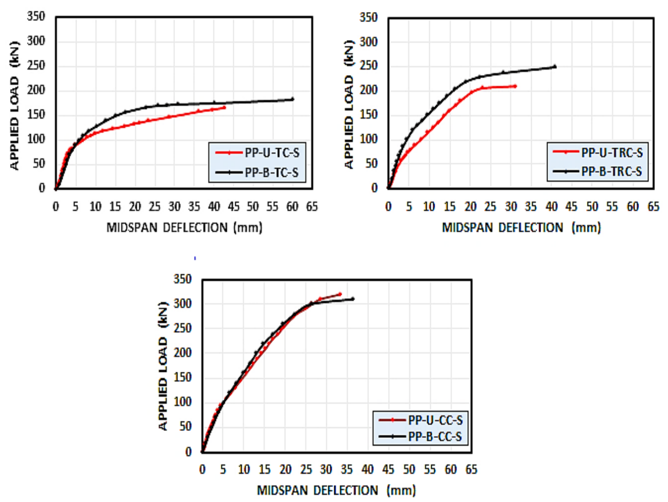

Fig. 3. Typical load-deflection response for beams exposed to monotonic static loading.

4.2. Cracking and ultimate load carrying capacities

Flexural cracks in all beams were initiated in the constant moment zone perpendicularly to the longitudinal axis of the beam and propagated up to the location of the tensile nonprestressed reinforcement. When the applied load increases, the pre-compressive stress in the concrete fibres at the soffit of the beam decreases and when the tensile stress at those fibres exceed the tensile strength of concrete, cracking develops. The visible cracks spread mainly along the beam length in the constant moment region at first and then propagate to other regions near supports. Table 4 shows the first cracking load and the ultimate load carrying capacity for each beam.

It should be pointed out that, increasing the intensity of non-prestressed reinforcement (i.e., increasing $\bar{\omega}$) slightly affected the cracking load. In addition, beams with bonded strands show higher ultimate load carrying capacities than the counterpart of beams with unbonded strands.

Table 4. First cracking and ultimate loads for beams of the first group.

Beam designation	PPR	$\bar{\omega}$	Cracking load P_{cr} , (kN)	Ultimate load P_u , (kN)	$\frac{P_{cr}}{P_u}$
PP-B-TC-S	0.771	0.185	67	182	0.368
PP-B-TRC-S	0.529	0.259	68	250	0.272
PP-B-CC-S	0.358	0.384	70	310	0.226
PP-U-TC-S	0.734	0.152	65	165	0.394
PP-U-TRC-S	0.409	0.271	57	210	0.271
PP-U-CC-S	0.339	0.318	70	320	0.219

4.3. Cracking behaviour for specimens under monotonic static loading

The behaviour of each tested beam was monitored from the moment of applying the external load up to failure. Primary cracks initiated at the region of the constant bending moment when the applied load reached the cracking load. As the applied load increased, additional cracks would appear. It was noticed that the number of cracks is stabilized at loading range between 60 and 80 percent of the ultimate load. A number of cracks, average crack spacing, and crack widths were measured and recorded throughout the test at each load increment. Table 5 illustrates a summary of the relevant test data for cracking at service load of 0.6 P_u.

Based on the experimental findings, the following observations may be summarised:

- In general, the number of cracks was the highest in beams with a high amount of nonprestressed steel. Cracks were propagated in these beams along the whole length to reach supports.
- For beams with high nonprestressed steel area, (low level of PPR and high amount of $\bar{\omega}$), more cracks were propagated and less crack spacing was observed comparing to beams, which have a high level of PPR. This is due to better overall bond achieved by the tensile nonprestressed steel with the surrounding concrete, which led to control the cracks in the tension zone of the beams.
- Comparing between the two beams (PP-B-TC-S and PP-U-TC-S), which have exactly the same amount of prestressed and nonprestressed steel, the difference was that, the first one has bonded strands while the other has unbonded strands. It was observed that the effect of bonding is very significant in controlling cracking behaviour (decreasing crack width from 0.15 mm to 0.03 mm).

In general, the presence of nonprestressed steel in prestressed concrete beams has a great benefit in controlling crack growth and it was one of the important reasons, (among others), to develop the partially prestressed concrete members. Figure 4 illustrates the progress of the mean crack width with respect to the applied load in all specimens of the first group.

These diagrams show that for beams with a high amount of nonprestressed steel (low level of PPR and high amount of $\bar{\omega}$), the crack width increasing rate is not significant with the load increasing until failure. The crack width for these beams (PP-B-CC-S and PP-U-CC-S) at failure is approximately 1.0 mm. The situation is different for beams with low nonprestressed steel (PP-B-TC-S and PP-U-TC-S). The crack width was increased slightly with the load increasing, and at a load level of 70 to 75 percent of the ultimate load, high increase in crack width was observed suddenly. The reason behind that related to the yielding of nonprestressed steel, which occurred at that level of loading.

Table 5. Number of cracks, mean crack spacing and maximum crack width at service load of $0.6\,P_u$.

