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DESIGN OF CENTRALIZED ROBUST PI CONTROLLER FOR A MULTIVARIABLE PROCESS

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Abstract

A simple robust control technique based on centralized Proportional Integral is proposed for a multivariable process. The study is undertaken to improve the performance of the multivariable system in presence of plant model mismatch. The method considers Steady State Gain Matrix and process parameters like time delay and time constant for the controller design. The robustness of the proposed technique is analysed with perturbation in all the three parameters namely process gain, time constant, and time delay. The servo and load disturbance rejection response is obtained. Also, the multiplicative uncertainty is checked with spectral radius criterion. The Integral Absolute Error values of main effect, interaction effect, and uncertainty in process parameters are also compared with the centralized controller designed based on Steady State Gain Matrix. The tuning parameters for the original plant and perturbed plant is kept constant throughout the analysis. Finally, the proposed control algorithm is validated using pilot plant binary distillation column.

Keywords: Centralized control, PI control, Process parameters, Multiplicative uncertainty, Spectral radius.

1. Introduction

The most important role of the control engineer in a process control is the tuning of controller. Almost all process industries have many manipulated and controlled variables. The major challenges in the Multiple Input Multiple Output (MIMO) system is because of the interaction effect among the loops, non- stationary behaviour, time delay and the disturbance added under operating condition. The interaction is mainly due to changes in one input effect with respect to several outputs.

Nomenclatures

G_c	Controller matrix
G_p	Plant transfer function
i, j	i^{th} row and j^{th} column
k	Process gain
K_C	Proportional gain
K_D	Derivative gain
K_I	Integral gain
L	Reflux
Q	Reboiler
r_1, r_2	Setpoint
T	Time constant
T_1	Tray 1 temperature in deg. C
T_5	Tray 5 temperature in deg. C
t_s	Settling time
y_1, y_2	Output response

Greek Symbols

Σ_A	Additive
δ_1, δ_2	Tuning parameters
Δ_A	Additive uncertainty
Δ_I	Input uncertainty
Δ_O	Output uncertainty
ρ	Spectral radius
τ	Time delay
τ_I	Integral time
ω	Frequency in rad/sec
Π_I	Input multiplicative
Π_O	Output multiplicative

Abbreviations

BA	Bat Algorithm
BFO	Bacterial Foraging Optimization
CF	Cost Function
FA	Firefly Algorithm
IAE	Integral Absolute Error
IMC	Internal Model Control
ISE	Integral Square Error
ISP	Industrial Scale Polymerization
MIMO	Multi Input Multi Output
OS	Overshoot
PI	Proportional-Integral
PID	Proportional-Integral-Derivative
PSO	Particle Swarm Algorithm
RGA	Relative Gain Array
RHP	Right Half Plane
SR	Spectral Radius
SSGM	Steady State Gain Matrix

Also, the main challenging task in the process industry is the process plant model mismatch. Even though there are many advanced control techniques, the Proportional-Integral-Derivative (PID) control technique remain as one of the effective method among all. Due to its effectiveness, adequate performance with simple structure, robustness against parameter uncertainties, and actuator/sensor uncertainties it is a topic of on-going research [1-4]. There are many control methods proposed in literature, to handle the process with multiple loops in practice. These methods include Internal Model Control (IMC) method, decoupler based full matrix controller, and synthesis method.

Sharma and Chidambaram [5] have proposed a centralized PI controller for non-square matrix with Right Half Plane (RHP) zeros using Steady State Gain Matrix (SSGM). Here, the tuning parameters δ_1 and δ_2 are in the region 0 to 1. But, in the method there is no role of the dynamic model parameters. The computation of SSGM is essential to avoid the sustained deviation of the process variable from the setpoint. Razali et al. [6] have presented combined control technique based on Davison, Penttinen-Koivo, and Maciejowski with the capacity of handling the dynamic process. Dhanyaram and Chidambaram [7] have presented a decoupler combined with diagonal PI controller based on SSGM. They have designed a decouple controller such that the resultant of the combined process and the decoupler becomes a diagonal system.

Further, they have identified the smaller time constant and large delay from the diagonal system. The resultant controller is obtained by combining SSGM and decouple controller. The tuning parameters, δ_1 and δ_2, are between the range 0.1 to 3 and 0.05 to1.5 respectively. A detailed review is given by Maciejowski [8] and, Skogestad and Postlethwaite [9] on the multivariable control with various multiplicative uncertainties. Liu et al and Spall. [10-11] discussed multivariate stochastic approximation by considering simultaneous perturbation in the process. Similar to this, authors have also carried out a simulation with multiple uncertainty in the process transfer function. Huusom et al. [12] have proposed iterative feedback tuning for the system in state space form which also includes uncertainty in one of the process parameter. In this paper, authors have used the transfer function approach to design a controller. Precup et al. [13] emphasized that future research must be focused on MIMO systems with iterative feedback tuning for the system in state space form.

Liu et al. [14] have proposed an analytical method of controlling multi-loop system with multiple time delay, based on PI/PID controller. The decoupler control strategy is used for the design of PI/PID controller with multiple time delays [14]. A direct synthesis approach is also used in literature [15-16], for controlling a high dimensional multivariable process in practice. An acceptable response is obtained with the capacity of handling loop interaction with direct synthesis approach. Several authors have presented [16-21] controllers based on the equivalent transfer function approach. Later, the standard form of PI/PID controller is obtained by using the Maclaurin series expansion.

Vu et al., Taiwo et al. and Naik et al. [22-24] have presented IMC based centralized control for multivariable systems. The interaction effect is identified based on the Relative Gain Array (RGA). Based on the RGA analysis, the best pairing of controlled and manipulated variables are formed. Further, the proposed Normalized Relative Gain Array (RNGA) technique showed improved response with good dynamic stability and performance compare to RGA. Chen et al. and Liu et al. [25-26] have discussed IMC based Smith delay compensator for controlling MIMO system with multiple time delay. The steady state gain matrix based decoupler controller is designed initially. Later, by comparing the approximated transfer function with standard PI structure the centralized controller is designed [25]. While Liu et al. [26] have used IMC directly in the Smith predictor structure without finding the standard PI controller. This method can handle problems due to static decoupling and model error effectively.

In this work, researcher develops a robust centralized PI control techniques based on larger delay and smaller time constant of the process transfer function matrix along with the SSGM. The tuning parameters are initially selected between the range 0 to 1. Further, the tuning parameter range is chosen based on the better closed loop response. Thus, the recommended range of the tuning parameters, δ_1 and δ_2, are between 0 to 1 and 1.5 to 3.5 respectively. The tuning parameters identified are also capable of handling uncertainty. The scope of the proposed method is to obtain improved main response and interaction response, compare to the recently presented technique. The servo and regulatory response is obtained. The process output response is also obtained with +30% perturbation in all the three process parameters (time constant, time delay, and process gain). Further, the control technique is analysed with multiplicative uncertainty to check for the sufficient and necessary conditions for robust stability. In the paper, robustness is also analysed with spectral radius criterion. The simulation results and analysis proves the effectiveness of the proposed control technique.

