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A FRACTIONAL MODEL PREDICTIVE CONTROL DESIGN FOR 2-D GANTRY CRANE SYSTEM

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Abstract

Fractional calculus has been a research focus since the last four decades to control the dynamical systems. Advances in fractional calculus such as Fractional Model Predictive Control had shown that system dynamics could be controlled even more effectively. This paper proposes Fractional Model Predictive Control design for a 2-D gantry crane system (a robotic manipulator). 2-D gantry crane system is an under-actuated system with two degrees of freedom and single control input. The model is derived using Euler-Lagrange formulation and the corresponding fractional model is embedded, further, this model is approximated using an Oustaloup-Recursive-Approximation for different fractional values. A Fractional Model Predictive Controller and a traditional Model Predictive Control for the 2-D Gantry crane system are designed using MPC toolbox of MATLAB. This controller gives a better response in terms of systems settling time and overshoot in the system. Further, the performances of these controllers are compared with other existing controllers. The purpose of these controllers here is to control the position of the trolley and the swing angle of the cable through which the load is suspended.

Keywords: 2-D gantry crane system, Fractional calculus, Fractional model predictive control; Model predictive control; Robustness.

1. Introduction

The roots of the fractional order calculus theory can be traced from [1]. Since then there have been several attempts by researchers to use fractional order calculus to deal with dynamical systems [2, 3-9]. Conventional integer order calculus follows differentiation and integration of only integer order, whereas fractional order calculus follows integration and differentiation of real order including integer order. Control of mechanical systems using fractional calculus is currently among one of the most active fields of research [10-15].

In the recent past, there has been a lot of focus on Model Predictive Control (MPC) [16-18]. Predictive control is based on controlling some criterion with the help of a model to predict the system behaviour for further action. An under-actuated system based on MPC has been studied by the group of researchers. For instance, simulation studies of performance and efficiency of the MPC controlled underactuated system have been presented in [19]. An adaptive control strategy is demonstrated in [20]. Furthermore, Fractional Model Predictive Control (FMPC) of fractional order systems is studied in [21-23]. In a similar study, robust MPC for the fractional system is presented in [24]. In these papers, well-defined fractional systems are considered to design the MPC.

Most recently Fractional Order Controllers are used in electric vehicle control [25], for magnetic lavation control with simplified fractional controller [26], implementation of Fractional Order Controller to DC motor control [27] and controlling the dynamic behaviour of power systems [28]. Recently Fractional Model Predictive Control attracts many researchers in designing control strategies. Paper [29] discusses MPC designing of discrete time fractional order systems. Paper [30] discuss designing of fractional MPC for temperature control in industrial processes. Hence, this shows that fractional MPC (FMPC) has a tremendous potential for research.

Motivated by the recent advances in FMPC, this paper discusses the models 2-D gantry crane system using Euler-Lagrange formulation. The corresponding fractional model is embedded for capturing the dynamics of the system. Finally, the FMPC is designed using Oustaloup-Recursive Approximation method of fractional order. Simulation results show that the performance of FMPC is better than its MPC counterpart. In addition, a robustness study by varying system parameters is carried out by changing the various parameters of the system. As per the author’s knowledge, there is no prior work on FMPC for the 2-D gantry crane system.

The motive of this paper is to present the design of an efficient controller, which can stabilize the consider system with the minimum amount of time with less error as compared to the existing controllers.

The structure of the paper organized as, Section 2 discuss the basic preliminaries about fractional order calculus. In Section 3, the model of the considered system is derived and further the fractional model is embedded in Section 4 and controller is also designed and compared with the traditional controllers in this section. In Section 5, the robustness of the controller is reported and the results are compared with existing controllers followed by a conclusion in Section 6.

1.1. Preliminary

Fractional Order Control (FOC) is a generalization of the Integer Order Control (IOC) to a real or may complex order. The fractional operator is introduced as follows:

$$ {}_aD_t^{\alpha} f(t) = \begin{cases} \dfrac{d^{\alpha}}{dt^{\alpha}} & \alpha > 0 \\ 1 & \alpha = 0 \\ \displaystyle\int_a^t d\tau^{\alpha} & \alpha < 0 \end{cases} $$

where $\alpha \in R$, a and t are the limits of the fractional operators.