Beam designation	Service load $0.6\,P_u$, (kN)	Number of cracks	Mean crack spacing a_{cs}, (mm)	Maximum crack width W_{max}, (mm)
PP-B-TC-S	109	9	175	0.03
PP-B-TRC-S	150	10	160	0.17
PP-B-CC-S	186	18	110	0.12
PP-U-TC-S	99	7	200	0.15
PP-U-TRC-S	126	13	125	0.10
PP-U-CC-S	192	17	120	0.30

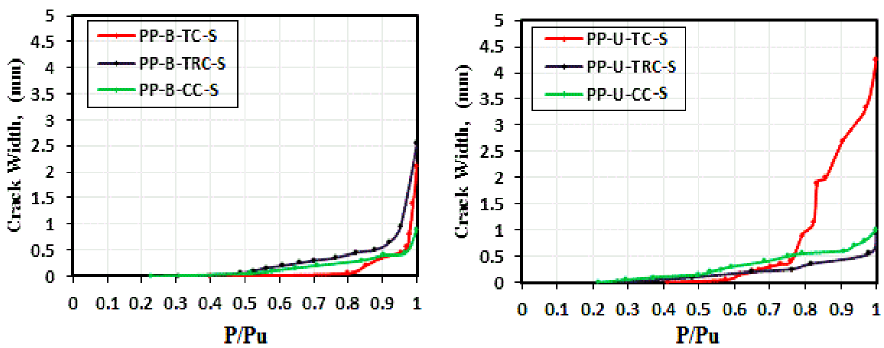

Fig. 4. Crack width versus applied load diagram for beams exposed to monotonic static loading.

4.4. Stress and strain in steel reinforcement and concrete

Uniaxial electrical resistance strain gauges were used to measure the strain in concrete as well as in steel. Two different dimensions of pre-wired strain gauges of 120 Ω resistance, made in Japan (TML company), were used in all the tests. Details of strain gauges are shown in Table 6. The strain in prestressed strands has been measured using two electrical strain gauges per each strand, which were fixed at midspan section. Strains in strand were automatically recorded in the computer for each load increment through the strain data logger. Accordingly, the stress was calculated from strain gauge readings using the actual stress-strain curves of the strand.

Table 7 shows the effective stresses and strains for beams exposed to monotonic static loading just before the test, the increase in strain at ultimate loading ($\Delta\varepsilon_{ps}$), the total strain at failure (ε_{ps}) and the stress at ultimate (f_{ps}) in the strand. The stress at ultimate was determined from the measured total strain using the adopted stress-strain curves of the prestressed steel. The following findings may be recorded from Table 7.

- As expected, the increase of strain in prestressed steel of beams with unbonded steel is less than the counterpart beams with bonded steel. For example, comparing beams (PP-B-TC-S) and (PP-U-TC-S), which have exactly the same reinforcement (prestressed and nonprestressed), the increased strain in bonded strands is higher than that in unbonded strands by about (53%).
- For the entire prestressed beams with bonded strands, the total stress in strands at the ultimate stage is beyond the yield stress. Furthermore, for prestressed beams with unbonded strands, no one of the strands reach the yield stress.

Strain gauges were fixed at extreme top concrete fibres and at both nonprestressed steels (in tension and compression zones). All these strain gauges were fixed at midspan location of the tested beams. The compressive strain at the extreme top fibre of concrete and the strain in nonprestressed steel for each beam was monitored and recorded automatically. This process has been done through the strain data logger and for each load increment starting from the moment of load application up to failure. Figure 5 illustrates load versus increased strain in extreme top concrete fibre, nonprestressed tensile steel, and prestressed steel.

Investigating Fig. 5, the following observations can be highlighted:

- The load-strain curve of nonprestressed reinforcement in the tension zone of (PP-B-TC-S, PP-B-TRC-S, PP-U-TC-S, and PP-U-TRC-S) has two distinct points. The first point represents the first crack initiation, while the second point is a well-defined yield point. This is a logic behaviour since these beams were designed as under reinforced section.
- Compression-controlled beams (PP-B-CC-S and PP-U-CC-S) failed by crushing of concrete in compression zone rather than yielding of steel. This is due to a large amount of the provided steel reinforcement in the tension zone of these beams. Therefore, no yield points are noticed in tensile nonprestressed steel.
- In beams with bonded strands (PP-B-TC-S, PP-B-TRC-S, and PP-B-CC-S), the strain curves of both steel (nonprestressed and prestressed) are almost identical until the appearance of the first crack. This is due to strain compatibility since the two types of reinforcement are very close to each. After that and due to cracks initiation, the compatibility ceases gradually and the behaviour became different. The situation is totally different in beams with unbonded strands (PP-U-TC-S, PP-U-TRC-S, and PP-U-CC-S). In these beams, the strain compatibility is absent between concrete and strand due to unbonded condition, therefore, the strain behaviour curve of both types of steel (nonprestressed and prestressed) is not similar from the beginning due to the bond effect, which reflected on the performance of unbonded prestressed steel clearly.

Table 8 shows the strain due to the applied external load only in extreme top concrete fibre $\varepsilon_{c,top}$, nonprestressed steel in tension ε_s, and compressions zones

ε_s' at ultimate load. To get the total strain at ultimate for concrete and steel the strain due to prestressing force, which was potential in each material before the exposure of the member to external an load, should be added. From the table, one can conclude that, the strain in nonprestressed steel in tension zone was proportionated inversely with the reinforcing index ϖ. At low levels of ϖ, this steel may attain yielding and vice versa.