2. Multivariable PI Control System Design

In general, a multivariate process as shown in Fig. 1. Here, '*r*' is the setpoint and '*y*' is the process output, whereas, '*d*' is the load disturbance.

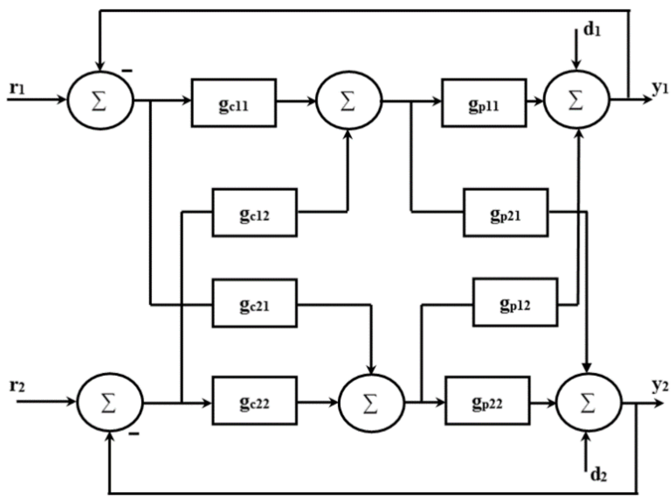


Fig. 1. Centralized control structure for multivariable system.

The '*m*' and '*n*' in Eq. 1 represents the size ($n \times m$) of the transfer function matrix of the plant, which is expressed by

$$G_p(s) = \begin{bmatrix} g_{p11}(s) & g_{p12}(s) & \cdots & g_{p1m}(s) \\ g_{p21}(s) & g_{p22}(s) & \cdots & g_{p2m}(s) \\ \vdots & \vdots & \ddots & \vdots \\ g_{pn1}(s) & g_{pn2}(s) & \cdots & g_{pnm}(s) \end{bmatrix} \tag{1}$$

In the process transfer function, each of the element is represented as a first order plus time delay model, which is represented by g_{ij}, where 'i' is the number of rows and 'j' is the number of columns in the system.

$$g_{ij} = \frac{k_{ij}}{T_{ij}s+1} e^{-\tau_{ij}s} \tag{2}$$

where, k_{ij}, τ_{ij} and T_{ij} are the process gain, time delay, and time constant, respectively.

$G_c(s)$ is the full-dimensional controller matrix with $n \times m$ dimensions, given by

$$G_c(s) = \begin{bmatrix} g_{c11}(s) & g_{c12}(s) & \cdots & g_{c1m}(s) \\ g_{c21}(s) & g_{c22}(s) & \cdots & g_{c2m}(s) \\ \vdots & \vdots & \ddots & \vdots \\ g_{cn1}(s) & g_{cn2}(s) & \cdots & g_{cnm}(s) \end{bmatrix} \tag{3}$$

$G_c(s)$ is calculated using an inverse steady state gain matrix and process parameters, given by

$$G_c(s) = K_C + \frac{K_I}{s} \times \left[G_p(s=0)\right]^{-1} \tag{4}$$

where, $[G_p(s=0)]$ is the steady state gain matrix, the Proportional gain (K_C), and Integral gain (K_I) are given by

$K_C = \delta_1\left(\frac{T}{\tau}\right)$ and $K_I = \frac{K_c}{\tau_I}$; Here, $\tau_I=\delta_2\tau$

The larger delay (i.e., τ=max $\tau_{ij}[G_p(s)]$) and smaller time constant (i.e., T=min $T_{ij}[G_p(s)]$) is considered here for the PI controller design. δ_1 and δ_2 are the tuning parameters with respect to the Proportional gain (K_C) and Integral time control (τ_I) respectively [27].

3.Multivariable System Stability Analysis

The uncertainties are unavoidable in a real process control industries. In process industries, these uncertainties is due to variations in feed flow rate, reboiler duty, or column temperature. The proposed PI control tuning is tested in the presence of various types of uncertainties. Thus, in section 4 designed controller is evaluated by considering the two extensively used process transfer function matrix in the literature. Moreover, robust stability analysis is evaluated with the presence of perturbation in all the three process parameters of the process transfer function. There are many structured or unstructured uncertainties in real time practice. The present study is based on additive, input multiplicative, and output multiplicative uncertainties, which are commonly encountered in process industries. The absolute error between the actual plant and the model is responsible for the additive uncertainties, while the relative error is responsible for the multiplicative uncertainties [28].

Usually, the additive uncertainties added into the process transfer function is described as the perturbation in the model parameters such as time constants, time delays, and steady state gains. It is shown in Fig. 2(a) and expressed as $\sum_A = G_p(s) + \Delta_A(s)$. The process input multiplicative uncertainties shown in Fig. 2(b), can be illuminated as the process actuator uncertainties, which are given by $\prod_I = G_p(s)[I + \Delta_I(s)]$. The process output multiplicative uncertainties, shown in Fig. 2(c), can be illuminated practically as process sensor uncertainties, which is given by $\prod_O = [I + \Delta_O(s)]G_p(s)$. Here, $\Delta_A(s)$, $\Delta_I(s)$ and $\Delta_O(s)$ are assumed to be stable [10-11].

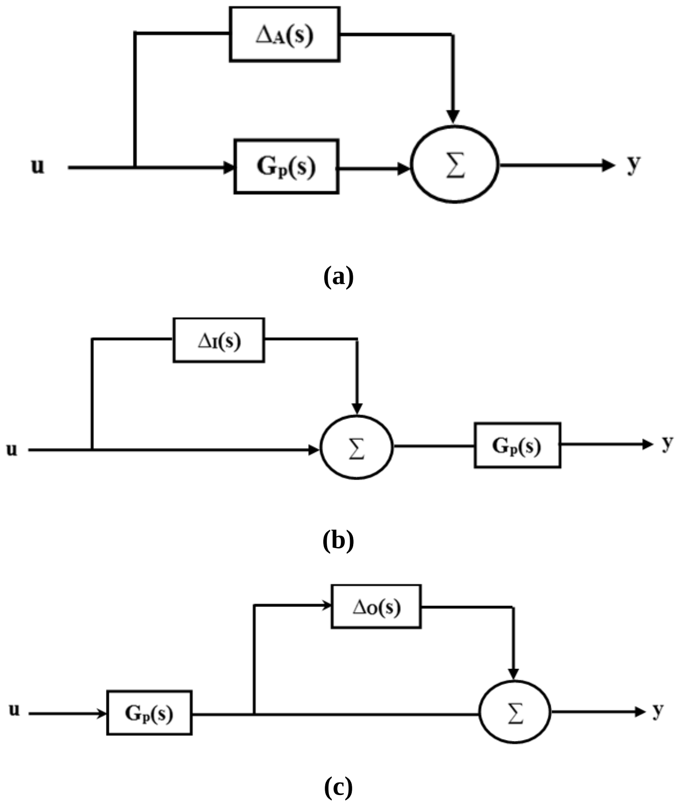


Fig. 2. (a) Process with additive (b) Input multiplicative (c) Output multiplicative uncertainties.