There are many definitions given by different researchers for fractional calculus. Out of them, important definitions for fractional derivatives are given below [31, 32].

- Riemann-Liouville (*RL*) definition:

For the case of $0 < \alpha < 1$ the expression of the fractional derivative is:

$$ {}_a^{\alpha}D_t f(t) = \frac{1}{\Gamma(n-\alpha)} \frac{d^n}{dt^n} \int_0^t \frac{f(\tau)}{(t-\tau)^{\alpha-n+1}} d\tau \tag{1} $$

where $\Gamma(.)$ is Gamma function, n is an integer, which satisfies the conditions n-1 < $\alpha < n$.

- Grunwald-Letnikov (GL) definition: If we consider $n = \frac{t-a}{h}$, where 'a' is real constant, which expresses a limit value we can write,

$$ {}_a^{\alpha}D_t f(t) = \lim_{l \to 0} \frac{1}{l^{\alpha}} \sum_{m=0}^{\frac{t-a}{l}} (-1)^m \frac{\Gamma(\alpha+1)}{m!\,\Gamma(\alpha-m+1)} f(t-ml) \tag{2} $$

- Caputo definition: The Caputo definition is used in many engineering applications. It is defined by,

$$ {}_a^{\alpha}D_t f(t) = \frac{1}{\Gamma(m-\alpha)} \int_0^t \frac{f^{(m)}(\tau)}{(t-\tau)^{\alpha-m+1}} d\tau \tag{3} $$

The Caputo and Riemann-Liouville formulation coincide when the initial conditions are zero.

2. Modelling of the 2-D Gantry Crane System

The 2-D Gantry crane system is one of the good examples of under-actuated systems. It is used to transport a load from one place to another with the help of a trolley and load system. These cranes can handle a huge amount of load and basically used in platforms, ships, depots, factories, etc. Therefore, the modelling and the control of these systems is the main aim of this paper.

The 2-D Gantry crane system is shown in Fig. 1. The mass of the trolley and the mass of load are denoted as M and m, respectively. The size of the cable assumed to be l meter. There is an *x*-directed force is applied on the trolley and denoted by *u(t)* the gravitational force *mg* be always acted on the load and *g* is acceleration due to gravity *x(t)* represents trolley position and $\theta(t)$ represents the tilt angle with referenced to the vertical direction.

The Euler-Lagrange formulation is used for model derivation. It is required to find the kinetic energy (*T*) and potential energy (*V*) of the system and then Lagrangian be the difference between *T* and *V*.

$$L = T - V \tag{4}$$

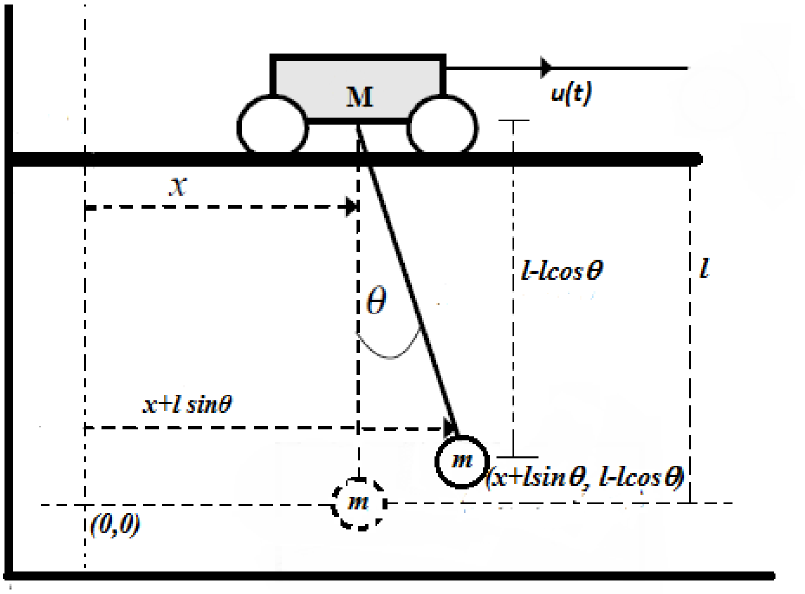

Fig. 1. 2-D Gantry crane system.