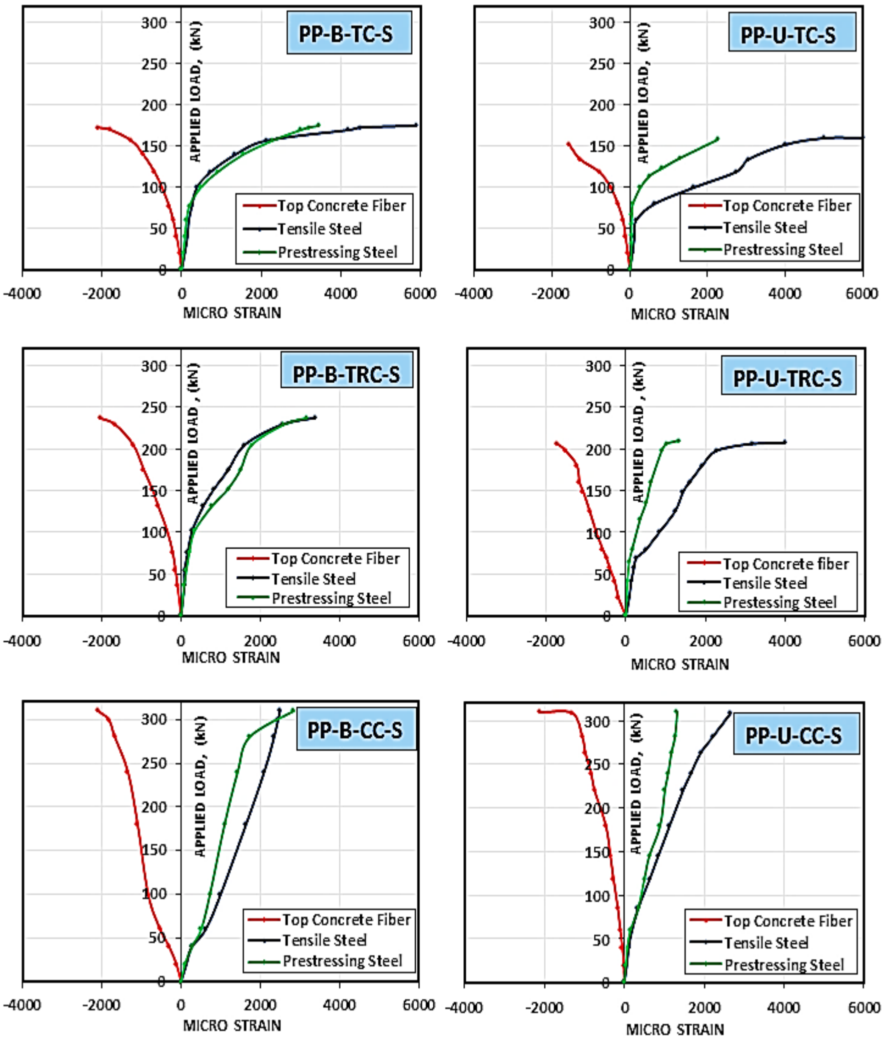

Fig. 5. Load versus strains of reinforcing steel and at top concrete fibre.

Table 6. Strain gauges details.

Model	Gauge size, (mm)		Gage factor	Gauge material	Setting location
	Length	Width			
FLA-5-11-3L	5	1.5	2.13±1%	Alloy foils	Steel
PL-60-11-3L	60	1	2.13±1%	Polyester wire	Concrete

Table 7. Experimental strain and stress in prestressed strands at ultimate load.

Beam designation	Effective prestress (f_{pe}), (MPa)	Effective prestrain (ε_{pe}) $\times 10^{-6}$	Increase of strain ($\Delta\varepsilon_{ps}$) $\times 10^{-6}$	Total strain (ε_{ps}) $\times 10^{-6}$	Total stress (f_{ps}), (MPa)
PP-B-TC-S	1091	+5526	+3451	+8977	1702
PP-B-TRC-S	1215	+6150	+3180	+9330	1722
PP-B-CC-S	1199	+6071	+2871	+8942	1695
PP-U-TC-S	1252	+6339	+2258	+8597	1662
PP-U-TRC-S	1175	+5951	+1329	+7280	1469
PP-U-CC-S	1122	+5681	+1326	+7007	1398

Table 8. Experimental strain in extreme top concrete fiber, nonprestressed steel in tension and compression zones at ultimate.

Beam designation	PPR	$\bar{\omega}$	Concrete strain ($\varepsilon_{c,top}$) $\times 10^{-6}$	Steel strain (ε_s) $\times 10^{-6}$	Steel strain (ε'_s) $\times 10^{-6}$
PP-B-TC-S	0.771	0.185	-2050	+12300	-320
PP-B-TRC-S	0.529	0.259	-2040	+3400	-770
PP-B-CC-S	0.358	0.384	-2130	+2400	-2430
PP-U-TC-S	0.734	0.152	-1540	+6300	-1080
PP-U-TRC-S	0.409	0.271	-1790	+5200	-850
PP-U-CC-S	0.339	0.318	-2110	+2700	-1950

4.5. Mode of failure for beams exposed to monotonic static loading

Depending on the intensity of steel reinforcement provided in the beams, two types of flexural failures were observed. The first type of failure is occurring when concrete in compression zone is crushing, preceded by yielding of nonprestressed steel. The failure of beams PP-B-TC-S, PP-B-TRC-S, PP-U-TC-S and PP-U-TRC-S were of this type.