The standard M-Δ structure is considered here for the robustness analysis as shown in Fig. 3. Here, Δ_A, Δ_I, and Δ_O are the transfer function forms of various uncertainties in the process, which can be expressed as

$$M_A = -G_c\left(I + G_p G_c\right)^{-1} \tag{5}$$

$$M_I = -G_c\left(I + G_pG_c\right)^{-1}G_p \tag{6}$$

$$M_O = -G_pG_c\left(I + G_pG_c\right)^{-1} \tag{7}$$

Here, M_A, M_I and M_O are stable for the given control system design. Thus, condition for robust stability constraints is given by small gain theorem [9] as

$$\left\|G_c\left(I + G_pG_c\right)^{-1}\right\|_\infty < \frac{1}{\left\|\Delta_A\right\|_\infty} \tag{8}$$

$$\left\|G_c\left(I + G_pG_c\right)^{-1}G_p\right\|_\infty < \frac{1}{\left\|\Delta_I\right\|_\infty} \tag{9}$$

$$\left\|G_pG_c\left(I + G_pG_c\right)^{-1}\right\|_\infty < \frac{1}{\left\|\Delta_O\right\|_\infty} \tag{10}$$

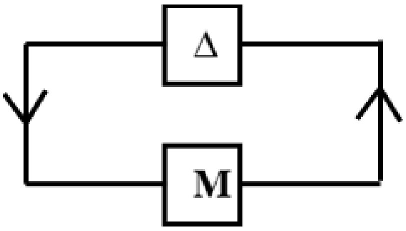


Fig. 3. General M-Δ structure.

For a multivariable system with the above expressions, Eqs. (8-10) are computed with H-infinity norm in a MATLAB environment. In the present study, spectral radius criteria is taken into consideration for the robustness analysis. It states that if *L(s)* is the stable loop transfer function of the given system, the condition for stability $\rho((L(j\omega))$ <1 should be satisfied for all ω. There is no phase information of the process taken into consideration for the study. The spectral radius implicate that the magnitude of the system gain is less than one for all eigen values, and also, for all frequencies; then, gradually the signal will die and the system will be stable. Thus, the spectral radius multivariable process criterion is obtained by $\left\|M\Delta\right\|_\infty < 1$, which implies that

$$\rho\left(M\Delta\right) < 1 \, \forall \, \omega \in [0, \infty] \tag{11}$$

The stability constraints for the robust stability of MIMO system can be rewritten as

$$\rho\left(G_c\left(I + G_pG_c\right)^{-1}\Delta_A\right) < 1 \;\; \forall \, \omega \in [0, \infty] \tag{12}$$

$$\rho\left(G_c\left(I + G_pG_c\right)^{-1}G_p\Delta_I\right) < 1 \, \forall \omega \in [0,\infty] \tag{13}$$

$$\rho\left(G_pG_c\left(I + G_pG_c\right)^{-1}\Delta_O\right) < 1 \, \forall \omega \in [0,\infty] \tag{14}$$

Thus, the robust stability is achieved by checking the spectral magnitude values of Eqs. (12-14), by observing whether it is within unity for all ranges of frequency ω.

4. Simulation Results and Analysis

4.1. Example 1: Wood and Berry Distillation Column

The Wood and Berry Distillation Column [7, 16] is extensively used in literature for the study of different control techniques. The transfer function model with two manipulated and controlled variables Wood and Berry Distillation column is given by

$$G_p(s) = \begin{bmatrix} \dfrac{12.8e^{-s}}{16.7s+1} & \dfrac{-18.9e^{-3s}}{21s+1} \\ \dfrac{6.6e^{-7s}}{10.9s+1} & \dfrac{-19.4e^{-3s}}{14.4s+1} \end{bmatrix} \tag{15}$$

The SSGM based centralized PI controller matrix is given by [7]

$$G_c(s) = \begin{bmatrix} 0.3140 + \dfrac{0.0471}{s} & -0.3058 - \dfrac{0.04587}{s} \\ 0.1068 + \dfrac{0.01602}{s} & -0.2072 - \dfrac{0.03108}{s} \end{bmatrix} \tag{16}$$

Using the Davison method, the inverse steady state gain matrix for Eq. (15) is obtained as

$$G_p(s=0)^{-1} = \begin{bmatrix} 0.1570 & -0.1529 \\ 0.0534 & -0.1036 \end{bmatrix} \tag{17}$$

For the proposed work, the PI controllers setting is based on larger time delay and smaller time constant of the transfer function, in the model. Accordingly, τ =7 min and T=10.9 min. The best main effect and interaction effect can be achieved with tuning parameters δ_1=0.6 and δ_2=2 for the proposed method. Thus,

$$K_C = \delta_1\left(\frac{T}{\tau}\right) = 0.6\times\left(\frac{10.9}{7}\right) = 0.9342$$

Further,

$$K_I = \frac{K_c}{\tau_I} \qquad \text{where } \tau_I=\delta_2\tau.$$

Considering, δ_2=2, τ_I=14

$$K_I = \frac{K_c}{\tau_I} = \frac{0.9342}{14} = 0.0667$$

Thus, from Eq. (4)