After putting the values of *T* and *V*, the Lagrangian equation will be:

$$L = \frac{1}{2}M\dot{x}^2 + \frac{1}{2}m(\dot{x}^2 + (l\dot{\theta})^2 + 2\dot{x}\dot{\theta}^2 lcos\theta - Mgl - mg(l - lcos\theta) \tag{5}$$

Taking the value of *L* from Eq. (5) into Euler-Lagrange equations below,

$$\frac{d}{dt}\left(\frac{\partial L}{\partial \dot{x}}\right) - \frac{\partial L}{\partial x} = u \tag{6}$$

and

$$\frac{d}{dt}\left(\frac{\partial L}{\partial \dot{\theta}}\right) - \frac{\partial L}{\partial \theta} = 0 \tag{7}$$

After some mathematical manipulation and using Eqs. (6) and (7) and then forming into its nonlinear state-space model with $z_1 = x, z_2 = \dot{x}, z_3 =, z_4 = \dot{\theta}$, the resulting nonlinear state space representation is expressed as follows:

$$\frac{d}{dt}\begin{pmatrix} Z_1 \\ Z_2 \\ Z_3 \\ Z_4 \end{pmatrix} = \begin{pmatrix} Z_2 \\ \frac{\left(u + m\, l\, {Z_4}^2 sin\,(Z_3)\right) + g\, m\, \sin(Z_3)\, cos\,(Z_3)}{M + m - m\, cos^2(Z_3)} \\ Z_4 \\ \frac{u\cos(Z_3) + ml\, {Z_4}^2 \sin(Z_3)\cos(Z_3) + g(M+m)\sin(Z_3)}{ml\, cos^2(Z_3) - (M+m)l} \end{pmatrix} \quad (8)$$

Linearizing the above state space model obtained in Eq. (8) at $(z_0, u_0) = (0, 0)$and linearized matrix we get after taking the parameter values of M = 2.5 kg, m=1 kg, l = 1 m, g =9.8 m/s^2 ,is

$$\frac{d}{dt}\delta \underline{z} = \begin{pmatrix} 0 & 1 & 0 & 0 \\ 0 & 0 & 3.92 & 0 \\ 0 & 0 & 0 & 1 \\ 0 & 0 & -13.72 & 0 \end{pmatrix} \delta \underline{z} + \begin{pmatrix} 0 \\ 0.4 \\ 0 \\ -0.4 \end{pmatrix} \delta u \quad (9)$$

The output matrix is,

$$y = \begin{pmatrix} 1 & 0 & 0 & 0 \\ 0 & 0 & 1 & 0 \end{pmatrix} \begin{pmatrix} x \\ \dot{x} \\ \theta \\ \dot{\theta} \end{pmatrix} \quad (10)$$

3. Controller design using MPC toolbox

The 2-D Gantry crane system is converted to its equivalent fractional 2-D Gantry crane system. Then, the fractional model is approximated using an Oustaloup recursive approximation [33]. Further, this model of 2-D Gantry crane system is used to design the Model Predictive Controller. Here, the fractional model is considered because the integer order model, which is the approximation of the fractional order model and almost all existing systems are fractional [34-36]. The concept behind using the traditional model (integer order model) was the non-presence of the solution methods for fractional order models [37], but at present, there are various methods available for the approximation of the fractional models (derivative and integral) [38, 39] by using definitions presented in Eqs. (1) to (3). It can be easily and widely used in the areas of control theory [40], system models [41], mechanical systems analysis [42], circuital analysis [43], etc.

