

NOVEL RECEIVER DIVERSITY COMBINING METHODS FOR SPECTRUM SENSING USING META-ANALYTIC APPROACH BASED ON P-VALUES

DEEPA N. REDDY^{1,2,*}, Y. RAVINDER³

¹Department of Electronics and Telecommunication, JSPM's Rajarshi Shahu College of Engineering, Savitribai Phule Pune University, Pune, India

²Department of Electronics and Communication, BMS Institute of Technology and Management, Visvesvaraya Technological University, Bengaluru, India

³Department of Electronics and Telecommunication, Pune Institute of Computer Technology, Savitribai Phule Pune University, Pune, India

*Corresponding Author: reddydeepa2680@gmail.com

Abstract

The need for efficient spectrum utilization with reduced error rates has brought a paradigm shift in wireless communication systems from a Single Input and Single Output (SISO) systems to Multiple Input Multiple Output (MIMO) systems. Conventional diversity combiners are used to boost the received Signal to Noise Ratio at the Cognitive Radio receiver. However, these methods require perfect estimation of the channel. This paper proposes a Meta-Analytic approach based on p-Values for combining the data received from a secondary user equipped with multiple antennas. The effect of the p-Value method as receiver diversity combiner is studied and is compared with the existing non-coherent combining schemes, which do not need channel state information. The weighted Z test and Fisher's method are used to combine the p-Values derived from the Anderson Darling (AD) and Jarque Bera (JB) test statistics. A ballpark figure of the merits of these diversity combining methods are provided in this study. Through extensive Monte Carlo simulations, it is shown that the weighted Z test using the Anderson Darling test statistic provides a probability of detection very close to the existing non-coherent diversity combiners. Hence, this novel statistical approach based on p-Values provides a promising solution to combine the test statistics from multiple receiver antennas.

Keywords: Diversity combiners, Goodness of fit tests, p-Value, Probability of detection, Spectrum sensing.

1. Introduction

Sensing the availability of the radio channel is one of the salient tasks in Cognitive Radio (CR). The sensing performance can be enhanced by either augmenting the Signal to Noise Ratio (SNR) measured at the cognitive radio device or increase the dimension of the received signal space. In a practical wireless environment, it is critical to increasing the received SNR, as the signal received at CR can be deeply faded and shadowed. The reliability of wireless communication system suffers when the signal received at the CR antenna is faded. To overcome this effect of small-scale fading in the detection of primary user activity, diversity techniques are employed to provide an improvement in the received SNR and hence, achieve a higher probability of detection [1, 2].

In the past decades, different kinds of diversity combiners have been exhaustively investigated in the literature. From the studies, the Maximal Ratio Combining (MRC), Equal Gain Combining (EGC), and Selection Combining (SC) are the most commonly used diversity combiners. These diversity combining techniques no doubt provide an improvement in the received SNR but they demand the learning of the Channel State Information (CSI). Hence, this increases the implementation complexity [3-5]. The MRC technique to maximize the output SNR is discussed in [5, 6] with the assumption that the exact channel information can be estimated at the receiver. But practically the perfect estimation of the channel cannot be achieved and hence, this estimation error decays the sensing accuracy.

To mitigate the impact of the channel estimation error on the detection performance several diversity combining techniques were proposed in the literature. The non-coherent combining schemes, which do not need the CSI are investigated in [7]. Under this category, the square law combiner (SLC) and square law selection (SLS) are studied, which produces the decision statistic using the outputs of the square-law devices available in each of the diversity branch. Akbari et al. [8] proposed the use of evolutionary algorithms on receiver diversity based on the Imperialistic Competitive Algorithm (ICA). It is shown that this combiner does not demand the CSI, and it provides superior performance compared to MRC.

In most of the studies, to test for the null hypothesis several independent tests are performed. In most instances, it is necessary to integrate these results from independent tests to decide on the presence of the null hypothesis. The results from such independent tests are combined using the meta-analytic approach. From the review of the existing literature, these approaches using p-Values have been widely used by evolutionary biologists to combine the results from different studies. The p-Value method is widely used as it provides the strength of evidence in disagreement with the null hypothesis. Studies in [9-13] have investigated on combining the p-Values from independent tests.

Hence, this paper adopts the above-discussed method and proposes the p-Value based approach to combine the data received from multiple branches of the CR receiver. The proposed statistical test is more robust as it is independent of the primary user signal. However, these tests require the noise distribution to be known a priori. The preliminary focus of this study is to analyse the effect of meta-analysis based approaches such as the p-Value method as receiver diversity combiner and compare it with the existing non-coherent combining schemes, which do not need CSI. Figure 1 gives the classification of the diversity combining algorithms. To overcome the effect of imperfect estimation of channel state information, the p-

Values approach based on statistical methods is the first of its kind for combining data from diversity branches. These methods depend only on the exact significance levels or p-Values and not on the form of the data. Hence, they are called non-parametric or omnibus tests.