$$G_c(s) = 0.0667 + \frac{0.0667}{s}\times\left[G_p(s=0)\right]^{-1} \tag{18}$$

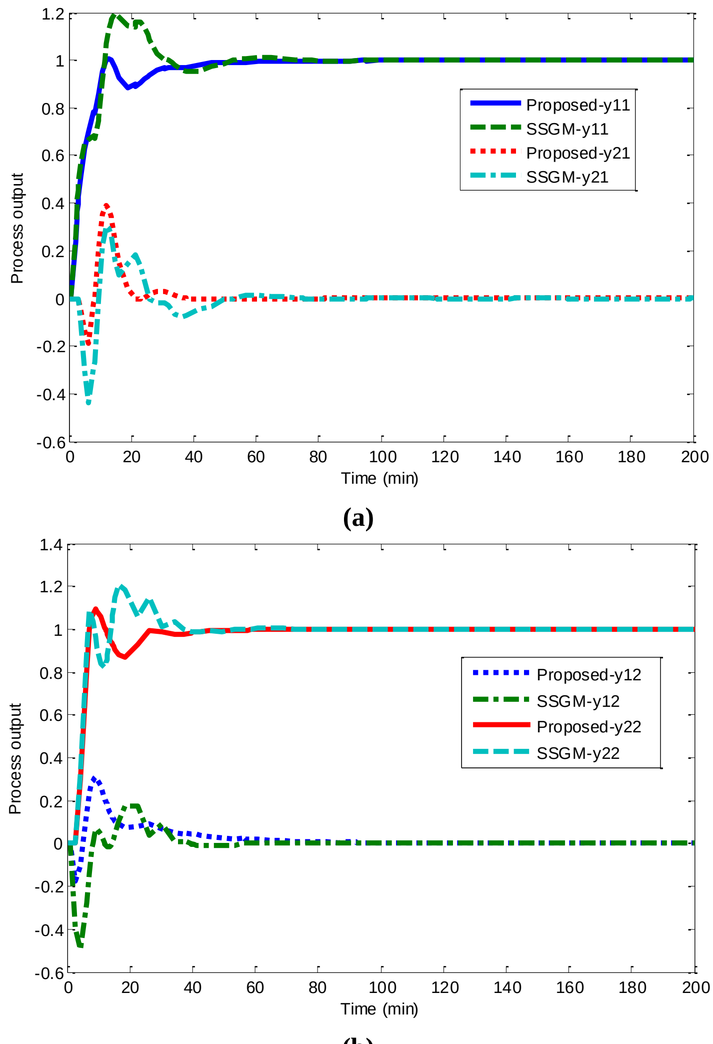
Since, there exist open loop diagonal dominance shown by RGA, the main effect is multiplied by its corresponding element of RGA value (λ_{11}=λ_{22}=2).

$$G_c(s) = 0.9342 + \frac{0.0667}{s}\begin{bmatrix} 0.3140 & -0.1529 \\ 0.0534 & -0.2076 \end{bmatrix}$$

Thus, the centralized PI controller is given by

$$G_c(s)=\begin{bmatrix}0.2933+\dfrac{0.02}{s} & -0.1428-\dfrac{0.01}{s}\\ 0.05+\dfrac{0.0036}{s} & -0.1936-\dfrac{0.0138}{s}\end{bmatrix} \tag{19}$$

Initially, the servo response is evaluated by unit step change in r_1 and r_2. The unit step change in r_1 is judged and the main response y_1 and interaction response y_2 are as shown in Fig. 4(a). The unit step change in r_2 is judged and the main response y_2 and interaction response y_1 are as shown in Fig. 4(b). Figure 4 compares the proposed method with the SSGM based centralized PI control method presented by Dhanyaram and Chidambaram [7]. The response shown in [7] is better compared to the recently proposed control algorithm by Kumar et al. [16] based on synthesis method.



(a)

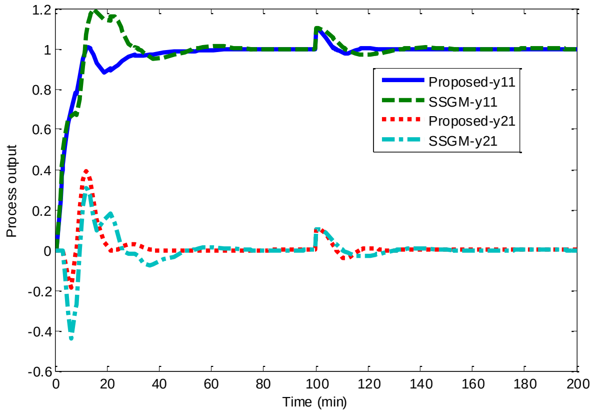
(b)

Fig. 4. Servo response comparison of example 1 with setpoint (a) r_1=1 and r_2=0 (b) r_1=0 and r_2=1.

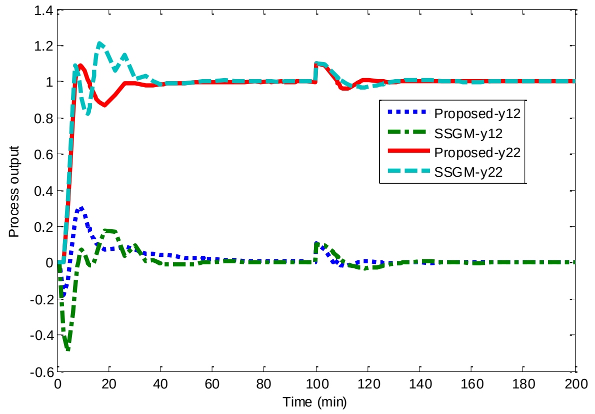
Both main response and interaction response significantly improved illustrated with Integral Absolute Error (IAE) values as shown in Table 1. The IAE values tabulated indicates that the controller is performing significantly effective in the presence of load disturbance and uncertainty in all the process parameters. Figure 5 shows the regulatory response with load disturbance rejection. Improved regulatory response can be achieved. Table 1 shows the sum of the IAE values for the main action and the interactions for the regulatory response. The interaction value is reduced with the same tuning parameters for the regulatory problem.

Table 1. IAE values for example 1.

Response	**Method**	y_{11}	y_{21}	y_{12}	y_{22}	**Main Action**	**Inter-action**	**Over-all**
Servo	Proposed	6.8	3.37	5.35	6.91	13.71	8.72	**22.43**
	SSGM [7]	8.16	5.42	4.49	7.62	15.78	9.91	**25.69**
Regulatory	Proposed	7.43	4.24	5.89	7.16	14.59	10.13	**24.72**
	SSGM [7]	9.36	6.57	5.67	8.76	18.12	12.24	**30.36**
Uncertainty (+30% in all process parameters)	Proposed	7.38	6.75	5.44	11.37	18.75	12.19	**30.94**
	SSGM [7]	**11.47**	**10.24**	**8.33**	**13.09**	**24.56**	**18.57**	**43.13**