The transfer function model is derived using the state space model, which is represented in Eqs. (9) and (10).

$$H(s) = \begin{bmatrix} \dfrac{0.4s^2 + 4}{s^4 + 13.7s^2} \\ \dfrac{-0.4s^2}{s^4 + 13.7s^2} \end{bmatrix} \quad (11)$$

The fractional transfer function for 2-D Gantry crane system is given by following transfer function. It is obtained by introducing the fractional order in the order of system [44, 45], which is represented by,

$$H(s^{\alpha})=\begin{bmatrix}\dfrac{0.4s^{2\alpha}+4}{s^{4\alpha}+13.7s^{2\alpha}}\\ \dfrac{-0.4s^{2\alpha}}{s^{4\alpha}+13.7s^{2\alpha}}\end{bmatrix} \tag{12}$$

By substituting the value of $\alpha = 1$ in Eq. (12), it is the same transfer function of 2-D Gantry crane system as represented in Eq. (11). Here using α, which non-integer order, capture more dynamics of the system. The value of α is between $0 < \alpha < 1$. The selection of α value is critical and important. Here, the various values of α are checked and validated using simulation results. Based on that, the FMPC are designed for this system. Now, the various values of α are considered and discussed.

Case 1: If $\alpha = 0.3$, the transfer function of Eq. (12) can be written as:

$$H(s^{0.3})=\begin{bmatrix}\dfrac{0.4s^{0.6}+4}{s^{1.2}+13.7s^{0.6}}\\ \dfrac{-0.4s^{0.6}}{s^{1.2}+13.7s^{0.6}}\end{bmatrix} \tag{13}$$

Equation (13) is the fractional model of 2-D Gantry crane system with $\alpha = 0.3$. Now by Oustaloup recursive approximation [33], the approximated model for 2-D Gantry crane system is given by,

$$\overline{H(s)}=\begin{bmatrix}\dfrac{1.1s+0.3}{s^{2}+3.6s+0.01}\\ \dfrac{-1.6s-0.4}{s^{2}+60.4s+14.4}\end{bmatrix} \tag{14}$$

Equations (13) and (14) represents the same system model; only the difference is that, Eq. (13) is approximated to Eq. (14). Using the above-approximated model, the corresponding controller is designed for the fractional model of 2-D Gantry crane system. Similarly, for a various order of fractional order system, the FMPC are designed and compared with each other (Case 2: $\alpha = 0.5$, Case 3: $\alpha = 0.8$, and Case 4: $\alpha = 1$, which is the original model of the 2-D Gantry crane system).

This is the first time when the FMPC controller strategy is implemented on the 2-D Gantry crane system. A MATLAB MPC toolbox [46] is used in the Windows 8 environment to design the Model Predictive Controller. The structure of the MPC toolbox is shown in Fig. 2. Considered 2-D Gantry crane system has single input and two outputs (SIMO system, single input and multiple outputs).

The input is acting on the trolley. The two outputs as shown in Fig. 2 are the position of the trolley and the swing angle of the suspended cable. There are two disturbances in this system, which are the mass of the trolley and load. Considering the first case, i.e., Case-1 and let us design the MPC using the above MPC toolbox, with a sampling interval of 0.1 seconds, prediction horizon to be 30 and control horizon to be 6. The aim is to control the trolley position *x(t)* and swing angle $\theta(t)$ with no overshoot and no oscillations.

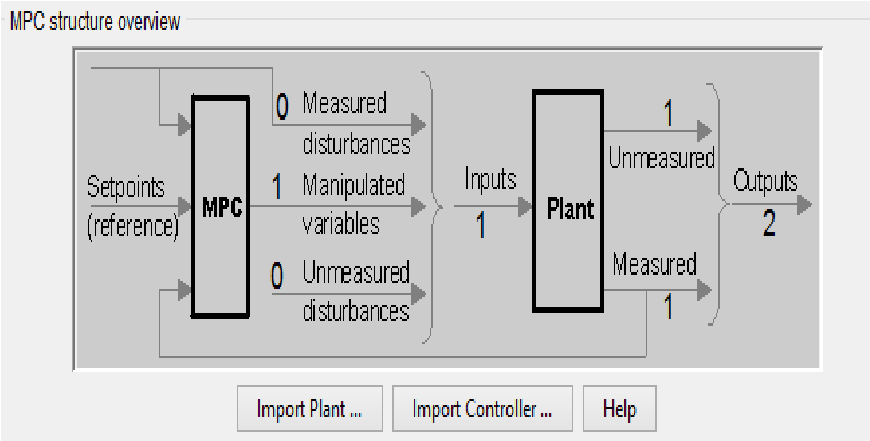

Fig. 2. MPC toolbox structure.