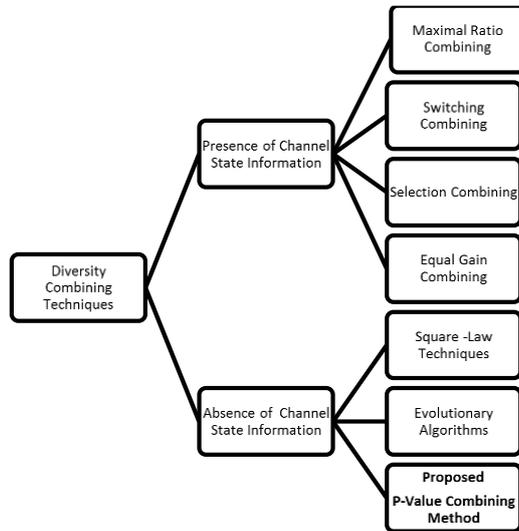


Fig. 1. Classification of diversity combining techniques.

The challenges of reliable sensing at very low signal-to-noise ratio in a fading environment are addressed by employing multiple sensing antennas at a CR receiver. Hence, this study considers SIMO (Single Input, Multiple Output) models and it is often used to reduce the impact of ionospheric fading and interference in wireless communication. The channel is considered to be affected by Additive White Gaussian noise (AWGN). The primary signal is considered as a sinusoidal pilot tone. Two significant blind sensing schemes considered for calculating the test statistic are 1) Energy detection 2) Goodness of Fit test. The efficacies of these methods are evaluated using Monte Carlo simulations. The results show that the proposed combining method provides detection very close to the existing non-coherent diversity combiners.

This paper is organized as follows: Section 2 gives the overview of blind sensing schemes for primary user detection, Section 3 discusses the proposed method, Section 4 discusses the results and Section 5 concludes the paper.

2.System Model

Consider the scenario of Single Input Multiple Output (SIMO) system with one transmit antenna and multiple receiver antennas. Assume that each CR contains M antennas. The M diversity branches are assumed to be sufficiently far from each other. Hence, this paper takes full advantage of this assumption that the received signals are statistically independent with negligible correlation. Corresponding to the signal received in the i^{th} antenna of the CR device the hypotheses H_0 and H_1 are defined as:

$$H_0: x_i[k]=v_i[k]$$

$$H_1: x_i[k]=h s[k]+v_i[k] \quad (1)$$

where h is the amplitude gain of the channel, i is the antenna index ($i=1, 2, \dots, M$) at each CR, $s[k]$ is the transmitted signal by PU and $v_i[k]$ is the AWGN noise component.

Two scenarios considered in the study are:

Case 1: The sample sizes received from each of the i^{th} antenna are same.

Case 2: The sample sizes received from each of the i^{th} antenna are different.

Two methods of detection of PU are detailed as follows:

2.1. Energy detection based sensing

Energy Detector (ED) is a blind sensing method with low computational and implementation complexities. Each individual branch at the receiver is provided with an energy detector to provide the instantaneous individual branch energy measurements. The energy of the received signal at the i^{th} branch is Y_i and N the sample size. The decision static Y_i is compared against a fixed threshold λ .

$$Y_i = \sum_{k=1}^N [x_i[k]]^2 \quad (2)$$

The simple hypothesis testing problem is formulated in Eq. (3). The probability of detection is investigated under the Neyman-Pearson (NP) criterion (using constant false alarm rate):

$$Y_i = \begin{matrix} H_1 \\ > \\ < \\ H_0 \end{matrix} \lambda \quad (3)$$

Non-coherent diversity combiners

The non-coherent combining schemes are more preferable to provide the diversity gain when the CSI is unavailable. One such method in this category is the square law techniques. The operation in Eq. (2) is executed using a square law device provided at each diversity branch of the CR receiver. The signal received from each i^{th} antenna is combined to form a better estimate of the primary user signal than using single antenna using Square Law Selection (SLS) and Square Law Combining (SLC) [7].

• Square-law selection

The energy vectors from M diversity branches, Y_1, Y_2, \dots, Y_M are used in SLS. The branch with the highest energy is selected. The test statistic is given in Eq. (4):

$$Y_{sls} = \max(Y_1, Y_2, \dots, Y_M) \quad (4)$$

• Square law combining

The energy vectors from M diversity branches, Y_1, Y_2, \dots, Y_M are gathered and combined in SLC to make a combined decision. The test statistic is given in Eq. (5):

$$Y_{slc} = \sum_{i=1}^M Y_i \quad (5)$$

2.2. Goodness of fit tests based sensing

The most well-known class of Gaussianity tests are used to determine whether a signal's samples are normally distributed or not. These tests check for the departures from the normal distribution. When the random variable X under consideration is normally distributed, the null hypothesis H_0 is declared [14]. The detection of the signal embedded in noise can be done by the Goodness of Fit Test (GoFT). It is a blind non-parametric hypothesis testing method, which decides on the null hypothesis if the received samples follow the noise Cumulative Distribution Function (CDF) denoted as F_0 .