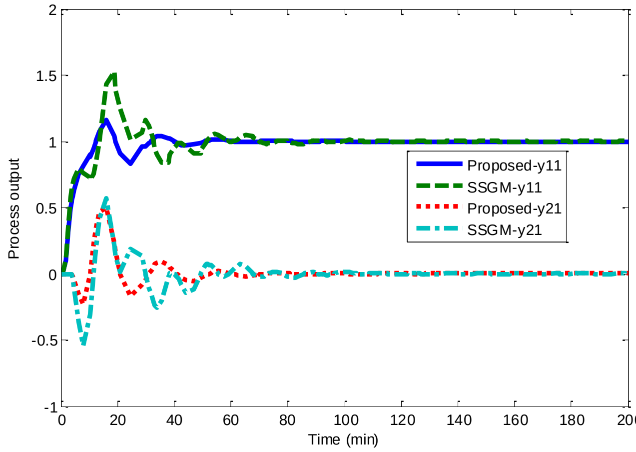
(a)



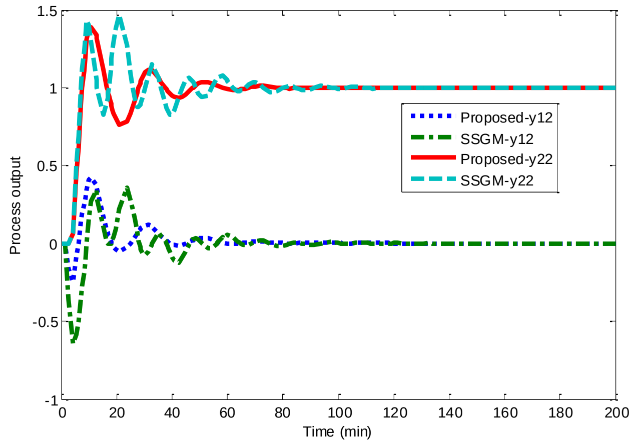
(b)

Fig. 5. Regulatory response comparison of example 1 with setpoint (a) r_1=1 and r_2=0 (b) r_1=0 and r_2=1.

The presented control technique is evaluated for +30% in all the process parameters. For the same tuning parameter values (δ_1=0.6 and δ_2=2) good performance is achieved as shown in Fig. 6.



(a)



(b)

Fig. 6. Robustness comparison with +30% uncertainty in all the process parameters for example 1 with setpoint (a) r_1=1 and r_2=0 (b) r_1=0 and r_2=1.

The IAE values are tabulated in Table 1. Also, the robustness is tested with multiplicative uncertainty. To show practical exhibition for the robust stability assume there exist a process additive multiplicative uncertainties Δ_A with 40% increase in process gain of each transfer function of the process transfer function matrix. Similarly, it can be demonstrated with 50% increase in the time constant of each element of the process transfer function matrix. Also, by assuming a process input multiplicative uncertainties the robustness against multiplicative uncertainties is tested.

$$\Delta_I = diag[(s+0.2)/(2s+1),(s+0.3)/(2s+1)]_{2\times 2} \qquad (20)$$

This is obtained by considering 50% uncertainty in the process first process input actuator in the high frequency region and 20% uncertainty in the lower frequency region. And, 50% uncertainty for the second process input in the high frequency region and 30% uncertainty in the lower frequency region as shown in Eq. (20). Similarly, the output multiplicative uncertainties can also be evaluated against robustness analysis, which is given by

$$\Delta_O = diag[(s+0.5)/(s+1),(s+0.1)/(2s+1)]_{2\times 2} \tag{21}$$

This is interpreted as 100% uncertainty in high frequency range and 50% uncertainty in lower frequency range for the first input actuator. And, 50% uncertainty in high frequency range and 10% uncertainty in lower frequency range for the second input actuator as represented in Eq. (21). Figure 7 shows the magnitude Spectral Radius (SR) plot, which represents the robust stability of the proposed control strategy. The peak value indicates the robust stability, i.e. smaller the peak value (<1) enhanced robust stability. Thus, it indicates that the proposed centralized PI controller preserves the robust stability well.

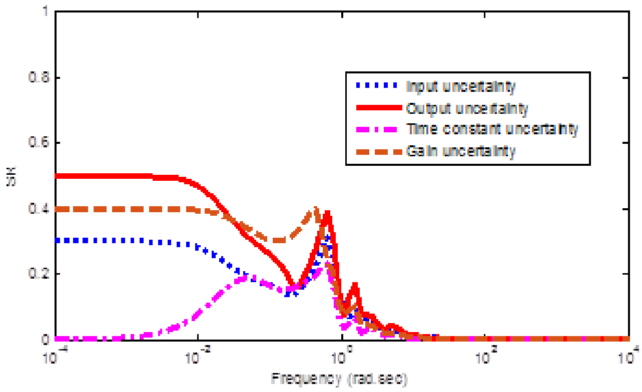


Fig. 7. Spectral radius magnitude plot for example 1.

The literature presents the various heuristic approaches to control multivariable system [30-32]. The optimal PI/PID controller values of various heuristics control algorithms for example 1 (Wood and Berry) model is tabulated in Table 2.

Table 2. Optimal PI/PID controller values of various heuristic control algorithms for example 1.

Control Algorithms	K_{C11}	K_{I11}	K_{D11}	K_{C22}	K_{I22}	K_{D22}
Bat Algorithm (BA) [31]	0.7882	0.0439	-	-0.0262	-0.0124	-
Firefly Algorithm (FA) [31]	0.9256	0.0283	-	-0.0385	-0.0133	-
Particle Swarm Optimization (PSO) [31]	0.8716	0.0262	-	-0.0185	-0.0105	-
Bacterial Foraging Optimization (BFO) [32]	0.6418	0.0528	0.0471	-0.0584	-0.0117	-0.0594

Table 3 illustrates the comparison of servo response Integral Absolute Error (IAE) and percentage overshoot (%OS) values of proposed PI control algorithm along with various heuristics control algorithms such as BA, FA, PSO and BFO (CF1) [31-32]. The time domain specification %OS is also determined and compared with proposed PI control algorithm. The $\%OS_1$ is determined for setpoint r_1=1 and r_2=0, $\%OS_2$ is determined for setpoint r_1=0 and r_2=1. It is observed that proposed PI control algorithm results with minimum IAE values compared to other heuristics control algorithms.

Table 3. Comparison of IAE and percentage overshoot for example 1 servo response.