It can be concluded from Fig. 3 that it is possible to design a Fractional Model Predictive Control, which can give the better output. So, it can be used to control the position of the trolley and swing angle of the cable for proper operation of 2-D Gantry crane. Let us consider case-4, an integer or regular model of the 2-D Gantry crane. For the same specification, let us design an MPC. Figure 4 shows the response of the integer or regular model of the 2-D Gantry crane. It can be observed from Figs. 3 and 4 that the settling time of FMPC of 2-D Gantry crane is much better than the integer or regular Model Predictive Controller.

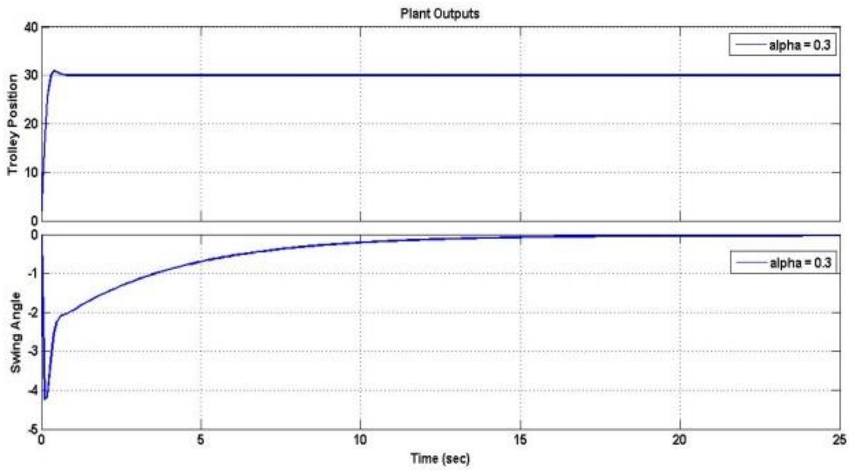

Fig. 3. The control output of the fractional 2-D Gantry crane system for $\alpha = 0.3$.

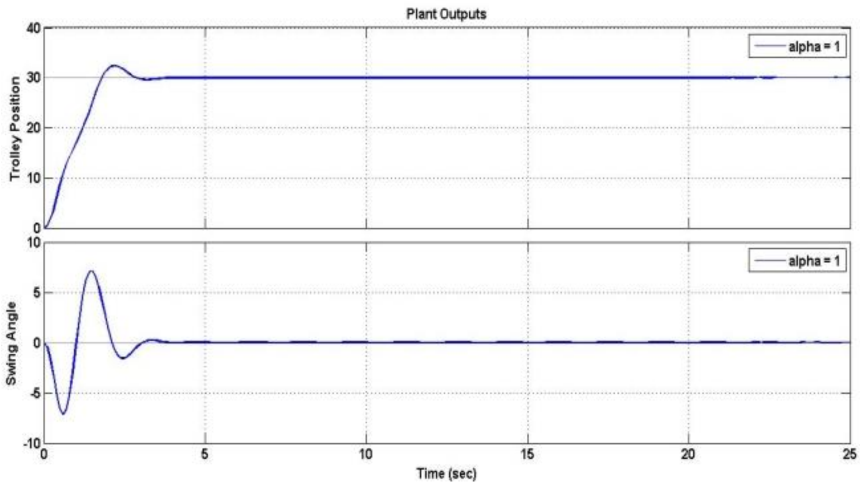

Fig. 4. The control output of the 2-D Gantry crane system for α =1.