Let $x[k]$ denote the set of N discrete time vector observations $k=1, 2, \dots, N$. The i^{th} component of $x[k]$ denoted as $x_i[k]$, $i=1, 2, \dots, M$. The signal detection in noise is given as a simple hypothesis testing problem in [15-17] and is expressed as

$$\begin{aligned} \text{Decide on } H_0: & \text{ if } F_n(x) = F_0(x) \\ \text{Decide on } H_1: & \text{ if } F_n(x) \neq F_0(x) \end{aligned} \tag{6}$$

where $F_n(x)$ is the empirical CDF of the received sample.

In statistical hypothesis testing, there are two categories of errors namely 1) False positive or type I error that occurs when H_0 is rejected when it is really true. 2) False negative or type II error that occurs when H_0 is erroneously failed to be rejected when it is really false. The type I error rate is also called the significance level and is usually denoted as alpha (α) and the latter is denoted as beta (β).

In most of the studies [10-13], experimenters have used either a significance level of 0.05 or 0.01. Lower significance levels require stronger sample evidence to be able to reject the null hypothesis. The 0.01 level is more conservative than the 0.05 level. Hence, this study considers type 1 error in signal detection in noise with $\alpha=0.05$ and the critical values are calculated using this assumption.

The goodness of fit tests can be broadly categorised into i) Empirical Distribution Function (EDF) Tests and ii) Tests based on descriptive measures. This paper features two important GoFTs one from each of the above-mentioned categories.

- **Empirical Distribution Function (EDF) Tests**

- **Anderson Darling Test**

Anderson Darling (AD) test is the best distance test for small samples. To test the normality of a random sample $x[k]$ the Anderson-Darling test statistic formulated in [18, 19] is given as:

$$A_n^2 = -N - \frac{\sum_{k=1}^N (2k-1)(\ln z_k - \ln z_{(N+1-k)})}{N} \tag{7}$$

with $y_k = (x_k - \check{x})/S$,

$$\check{x} = \sum x_k / N \text{ and } S^2 = \sum (x_k - \check{x})^2 / (N - 1) \tag{8}$$

According to D'Agostino and Stephens [20], when the mean and variance of the sample are unknown, the adjusted AD statistic is

$$A = A_n^2 \left(1 + \frac{0.75}{N} + \frac{2.25}{N^2} \right) \quad (9)$$

where $z_k = F_0(y_k)$ is the assumed distribution, N denotes the sample size, \ln is the natural logarithm.

The spectrum sensing problem is expressed as:

$$H_0: A \leq \lambda_{cv} \quad (10)$$

$$H_1: A > \lambda_{cv}$$

where λ_{cv} is a critical value. If the computed value of A exceeds the critical value then H_0 is rejected. A table of thresholds for different values of P_f is given in [14].

• Tests based on descriptive measures

Jarque and Bera test

The Jarque and Bera (JB) test is another goodness-of-fit test originally proposed by Bowman and Shenton [21] to check for normal distribution. It uses the skewness and kurtosis to determine whether the sample data is from a normal distribution. The data is declared to follow a normal distribution if the JB test statistic asymptotically has a chi-squared distribution with two degrees of freedom [22, 23].

The JB test statistic is the combination of the squares of normalized skewness and kurtosis and is given as follows:

$$J = \frac{N}{6} \left(\gamma_1^2 + \frac{(\gamma_2 - 3)^2}{4} \right) \quad (11)$$

where γ_1 is the skewness and γ_2 is the kurtosis and N is the number of samples. The spectrum sensing problem using JB test can be expressed as

$$H_0: J \leq \lambda_{cv} \quad (12)$$

$$H_1: J > \lambda_{cv}$$

The critical values of the JB test for different sample sizes are given in [22]. The primary user signal is declared present if the Jarque Bera test statistic is greater than the critical value and is declared as noise otherwise.

3. Proposed Method

In data analysis, it is generally insufficient using only a single one-dimensional summary statistic. Hence, this paper proposes a meta-analysis approach based on p-Values to combine data from independent tests to perform the overall assessment. The block diagram of the proposed method is given in Fig. 2. The M independent samples $x_i[k]$ $i=1, 2, \dots, M$ from M diversity branches of the CR receiver are tested for normality using AD test or JB test. The test statistic is used to determine the p-Value using the formula mentioned in Table 1 and the interpretation of the test results are given in Table 2. The new test statistic is obtained by combining the p-Values using the methods discussed in Section 3.2.