Method	IAE	$\%OS_1$	$\%OS_2$
Proposed	22.41	0.6	9.1
BA	27.93	11.34	8.92
FA	29.85	16.96	7.42
PSO	35.12	13.01	9.47
BFO(CF1)	23.15	5.37	2.43

4.2. Example 2: Industrial-scale polymerization reactor

The second example considered for the study is the Industrial-Scale Polymerization (ISP) reactor. The transfer function matrix is given by [7, 21]

$$G_p(s) = \begin{bmatrix} \frac{22.89e^{-0.2s}}{4.572s+1} & \frac{-11.64e^{-0.4s}}{1.807s+1} \\ \frac{4.689e^{-0.2s}}{2.174s+1} & \frac{5.8e^{-0.4s}}{1.801s+1} \end{bmatrix} \tag{22}$$

The SSGM based centralized PI controller matrix is given by [7]

$$G_c(s) = \begin{bmatrix} 0.155+\frac{0.0465}{s} & 0.3105+\frac{0.09315}{s} \\ -0.125-\frac{0.0375}{s} & 0.611+\frac{0.1833}{s} \end{bmatrix} \tag{23}$$

Using the Davison method, the inverse steady state gain matrix for Eq. (19) is obtained as

$$G_p(s=0)^{-1} = \begin{bmatrix} 0.0310 & 0.0621 \\ -0.0250 & 0.1222 \end{bmatrix} \tag{24}$$

For the proposed work, larger time delay and smaller time constant of the transfer function is considered for the PI settings in the model. Accordingly, τ=0.4 hr and T=1.801 hr. Thus, the corresponding tuning parameters are δ_1=0.9 and δ_2=3.3 for good control performance. Thus,

$$K_C = \delta_1\left(\frac{T}{\tau}\right) = 0.9\times\left(\frac{1.801}{0.4}\right) = 4.05$$

Further, $K_I = \frac{K_c}{\tau_I}$; where $\tau_I=\delta_2\tau$. Considering, δ_2=3.3, τ_I=1.32

$$K_I = \frac{K_c}{\tau_I} = \frac{4.05}{1.32} = 3.068$$

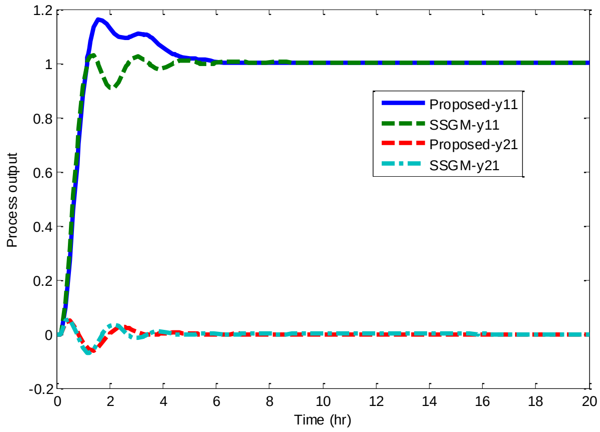
Thus, from Eq. (4)

$$G_c(s) = 4.05 + \frac{3.068}{s} \times \left[G_p(s=0)\right]^{-1} \tag{25}$$

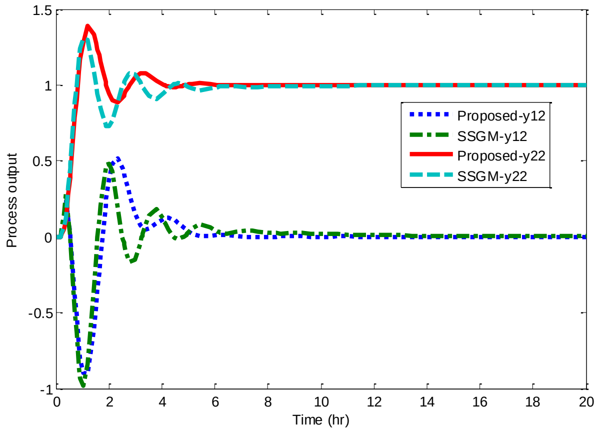
Substituting these in Eq. (4), the centralized PI control system is given by

$$G_c(s) = \begin{bmatrix} 0.12555 + \frac{0.0951}{s} & 0.2515 + \frac{0.19052}{s} \\ -0.10125 - \frac{0.0767}{s} & 0.49491 + \frac{0.3749}{s} \end{bmatrix} \tag{26}$$

Evaluated the servo response of the present method along with that of SSGM method [7] as shown in Fig. 8. The main action and interaction values are tabulated in Table 4 for both the methods. Similarly, the regulatory response for load disturbance rejection for both the methods is shown in Fig. 9. The sum of the interaction effect is reduced for both servo and regulatory response. The robustness with +30% parameter variations in all the process parameters is analysed and the IAE values are tabulated in Table 4. Both the main effect and the interaction effect are significantly improved with the present method.

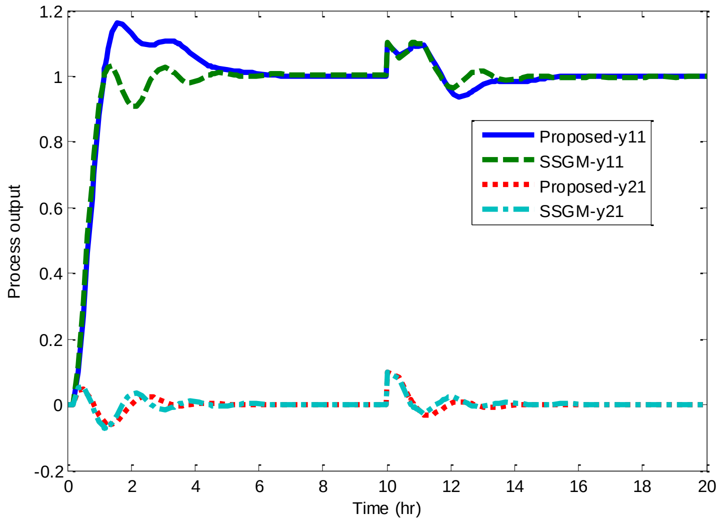


(a)

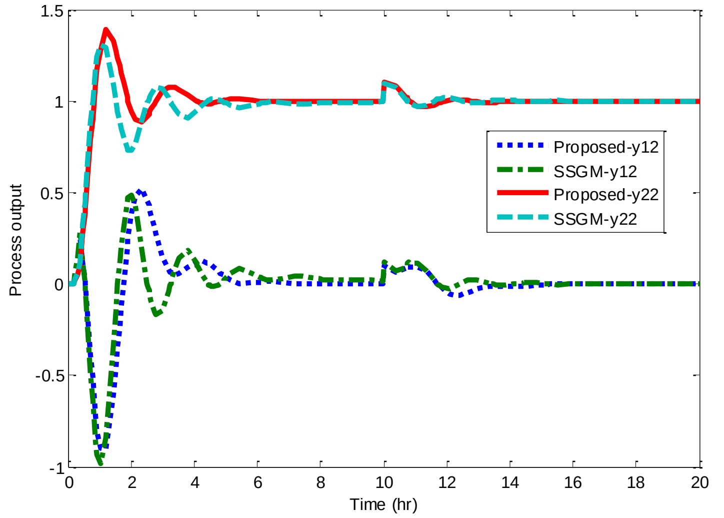


(b)

Fig. 8. Servo response comparison of example 2 with setpoint (a) r_1=1 and r_2=0 (b) r_1=0 and r_2=1.