The second type of failure is occurring by crushing of concrete before yielding of nonprestressed steel. This mode of failure characterized beams PP-B-CC-S and PP-U-CC-S. It should be pointed out that, no slip between strands and concrete was recorded for all the beams and no fracture of any type of reinforcement was observed. Figure 6 illustrates the tension-controlled and the compression-controlled beams at failure.

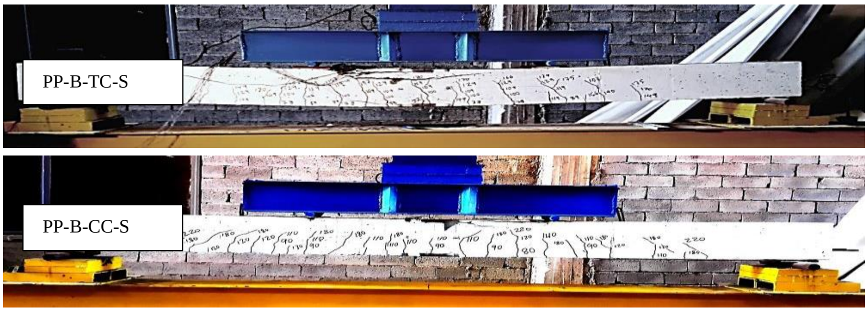

Fig. 6. Tension-controlled and compression-controlled beams at failure.

5. Behaviour of Experimental Beams under Repeated Loading

The second part of the experimental program includes testing six beams under limited cycles of repeated loading (*R*). The load test was carried out in three stages according to the s following sequence:

- At the beginning specimens of the second group were subjected to ten cycles of repeated loading. The minimum P_{min} and maximum P_{max} levels of repeated loading were taken to be 40 and 60 percent of the similar specimen, from the first group, failure load as recorded from the monotonic static test. Starting from P_{min} the load was applied monotonically with increments of five percent of ultimate load of the accompanying, from the first group, specimen up to P_{max} and then unloaded to P_{min} by the same sequence of decrements. Ten cycles of loading and unloading between P_{max}andP_{min}were implemented. The deflections, strains in steel and concrete, crack widths, crack spacing, crack numbers, and strand slip were recorded for each load increment or decrement.
- After ten cycles of loading, the load was released to zero. The residual deflection, strains in steel, concrete and strand slip was recorded.
- Later after releasing the external load, the beam was reloaded under monotonic static loading up to failure with load increment of approximately five percent of load capacity of the accompanying specimen from the first group. During this stage, the same measurements were taken for each loading stage. The completely repeated load test took an average between 10 to 14 hours.

5.1. Change of midspan deflection at the maximum loading range P_{max}

Figure 7 shows the change of midspan deflection for all experimental beams, which exposed to limited repeated loading during ten cycles at the maximum load range of 0.6 P_u. It was observed that, after five or six cycles of loading, the deflection increase becomes not significant comparing to the first and second cycles of loading.

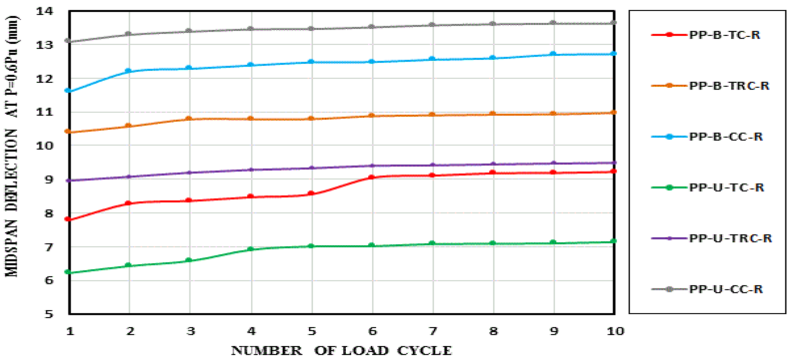

Fig. 7. Change of midspan deflection for experimental beams during ten loading cycles.

5.2. Cracking behaviour for specimens under repeated static loading

As it was implemented in monotonic static testing, number of cracks, average crack spacing and crack widths were measured and recorded throughout the repeated test at every loading increment. Table 9 shows these parameters at service load of 0.6 P_u for the first and tenth load cycles. Figure 8 shows the change of the number of cracks versus the number of load cycles for all the tested beams at a maximum load level of 0.6 P_u.

The main conclusions, which may be observed from Fig. 8, are:

- In general, the number of cracks is stabilized in the fifth cycle.
- In beams (PP-B-CC-R) and (PP-U-CC-R), the number of cracks is stabilized from the first cycle. This behaviour may be interpreted due to the high intensity of nonprestressed reinforcement in each of them (high level of $\overline{\omega}$).
- There is a sudden rise in the number of cracks for the beam (PP-B-TRC-R) at the second cycle, but the rise was steady for the rest beams.