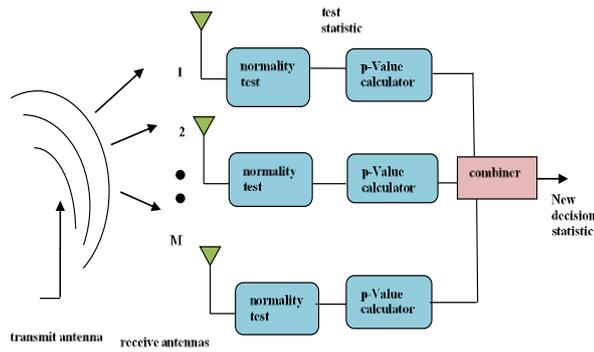


Fig. 2. Block diagram of the proposed method.

Table 1. p-Value formula for Anderson Darling test as given in [20].

AD statistic	p-Value formula
$A > 153.467$	$p = 0$
$0.6 < A \leq 153.467$	$p = e^{(1.2937 - 5.709 * A + 0.0186 A^2)}$
$0.34 < A \leq 0.60$	$p = e^{(0.9177 - 4.279 * A - 1.38 A^2)}$
$0.20 < A \leq 0.34$	$p = 1 - e^{(-8.318 + 42.796 * A - 59.938 A^2)}$
$A \leq 0.20$	$p = 1 - e^{(-13.436 + 101.14 * A - 223.73 A^2)}$

Table 2. Decision table.

Method	Condition	Decision
Classical test	If (test statistic > critical value)	H_0 is rejected
Classical test	If (test statistic < critical value)	H_0 cannot be rejected
p-Value	(p-Value < α)	H_0 is rejected
p-Value	(p-Value > α)	H_0 cannot be rejected

3.1. Significance of p-Value

Fisher justified that the p-Value can be viewed as an index of the “strength of evidence” against H_0 , with small p indicating an unlikely hypothesis [24].

The steps involved in hypothesis testing using p-Values as specified in [24] are given as follows:

- Define the null and alternative hypotheses.
- Determine the test statistic from the sample data.
- Calculate the p-Value using the value of the test statistic obtained from step 2.
- Fix the significance level $\alpha=0.05$ and interprets the results using Table 2.

Thus using the p-Value the compatibility of the data with the null hypothesis is measured but this value does not provide the probability on the correctness of the null hypothesis.

3.2. p-Value based diversity combiner

From the samples received from M diversity branches of the CR receiver, the test statistics (A_1, A_2, \dots, A_M) and its corresponding p-Values (p_1, p_2, \dots, p_M) are computed. This paper adopts the following statistical methods to integrate the p-Values from independent tests [9-13] to have an overall assessment on the detection of the primary user signal activity.

- **Fisher's test**

Fisher [25] proposed one popular method of combining the p-Values. Let p_1, p_2, \dots, p_M be the significance probabilities of the test statistic A or J in the i^{th} sample received from each diversity branch of the CR receiver. The joint assessment of the normality is based on the M values of the statistic. The different significance probabilities obtained from M diversity branches are combined using Fisher's method as given in Eq. (12).

$$F_T = -2\ln(\sum_{i=1}^M p_i) \quad (12)$$

- **Z test**

Stouffer et al. [26] proposed another approach called the z test to combine these p-Values. This method primarily converts to z values using the relation $z_i = F^{-1}(p_i)$, where F^{-1} is the inverse CDF of standard Gaussian distribution. The Z test statistic for CR receiver equipped with M antennas is formulated as

$$Z_T = \left(\sum_{i=1}^M z_i / \sqrt{M} \right) \quad (13)$$

- **Weighted Z test**

Mosteller and Bush [27] generalised the Z-test by giving weight w to each Z-Value. Under the null hypothesis, Z_w follows normal distribution described with parameters $\mu = 0$ and $\sigma^2 = \sum_{i=1}^M w_i^2$. The weights are usually taken as the sample sizes. The weighted z -test is defined as:

$$Z_W = \left(\frac{\sum_{i=1}^M w_i z_i}{\sqrt{\sum_{i=1}^M w_i^2}} \right) \quad (14)$$

Algorithm 1. p-Value based diversity combining

1. Obtain M observation samples from each of the diversity branches of the CR node.
 2. Let Z_i ($i = 1 \dots M$) be the observation vector. Sort the observations from each branch in ascending order.
 3. Calculate the AD test or JB test statistic using Eqs. (7) - (11)
 4. Let A_i ($i = 1 \dots M$) denote the test statistic obtained for M diversity branches.
 5. Using the formula given in Table 1 calculate the p-Value p_1, p_2, \dots, p_M
 6. The p values are combined using Eqs. (12 to 14) to obtain the new decision statistic.
 7. Reject null hypothesis if the new decision statistic is less than the predefined significance level.
-

4. Results and Discussion

4.1. Monte Carlo simulations

The performance analysis of spectrum sensing using receiver diversity in a CR environment are carried out using 1) Energy detection and 2) Goodness of Fit Test. The detection probability is used as a standard of measurement to determine the sensing accuracy. The following assumptions are made in the simulations.