(a)



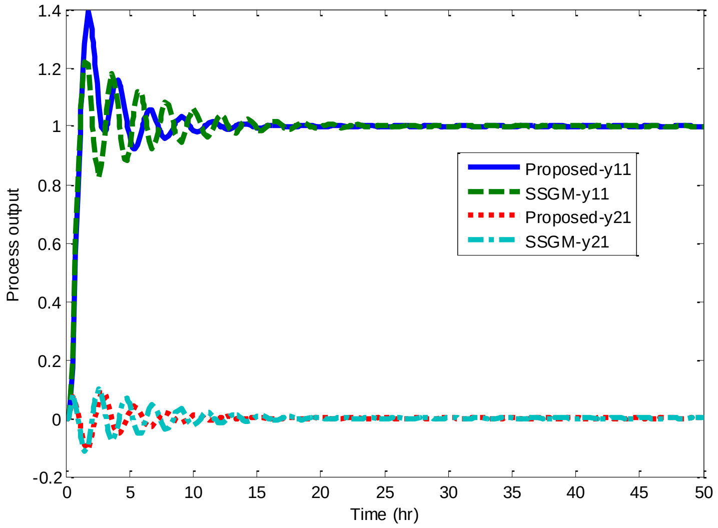
(b)

Fig. 9. Regulatory response comparison of example 2 with setpoint (a) r_1=1 and r_2=0 (b) r_1=0 and r_2=1.

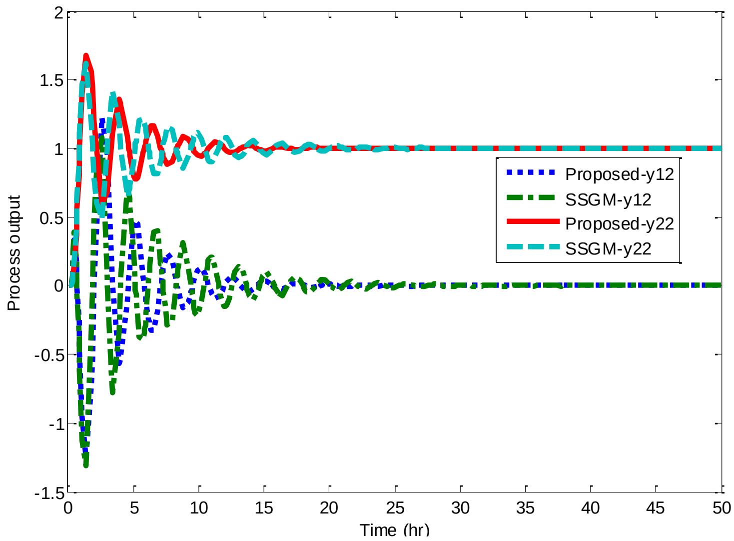
Table 4. IAE values for example 2.

Response	Method	y_{11}	y_{21}	y_{12}	y_{22}	Main Action	Inter-action	Over-all
Servo	Proposed	1.04	0.09	1.39	0.98	2.02	1.48	**3.5**
	SSGM [7]	0.79	0.11	1.42	1.05	1.84	1.53	**3.37**
Regulatory	Proposed	1.26	0.18	1.60	1.07	2.33	1.78	**4.11**
	SSGM [7]	0.95	0.19	1.54	1.12	2.07	1.73	**3.8**
Uncertainty (+30% in all process parameters)	Proposed	1.52	0.35	4.16	2.60	4.12	4.51	**8.63**
	SSGM [7]	**1.65**	**0.51**	**5.57**	**3.29**	**4.94**	**6.08**	**11.02**

Figure 10 shows that for the same tuning parameters value (δ_1=0.9 and δ_2=3.3) robustness is achieved significantly satisfied. Following the steps as explained in example 1, the multiplicative uncertainty is verified with the proposed method. To demonstrate additive uncertainties, 40% increase in process gain is considered initially. Further, 30% increase in time constant is considered for the illustration of additive uncertainties. Also, the input and output multiplicative uncertainties as represented in Eq. (27) and (28) is assumed, to verify the robust stability criterion.



(a)



(b)

Fig. 10. Robustness comparison with +30% uncertainty in all the process parameters for example 2 with setpoint (a) r_1=1 and r_2=0 (b) r_1=0 and r_2=1.

The spectral radius magnitude plot is as shown in Fig. 11. The consolidated time domain specifications such as settling time (t_s) and percentage overshoot (OS) for both the example are tabulated in Table 5.

$$\Delta_I = diag[(s+0.2)/(2s+1),(s+0.1)/(2s+1)]_{2\times2} \tag{27}$$

$$\Delta_O = diag[(s+0.5)/(2s+1),(s+0.1)/(1.33s+1)]_{2\times2} \tag{28}$$

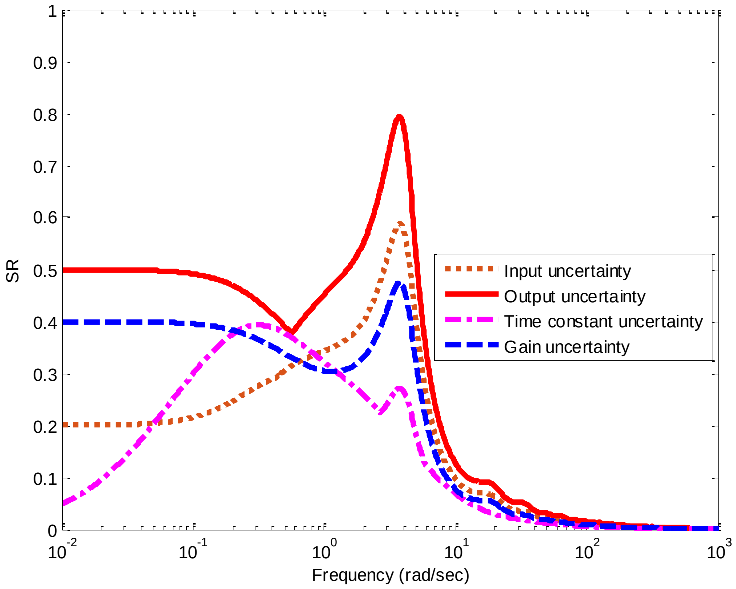


Fig. 11. Spectral radius magnitude plot for example 2.

Table 5. Settling time and percentage overshoot of example 1 and 2.