Table 9. Progress of a number of cracks, mean crack spacing and maximum crack width during repeated loading.

Beam designation	Number of cracks		Mean crack spacing a_{cs}, (mm)		Maximum crack width w_{max}, (mm)	
	1st cycle	10th cycle	1st cycle	10th cycle	1st cycle	10th cycle
PP-B–TC-R	10	13	140	115	0.10	0.15
PP-B–TRC-R	11	16	125	120	0.15	0.15
PP-B–CC-R	19	19	115	115	0.10	0.10
PP-U-TC-R	5	6	275	225	0.00	0.30
PP-U-TRC-R	12	13	145	135	0.10	0.10
PP-U-CC-R	19	19	110	110	0.35	0.40

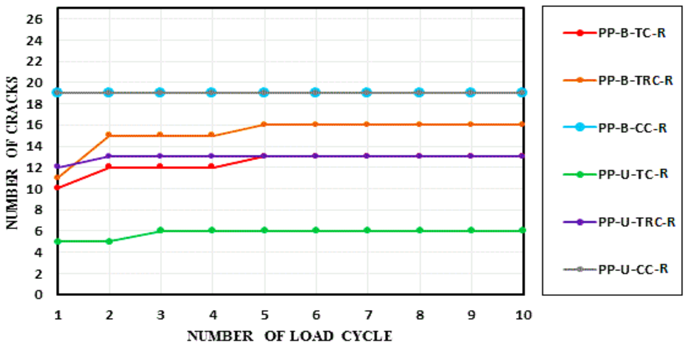

Fig. 8. Change of a number of cracks for experimental beams during ten loading cycles.

5.3. Change of strain in steel reinforcement and concrete under repeated static loading

Strains at extreme top concrete fibres and in steel reinforcement (nonprestressed and prestressed) were monitored throughout the test for each load increment during the ten loading cycles. Figures 9 to 11 highlight the change of strain, for all experimental beams, which exposed to limit repeated loading during ten cycles at a maximum load level of 0.6 P_u, at the extreme top concrete fibers, nonprestressed steel in tension zone, and prestressed strands, respectively.

The net percentage change in strains between the first and the tenth cycles of loading for these three components indicated in Table 10.

Investigating Figs. 9 to 11, the following observations may be recorded:

- Almost the strain in extreme top concrete fibres decreased gradually with the successive cycles of repeated loading as in Fig. 9.
- The strain in nonprestressed steel of the tension zone of the section increased with the increase of cycles approximately linearly for most beams as in Fig. 10. The rate of increase in strain was the largest in the beam (PP-B-TC-R).
- The same behaviour was noticed for prestressed steel. The strain was increased with the progress of cycles for all beams as in Fig. 11. In comparison with nonprestressed steel, the rate of increasing of the strain was the less. The maximum increase in strain was in the beam (PP-U-TC-R), while, the minimum was in the beam (PP-B-CC-R).
- The decrease of strain in concrete and the increase of strain in nonprestressed and prestressed steel with the successive increase of loading cycles attributed to the strain redistribution across the member depth due to the cyclic rapid creep process of concrete in compression.
- Comparing the progressive increase in strain due to cyclic rapid creep in prestressed and nonprestressed steel, it is very interesting to observing in Table 10 that almost the increase was more pronounced in the steel of small area in the cross-section.
- Comparing between the two beams (PP-B-TC-R) and (PP-U-TC-R), which were reinforced identically and have close values of PPR and $\bar{\omega}$. The progressive decrease in strain due to cyclic rapid creep in top concrete fiber after ten cycles was approximately the same but the progressive increase in strain in the mild steel of the tension zone was 31.8% for the beam (PP-B-TC-R), meanwhile, it was 13.6% for the beam (PP-U-TC-R).

This difference between the two beams can be attributed to the fact that, the first beam has bonded strands and the second has unbonded strands. It is worth to notice that, the opposite situation was monitored for the prestressed steel, where the progressive increase in strain in this steel was 29.6% and 65.2%, for the mentioned beams respectively. Accordingly, in bonded and unbonded beams with close values of both PPR and $\bar{\omega}$, the progressive increase in strain in nonprestressed steel, due to cyclic rapid creep of concrete, is more than the increase in strain in prestressed steel. While, in unbonded beams, the increase in strain in prestressed steel is more than the increase in strain in nonprestressed steel.

Table 10. Change in strain occurred between the first and tenth loading cycles at the level of $0.6\ P_u$.