- The system model has Single Input Multiple Output.
- The primary transmitter signal is a sinusoidal pilot signal of known frequency.
- Additive White Gaussian Noise with $\mu = 0$ and $\sigma^2 = 1$.
- The significance level (type I error) is set to $\alpha = 0.05$.
- The test statistic and hence, the p-Values are independent as they are calculated from samples received from different diversity branches, which have a negligible correlation.
- Sample sizes in the study:

Scenario 1: The sample size received from each of the i^{th} antenna is held constant, i.e., 100 samples.

Scenario 2: The sample size received from each of the i^{th} antenna is varied, the sample sizes are taken as 10, 50, 240 samples.

In Fig. 3 probability of detection vs. SNR using the square-law techniques for $N = 100$ samples, $M = 3$ and $P_f = 0.05$ under scenario 1 are studied. As evident from the Eqs. (4, 5), the SLC method of diversity combining outperforms SLS. This is supported by the simulation results presented in Fig. 3. The Fisher’s method, Z test and the weighted Z test are investigated as diversity combiners in the context of spectrum sensing. The detection probability using the above-mentioned diversity combiners are studied using AD test and JB test in Figs. 4 and 5 respectively for sample lengths considered in scenarios 1 and 2. Detection probability is used as a metric to evaluate the performance of the earlier mentioned tests as diversity combiners.

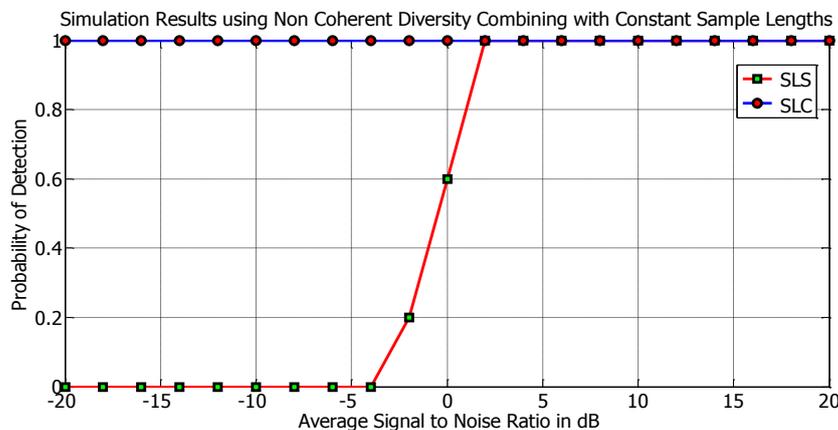
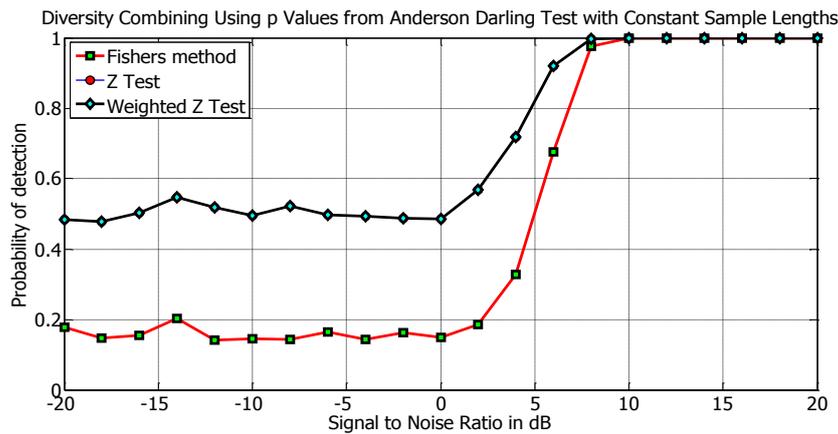


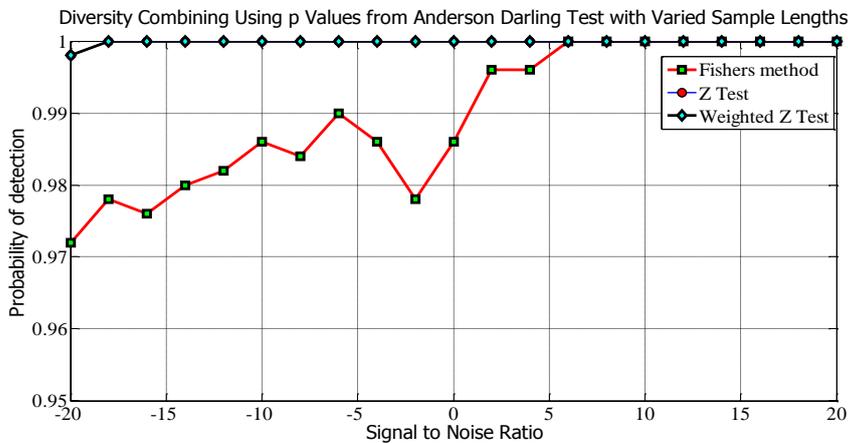
Fig. 3. Probability of detection vs. SNR for sinusoidal pilot signal for $N = 100$ samples, $M = 3$ and $P_f=0.05$ under scenario 1.