Now, comparing various cases α = 0.3, 0.5, 0.8 and 1 for the performances of the FMPC is explored. Figure 5 clearly gives the comparison of the MPC controller for 2-D Gantry crane

It can be concluded from the comparison responses of the MPC, that the FMPC gives better performance than the traditional MPC if we choose the value of α in between $0 < \alpha \leq 1$. Hence, the model corresponding to these values are the valid models for FMPC controller design. The comparison summary shown in Table 1 is also provided for better understanding of the Fractional Model Predictive Control vs. traditional Model Predictive Control.

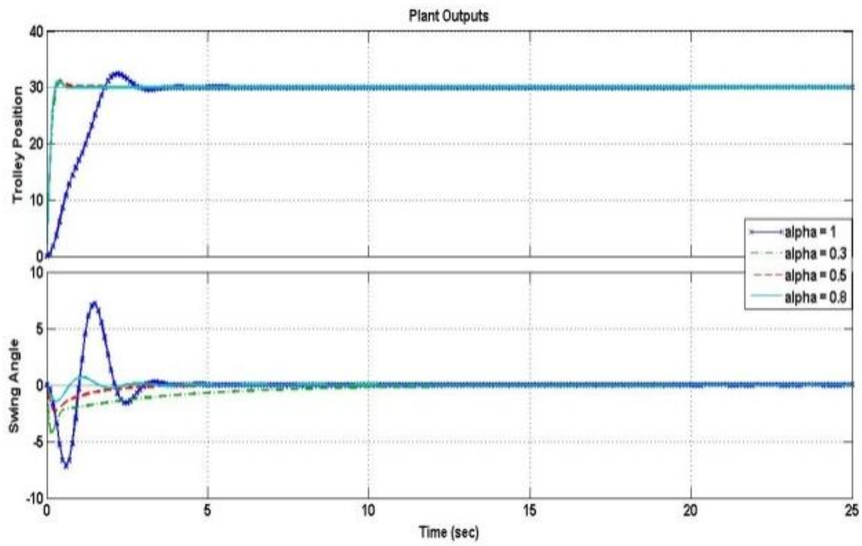

Fig. 5. Comparison of control outputs of the 2-D Gantry crane system for α = 0.3, 0.5 and 1.

From Table 1, it can be concluded that FMPC shows better response and if α = 0.5 then FMPC controller gives the best result as compare to traditional MPC controller for 2-D Gantry crane system. Hence, this paper will consider α = 0.5 as the optimum fractional order model of the 2-D Gantry crane system. Let us check the robustness of the FMPC with the model corresponding to α = 0.5.

Table 1. Comparison table for various values of α.

Specifications	Settling time (seconds)		Overshoot		Oscillations	
	x(t)	*θ(t)*	*x(t)*	*θ(t)*	*x(t)*	*θ(t)*
0.3	0.8	16	No	No	No	No
0.5	**0.8**	**3.5**	**No**	**No**	**No**	**No**
0.8	0.8	5	Yes	Yes	No	Yes
1	**3.8**	**4**	**Yes**	**Yes**	**Yes**	**Yes**

4. Conclusion

The main aim of designing a Fractional Model Predictive Control (FMPC) for 2-D Gantry crane system is successfully achieved. The proposed FMPC gives better results when it is compared in terms of settling time and overshoot of the system with traditional MPC controller. It is observed in this paper that, if the "α" values of the model are chosen in between $0 < \alpha < 1$, FMPC controller gives better response for the considered system, i.e., 2-D Gantry crane system. This method of designing controllers can also be extended to different robotic manipulators, under-actuated systems, etc.

Nomenclatures

L	Cable length, m
M	Mass of the trolley/cart, kg
m	Mass of the load, kg
T	Kinetic energy, J
u	Input to the system
V	Potential energy, J
x(t)	Represents trolley position, m

Greek Symbols

α	Fractional values
θ(t)	Represents the tilt angle, deg.

Abbreviations

FMPC	Fractional Model Predictive Control
FOC	Fractional Order Control
IOC	Integer Order Control
MPC	Model Predictive Control

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