Beam designation	Change in strain between 1st and 10th loading cycles, (%)		
	Extreme top concrete fiber	Nonprestressed steel in tension zone	Prestressed steel
PP-B-TC-R	-9.0	+31.8	+29.6
PP-B-TRC-R	-4.1	+12.1	+23.7
PP-B-CC-R	-5.9	+18.8	+12.1
PP-U-TC-R	-8.6	+13.6	+65.2
PP-U-TRC-R	+6.1	+24.3	+25.4
PP-U-CC-R	-35.9	+14.7	+15.7

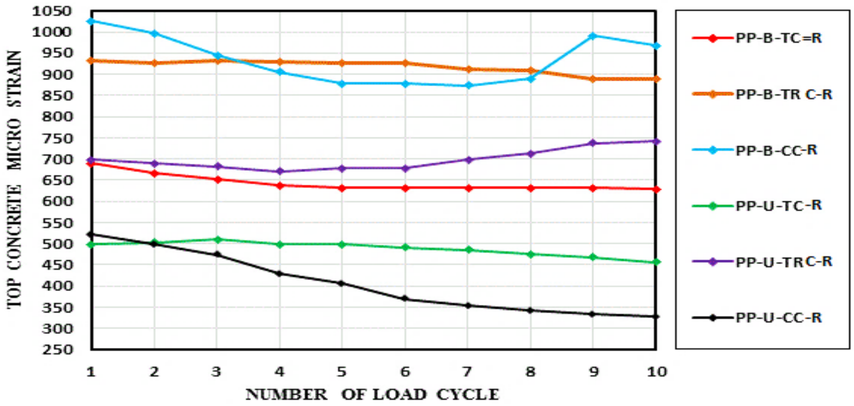

Fig. 9. Change of strain at extreme top concrete fibres for experimental beams during ten loading cycles.

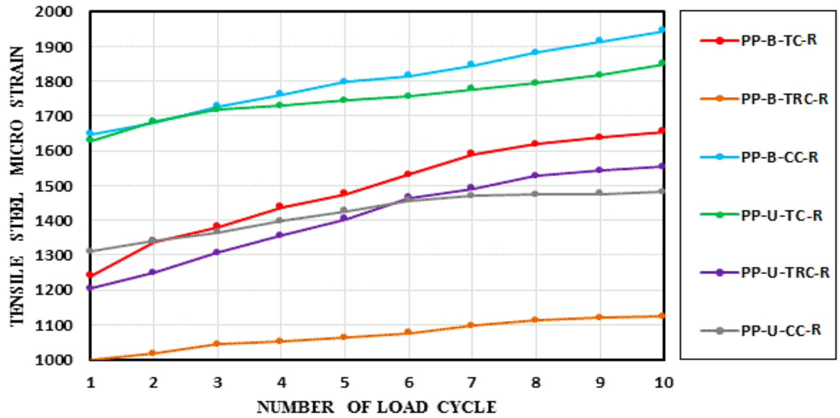

Fig. 10. Change of strain in nonprestressed steel in tension zone for experimental beams during ten loading cycles.

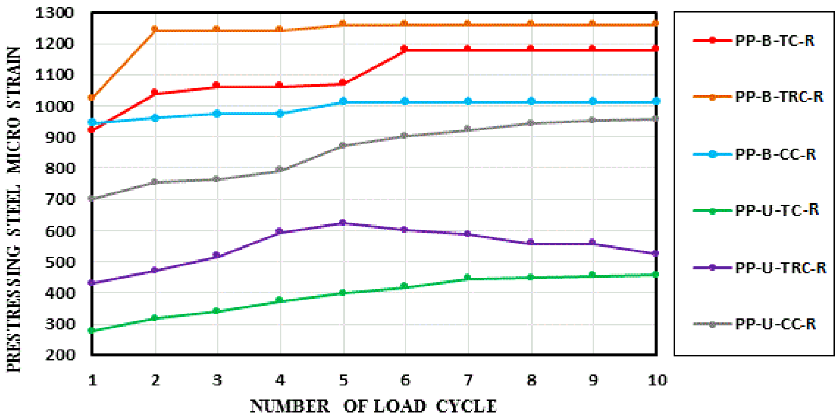

Fig. 11. Change of strain in prestressed strands for experimental beams during ten loading cycles.

5.4. Mode of failure

It was observed that the mode of failure for each beam subjected to repeated static loading was exactly the same mode of failure for the counterpart beam under monotonic static loading.

As in monotonic static test, no slip between strands and the surrounding concrete was observed. In addition, no rupture of any type of reinforcing bars (nonprestressed and prestressed) in any beams was noticed throughout all the repeated loading tests.

6. Comparative Study: Monotonic and Repeated Static Loading Results

A comparison between the flexural behaviour of the beams tested under monotonic static loading and the beams subjected to repeated static loading has been conducted.

The goal of this comparison is to investigate the effect of limited cycles of repeated loading on the performance of partially prestressed concrete beams. This comparison involves load-deflection curves, number of cracks, crack widths and crack spacing.

6.1. Load-deflection response

Figure 12 includes the load-deflection curves for the beams under monotonic static loading and the complete load-deflection curves for the counterpart beams, which created under the following sequence of loading: cycles of repeated loading (ten cycles of loading and unloading between P_{max} and P_{min} ranged between 40 to 60 percent of ultimate static load), after that the load was released totally, and then the beam was reloaded with monotonic static loading until failure.

The complete load-deflection curve, which illustrated in Fig. 12, simulates just the reloading process with monotonic static loading (i.e., the behaviour with a reduced flexural stiffness as cracks in tension zone tended to reopen successively with the increasing of the applied load).