From Figs. 4 and 5, the following observations are made:

- It is observed that the AD test shows a higher probability of detection compared to the JB test.
- The weighted Z test performs better than the Z test and Fisher's test in both AD and JB test studies for case 2, i.e., when the numbers of individuals in a sample are different. It provides detection very close to the existing square law combiners.
- When the sample sizes are equal, the performance of the weighted Z test and Z test is identical.
- In the low SNR regimes, the weighted Z test works as a better diversity combiner compared to the other two methods as it provides a higher probability of detection.

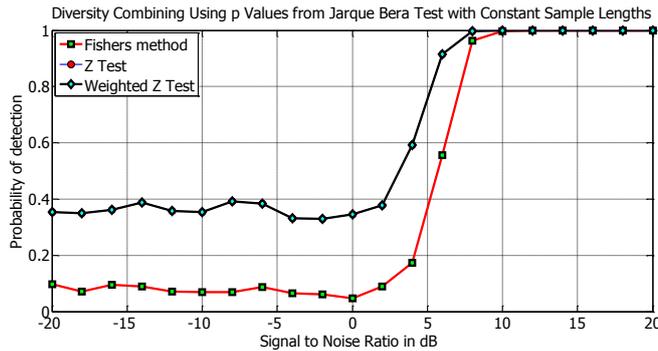


(a) Scenario 1.

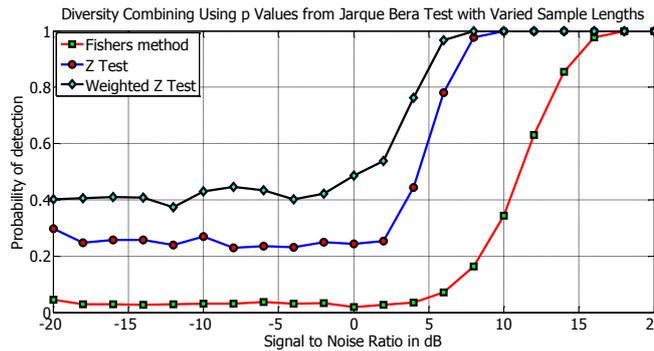


(b) Scenario 2.

Fig. 4. Probability of detection vs. SNR of the proposed method using AD test with sinusoidal pilot signal, $M = 3$ and $\alpha = 0.05$, $\mu = 0$.



(a) Scenario 1.



(b) Scenario 2.

Fig. 5. Probability of detection vs. SNR of the proposed method using JB test with sinusoidal pilot signal, $M = 3$ and $\alpha = 0.05$, $\mu = 0$.

Table 3 gives the tabulation of p-Values, Fisher’s test, weighted Z test and Z test statistic for scenario 1 and scenario 2 for $M=3$. The p-Values from M diversity branches are combined using Eqs. (12-14) to obtain the F_T , Z_T and Z_W test statistic.

From Fig. 6 and the tabulation, the following can be inferred.

- As the number of samples used for hypothesis testing grows, it yields smaller p-Values hence, increases the probability of detection of the primary user.

As the mean of the random noise increases, the power of the combined test statistic also grows [13]. From the scatter plot in Fig. 7, it can be inferred that as the SNR increases there is a linear decrease in the p-Value, which helps in validating the decision of rejection of null hypothesis as the SNR increases (or decision on the presence of the PU signal).

Table 3. Comparison of the p-Value combining techniques for $M = 3$ receiver diversity for SNR of -20 dB.

Study	Sample length	μ	Z-test	WZ-test	Fisher’s test	p-Value from M diversity branches		
						p_1	p_2	p_3
Scenario 1	100	0	-0.5915	-0.5915	-1.0255	0.96	0.44	0.26
	100	1	0.0106	0.0106	-0.8040	0.59	0.54	0.35
Scenario 2	100	0	-0.5852	0.1667	-0.9826	0.54	0.98	0.10
	100	1	-0.0399	1.2971	-0.9614	0.67	0.88	0.05

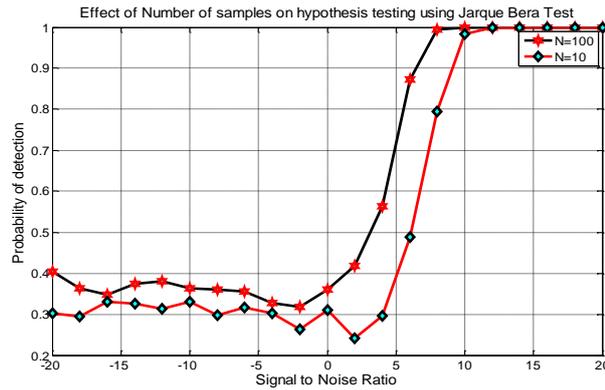


Fig. 6. Effect of number on samples on probability of detection for $M = 3$, using weighted Z-test.