Response	Setpoint	Method	Example 1: WB		Example 2: ISP	
			t_s(min)	%OS	t_s(hr)	%OS
Servo	r_1=1; r_2=0	Proposed	72.6	0.6	7.87	16.2
		SSGM [7]	77.88	19.8	9.5	3.2
	r_1=0; r_2=1	Proposed	60.76	9.1	5.82	38.8
		SSGM [7]	72.76	20.9	8.5	30.6
Regulatory	r_1=1; r_2=0	Proposed	118.1	0.6	16.1	16.2
		SSGM [7]	153	19.8	14.64	3.2
	r_1=0; r_2=1	Proposed	127.4	9.1	14.13	38.8
		SSGM [7]	132.8	20.9	13.34	30.6
Uncertainty (+30% in all process parameters)	r_1=1; r_2=0	Proposed	63.27	16.3	18.03	39.2
		SSGM [7]	106.7	52.8	24.43	22.1
	r_1=0; r_2=1	Proposed	86.78	38.6	17.33	67.7
		SSGM [7]	114.5	43.1	26.7	62.1

Further, the experimental setup of distillation column is fabricated. The experiment is conducted on a pilot scale binary distillation column and the setup for the distillation column is as shown in Fig. 12.

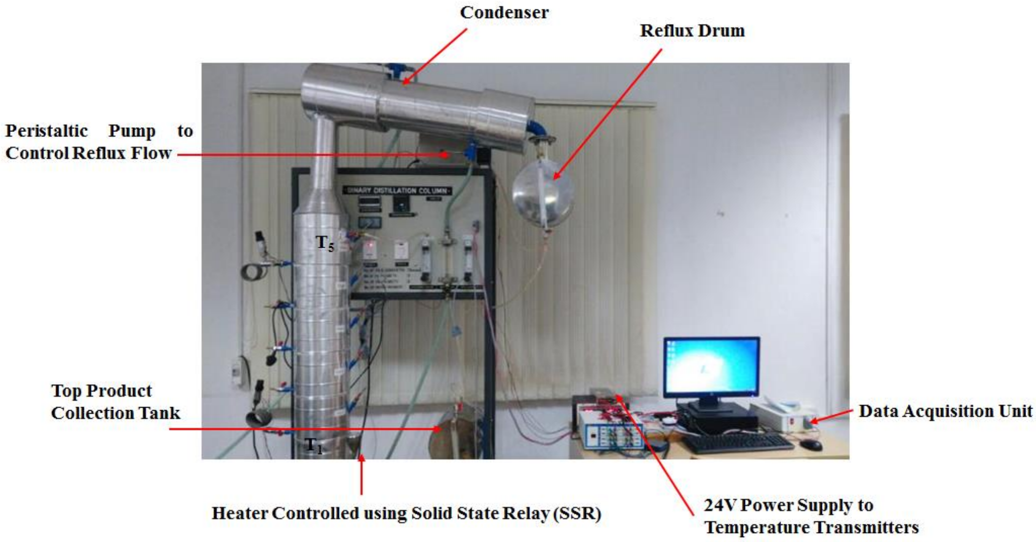


Fig. 12. Experimental setup of pilot plant distillation column.

The open loop test is performed on the distillation column for model identification [33]. The step change is applied to the manipulated variables, reflux (L), and the reboiler (Q). Later responses of tray temperatures T_5 and T_1 are recorded. Then, the empirical FOPTD model is identified by Vinaya and Arasu, as shown in Eq. (29).

$$\begin{bmatrix} T_5 \\ T_1 \end{bmatrix} = \begin{bmatrix} \frac{-0.13e^{-0.03s}}{1.14s+1} & \frac{0.18e^{-0.03s}}{0.64s+1} \\ \frac{-0.34e^{-1.22s}}{1.23s+1} & \frac{0.18e^{-0.03s}}{0.32s+1} \end{bmatrix} \begin{bmatrix} L \\ Q \end{bmatrix} \tag{29}$$

Here, the process gain measured in *ºC/%*, the time constant and the time delay both are measured in hours [34].

The controller for the Eq. (29) is designed based on proposed method is given by

$$G_c(s) = \begin{bmatrix} 1.8733 + \frac{1.1809}{s} & -1.8733 - \frac{1.1809}{s} \\ 3.5385 + \frac{2.231}{s} & -1.3529 - \frac{0.8529}{s} \end{bmatrix} \tag{30}$$

The proposed control algorithm is implemented on a pilot plant distillation column and its response is as shown in Fig. 13.

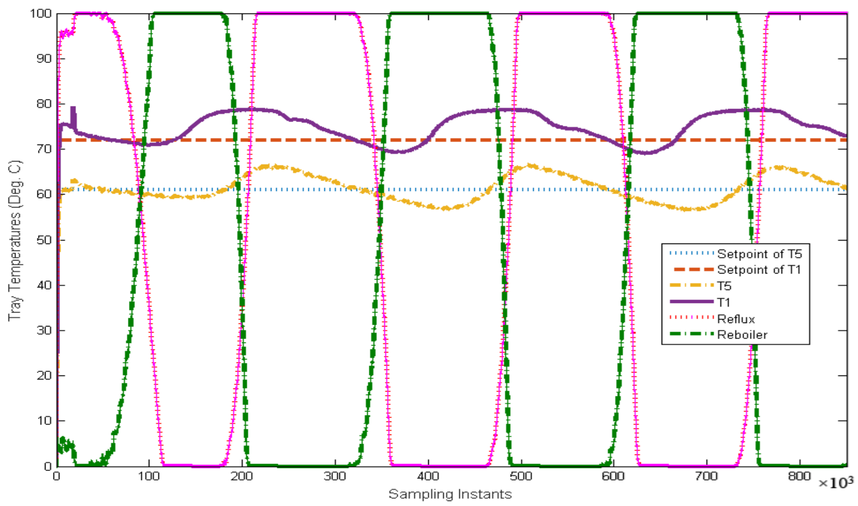


Fig. 13. Real-time implementation of proposed control algorithm on a pilot plant binary distillation column with the setpoint of T_1=72 deg. C. and T_5=61 deg. C.

5. Conclusions

The proposed centralized robust PI controller, based on steady state gain matrix along with time constant and time delay of the process transfer function. The effectiveness of the presented technique is evaluated with the IAE values for both regulatory and servo response. Better interaction response and main response is achieved with the proposed controller. The controller is illustrated with two simulated examples. For the same tuning parameters, δ_1 and δ_2 robustness is evaluated. The robustness stability studied with +30% perturbations in all the process parameters. The performance is significantly improved with plant model mismatch. The multiplicative uncertainty in process plant transfer function is also satisfying the spectral radius criterion. Further, the proposed control algorithm is validated using pilot plant distillation column. A possible direction for future work is to design and validation of control algorithm such as centralized, decentralized, dynamic matrix control, etc. in real-time. This topic is currently under investigation by the authors.

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