From these diagrams, the following observations can be noticed:

- Although the load-deflection curves for identical beams are almost similar there is no clearly recognized cracking point for beams, which exposed to repeated loading because in all these specimens cracks initiated during the first load cycle to P_{max} , where the mentioned figure did not show the details of the loading cycles.
- Ultimate deflections for beams, which subjected to repeated cyclic loading were of the same order of magnitude as those for specimens, which exposed to monotonic static loading. The difference in deflection values between the identical beams was not significant.
- The residual deflection, after 10 cycles of repeated loading followed by loading release, was very small for PP-B-TC-R and PP-U-TC-R beams. It was 0.85 mm and 0.64 mm, respectively. For other beams, the values were almost equal to 1.5 mm.

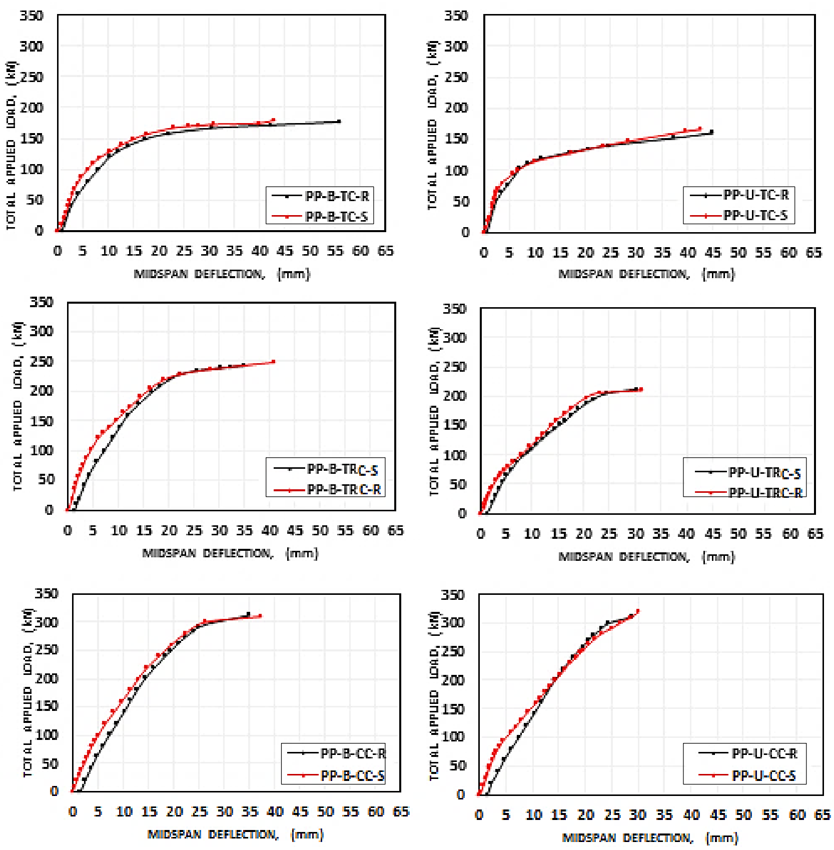

Fig. 12. Load-deflection curves of beams under monotonic static loading and repeated static loading after (10) cycles.

6.2. Cracking behaviour

Table 11 shows the number of cracks, crack width and mean crack spacing at load stage of 60 percent at the ultimate load for beams under monotonic static and beams under repeated static loading.

For the beams under repeated loading, Table 11 represents results after 10 cycles of repeated loading. The following conclusions may be recorded:

- The number of cracks for identical beams is increased for beams with bonded strands while for beams with unbonded strands, the number of cracks is almost the same under both types of testing.
- In general, repeated loading significantly affecting crack width especially for beams with unbonded strands rather than beams with bonded strands.

Figure 13 shows the tested specimens at failure. Each identical specimens were put together to simplify the comparison between them from crack propagation point of view.

In general, limited repeated loading has an insignificant effect on a number of cracks, cracks propagation and crack alignment.

Table 11. Cracking parameters for beams under monotonic static test and for beams after ten cycles of repeated loading at $0.6\ P_u$.

Beam designation	Number of cracks		Mean crack spacing a_{cs}, (mm)		Max. crack width w_{max}, (mm)	
	Static test	After ten cycles	Static test	After ten cycles	Static test	After ten cycles
PP-B-TC-S, PP-B-TC-R	9	13	175	115	0.03	0.12
PP-B-TRC-S, PP-B-TRC-R	10	16	160	120	0.17	0.15
PP-B-CC-S, PP-B-CC-R	18	19	110	115	0.12	0.12
PP-U-TC-S, PP-U-TC-R	7	7	200	225	0.15	0.28
PP-U-TRC-S, PP-U-TRC-R	13	13	125	135	0.15	0.25
PP-U-CC-S, PP-U-CC-R	17	19	120	110	0.30	0.45