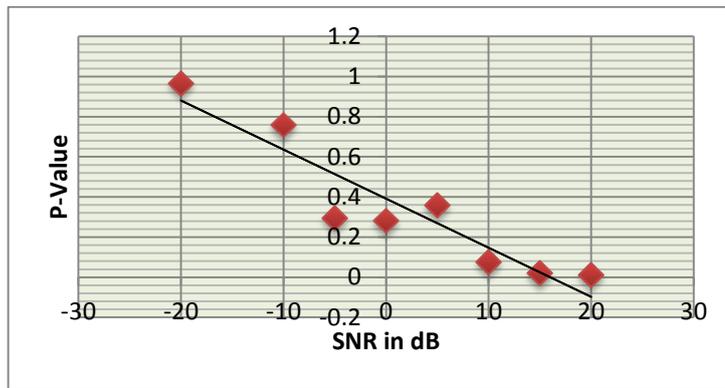


Fig. 7. Scatter plot of SNR vs. p-Value for $M = 3$, $\alpha=0.05$, $N = 100$ and sinusoidal pilot signal.

5. Conclusions

A novel meta-analytic approach to combine the data received from multiple diversity branches of the CR receiver is proposed. A ballpark figure of the merits of these diversity combining methods are provided in this study. Results show that AD test shows superior performance compared to JB test. The weighted Z test using the AD statistic is preferable compared to other methods as it shows superior detection compared to the Z test and Fisher’s test even when the samples received from each antenna is varied. Also, the algorithm proposed improves the detection of the PU in low SNR regimes. Through extensive Monte Carlo simulations, it is shown that the Weighted Z test using the Anderson Darling test statistic provides primary user detection very close to the existing non-coherent diversity combiners. It is also observed that the probability of detection obtained with the proposed method is higher than the functional requirements of obtaining a detection probability of 0.9 as specified in the cognitive radio IEEE 802.22 Wireless Regional Area Network (WRAN) Standard [28]. Hence, this novel statistical approach based on p-Values provides a promising solution to combine the test statistics from

multiple receiver antennas. Furthermore, this study can be further extended to detection when the p-Values of individual diversity branches are correlated.

Nomenclatures	
A	Anderson Darling test statistic
h	Amplitude gain of the channel
H_0	Null hypothesis
H_1	Alternate hypothesis
i	Antenna index
J	Jarque Bera test statistic
k	Sample index
M	Number of antennas in each CR
N	Sample size
P_d	Probability of detection
P_f	Probability of false alarm
$s[k]$	Transmitted signal
$v[k]$	Noise component
$x[k]$	Received signal
Y	Energy measurement
Greek Symbols	
α	Type I error rate
B	Type II error rate
γ_1	Measure of skewness
γ_2	Measure of kurtosis
λ	Fixed Threshold
λ_{cv}	Critical Value
μ	Mean
σ^2	Variance
Abbreviations	
CSI	Channel State Information
SIMO	Single Input Multiple Output
SNR	Signal to Noise Ratio

References

1. Yucek, T.; and Arslan, H. (2009). A survey of spectrum sensing algorithms for cognitive radio applications. *IEEE Communications Surveys & Tutorials*, 11(1), 116-130.
2. Meitzner, J.; Schober, R.; Lampe, L.; Gerstacker, W.H.; and Hoeher, P.A. (2009). Multiple-antenna techniques for wireless communications - a comprehensive literature survey. *IEEE Communications Surveys & Tutorials*, 11(2), 87-105.
3. Wang, P.; Fang, J.; Han, N.; and Li, H. (2010). Multiantenna-assisted spectrum sensing for cognitive radio. *IEEE Transactions on Vehicular Technology*, 59(4), 1791-1800.

4. Digham, F.F.; Alouini, M.-S.; and Simon, M.K. (2007). On the energy detection of unknown signals over fading channels. *IEEE transactions on communications*, 55(1), 21-24.
5. Annavaajjala, R.; and Milstein, L.B. (2005). Performance analysis of linear diversity-combining schemes on rayleigh fading channels with binary signaling and gaussian weighting errors. *IEEE Transactions on Wireless Communications*, 4(5), 2267-2278.
6. Herath, S.P.; and Rajatheva, N. (2008). Analysis of equal gain combining in energy detection for cognitive radio over Nakagami channels. *Proceedings of the IEEE Global Telecommunications Conference*. New Orleans, Louisiana, United States of America, 1-5.
7. Herath, S.P.; Rajatheva, N.; and Tellambura, C. (2009). On the energy detection of unknown deterministic signal over Nakagami channels with selection combining. *Proceedings of the IEEE Canadian Conference on the Electrical and Computer Engineering*. St. John, Newfoundland, Canada, 745-749.
8. Akbari, M.; Manesh, M.R.; El-Saleh, A.A.; and Reza, A.W. (2014). Receiver diversity combining using evolutionary algorithms in rayleigh fading channel. *The Scientific World Journal*, Article ID 128195, 11 pages.
9. Pettitt, A.N. (1977). Testing the normality of several independent samples using the Anderson-Darling statistic. *Journal of Royal Statistical Society, Series C (Applied Statistics)*, 26(2), 156-161.
10. Zaykin, D.V. (2011). Optimally weighted z-test is a powerful method for combining probabilities in meta-analysis. *Journal of Evolutionary Biology*, 24(8), 1836-1841.
11. Rosenthal, R. (1978). Combining results of independent studies. *Psychological Bulletin*, 85(1), 185-193.
12. Bhandary, M.; and Zhang, X. (2011). Comparison of several tests for combining several independent tests. *Journal of Modern Applied Statistical Methods*, 10(2), 436-446.
13. Whitlock, M.C. (2005). Combining probability from independent tests: The weighted z-method is superior to fisher's approach. *Journal of Evolutionary Biology*, 18(5), 1368-1373.
14. Stephens, M.A. (1974). EDF statistics for goodness of fit and some comparisons. *Journal of the American Statistical Association*, 69(347), 730-737.
15. Wang, H.; Yang, E.-H.; Zhao, Z.; and Zhang, W. (2009). Spectrum sensing in cognitive radio using goodness of fit testing. *IEEE Transactions on Wireless Communications*, 8(11), 5427-5430.
16. Tegui, D.; Le Nir, V.; and Scheers, B. (2014). Spectrum sensing method based on goodness of fit test using chi-square distribution. *Electronics Letters*, 50(9), 713-715.
17. Tegui, D.; Le Nir, V.; and Scheers, B. (2015). Spectrum sensing method based on likelihood ratio goodness-of-fit test. *Electronics Letters*, 51(3), 253-255.
18. Gurugopinath, S.; and Samudhyatha, B. (2015). Multi-dimensional Anderson-Darling statistic based goodness-of-fit test for spectrum sensing. *Proceedings of the IEEE Seventh International Workshop on Signal Design and its Applications in Communications (IWSDA)*. Bengaluru, India, 165-169.

19. Romeu, J.L. (2003). Anderson Darling: A goodness of fit test for small sample assumptions. *RAC Start*, 10(5), 6 pages.
20. D'Agostino, R.B.; and Stephens M.A. (1986). *Goodness-of-fit techniques*. Statistics: Text books and monograph. Boca Raton, Florida: CRC Press.
21. Bowman, K.O., and Shenton, L.R. (1975). Omnibus test contours for departures from normality based on $\sqrt{b_1}$ and b_2 . *Biometrika*, 62(2), 243-250.
22. Thadewald, T.; and Buning, H. (2007). Jarque-Bera test and its competitors for testing normality-A power comparison. *Journal of Applied Statistics*, 34(1), 87-105.
23. Adefisoye, J.O.; Golam Kibria, B.M.; and George, F. (2016). Performances of several univariate tests of normality: An empirical study. *Journal of Biometrics & Biostatistics*, 7(4), 1-8.
24. Berger, J.O. (2003). Could Fisher, Jeffreys and Neyman have agreed on testing? *Statistical Science*, 18(1), 1-32.
25. Fisher, R.A. (1932). *Statistical methods for research workers (4th ed.)*. Edinburgh, Scotland: Oliver and Boyd.
26. Stouffer, S.A.; Suchman, E.A.; DeVinney, L.C.; Star, S.A.; Williams Jr., R.M.; (1949). *The American soldier: Adjustment during army life*. (Studies in social psychology in World War II), Vol. 1. Oxford, England: Priceton University Press.
27. Mosteller, F.; and Bush, R.R. (1954). *Selected quantitative techniques*. Handbook of social psychology, vol. 1 (G. Lindzey, ed.). Cambridge, Massachusetts: Addison-Wesley.
28. Stevenson; C.R.; Chouinard, G.; Lei, Z.; Hu, W.; Shellhammer, S.J.; and Cardwell, W. (2006). IEEE 802.22: The first cognitive radio wireless regional area network standard. *IEEE Communications Magazine*, 47(1), 130-138.