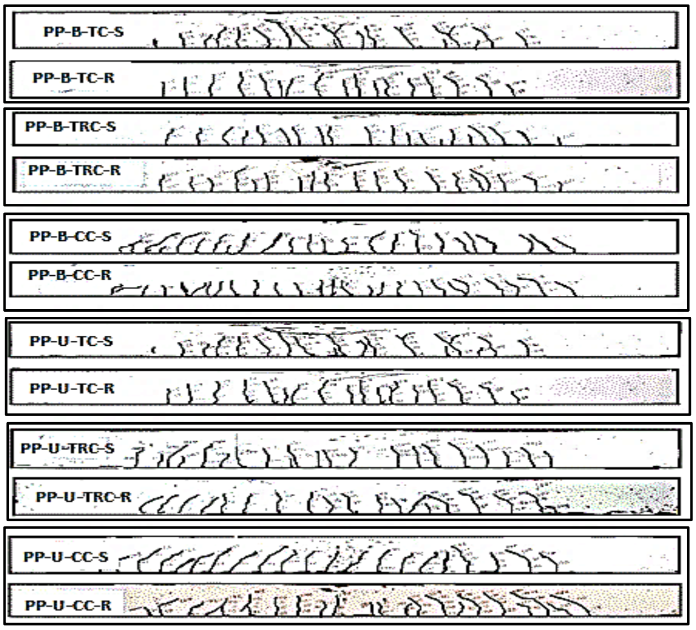

Fig. 13. Crack propagation of tested specimens.

7. Conclusions

The following conclusions can be drawn concerning the effects of repeated loading on the partially prestressed concrete beams.

- Beams with bonded strands were stiffer than the counterpart with unbonded strands. This is very obvious for beams (PP-B-TC-S and PP-U-TC-S), which have exactly the same amount of prestressed and nonprestressed steel. The ultimate strength of PP-B-TC-S was 182 kN, while for PP-U-TC-S was 165 kN. Meanwhile, beams with a low amount of nonprestressed steel exhibited more ductile behaviour than those with a high amount of nonprestressed steel.
- Increasing nonprestressed steel has little effect on first cracking load but it has a significant effect on the ultimate load. In addition, a number of cracks is proportional to the amount of nonprestressed steel. On the other hand, the

nonprestressed steel contributed to decreasing crack width and controlling the cracks in the beams.

- The increase in stress of bonded prestressed steel is higher than that in unbonded prestressed steel. The entire bonded prestressed strand reaches the yield (f_{ps}>0.9f_{pu}) while no unbonded strands reach yielding.
- In a repeated loading test, comparing between results of the first and tenth load cycles for the tested beams shows that, the increased range in midspan deflection was between 4.5% to 56% while the maximum crack width increase range was between 14% to 300%.
- The rate of increasing deflection and crack width was significant at the first five load cycles. Stabilization of deflection, number of cracks and crack width was observed for the second five loads cycles.
- The load carrying capacities are very close for identical beams for both types of test (static and repeated).
- All beams were failed in a flexural mode and no slip between any type of steel (prestressed and nonprestressed) and concrete was noticed.
- Repeated loading has little effect on beams with a high amount of nonprestressed steel but it has a significant effect on beams with a little amount of steel especially for crack width increasing. For example, crack width increased by about 50% for PP-B-TC-R beam between load cycle 1 and 10, while it still the same for PP-B-CC-R beam. The same thing may be noticed for PP-U-TC-R, the crack width increased by about 300% between load cycle 1 and 10, while it was increased by only 14% for PP-U-CC-R beam.

The following studies can be recommended for future works:

- Investigate the partially prestressed concrete beams under repeated loading of load range between 40% to 80% of ultimate monotonic static loading.
- Studying the same types of tested beams under reversal repeated loading with different loading ranges.

Investigate the partially prestressed concrete beams under repeated loading with initial prestressing stress less than 0.7 P_u such as 0.5 P_u or less.

Nomenclatures

A_{ps}	Area of prestressed steel in tension zone, mm^2
A_s	Area of nonprestressed steel in tension zone, mm^2
A'_s	Area of nonprestressed steel in compression zone, mm^2
a	Depth of equivalent compressive stress block of concrete, mm
a_{cs}	Mean crack spacing, mm
b	Width of compression face of the member, mm
c	Depth of neutral axis, mm
d_p	Distance from the extreme compressive fiber of concrete to the centroid of prestressed steel in tension zone, mm
d_s	Distance from the extreme compression fiber of concrete to the centroid of nonprestressed steel in tension, mm
d_t	Distance from the extreme compression fiber of concrete

	to the extreme layer of tension steel, mm
f_c'	Specified compressive strength of concrete at 28 days age, MPa
f_{ps}	Stress in prestressed steel at nominal flexural strength, MPa
f_{pe}	Effective stress in prestressing reinforcement, after allowance for all prestress losses, MPa
f_y	Specified yield strength of nonprestressing tensile steel, MPa
f_y'	Specified yield strength of nonprestressing compression steel, MPa
M_n	Total nominal moment capacity, kN.m
M_{np}	Nominal moment capacity provided by prestressed steel, kN.m
P_{min}	Minimum load range of repeated loading, kN
P_{max}	Maximum load range of repeated loading, kN
Greek Symbols	
ρ	Nonprestressed tensile steel ratio
ρ'	Compression steel ratio
ρ_p	Prestressed steel ratio
ω	Nonprestressed tensile steel reinforcing index
$\bar{\omega}$	Global reinforcing index
ω'	Compression steel reinforcing index
ω_p	Prestressed steel reinforcing index
Abbreviations	
ACI	American Concrete Institute
PPR	Partial Prestressing Ratio

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