

MEASUREMENT OF THE ABSOLUTE EFFICIENCY OF A SEMICONDUCTOR DETECTOR USING AN ISOTROPIC SPHERICAL SOURCE

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Abstract

The calibration of hyper pure germanium semiconductor detectors is required to calculate the activity of radioactive sources. To find out the activity, the detector full-energy peak efficiency is required. The full-energy peak efficiency of HPGe semiconductor detector for an isotropic radiating spherical source is obtained experimentally. We compared the measured efficiency values with theoretical calculations. Furthermore, the variation of the window materials of the detector on the full-energy peak efficiency have been studied. By comparison, the theoretical and experimental full-energy peak efficiency values are in good agreement. The difference percentage between the measured efficiency values and the calculated ones is less than 4%.

Keywords: Closed-end HPGe detector, Gamma-ray spectrometry, Full-energy peak efficiency, Spherical sources.

1. Introduction

The efficiency of germanium gamma-ray detectors has been treated by several authors [1-24]. In the present work, the author will illustrate how to calibrate HPGe detectors used to determine the source activity for radioactive gas spheres. The experimental efficiencies are compared to the calculated one using an analytical method based on the direct mathematical method reported by Abbas [12-20]. The effect of the source self-absorption on the discrepancies of the calculated efficiencies is also introduced. Furthermore, the variation of the side wall and the window materials of the detector on the photopeak efficiency have been studied.

Nomenclatures

A_S	Activity
$N(E)$	Net peak area in counts
$P(E)$	Emission probability

Greek Symbols

λ	Decay constant
ρ	Lateral displacement
σ_ε	Uncertainty
Ω	Solid angle

Abbreviations

FEPE	Full-energy peak efficiency
HPGe	Hyper pure germanium

2. Experimental Setup

The full-energy-peak efficiencies were determined for an n-type GMX Ortec HPGe detectors. The detector model 10180-S with relative efficiency 9% and of 51 cm³ active volume, as shown in Fig. 1, at one source-to-detector distance using an isotropic radiating spherical source, as shown in Fig. 2. The type of source is a 32 cm³ Pyrex sphere with 3.9 cm inner diameter and a wall thickness of 0.2 cm contains ⁸⁵Kr and ¹³³Xe Noble gases with activities $(4.725 \pm 0.059) \times 10^7$ Bq and $(1.969 \pm 0.016) \times 10^7$ Bq, respectively. ⁸⁵Kr is an inert gas and a beta emitter with a half-life of 10.76 years, while ¹³³Xe is also inert and emits both beta and gamma radiation, with a half-life of 5.248 days. The gamma-ray energies and emission probabilities are those listed in the Evaluation Nuclear Structure Data File (ENSDF) tables available in the National Nuclear Data Centre Web Page or in the IAEA website, see Table 1. The source-to-detector distance is 40.5 cm as measured from the centre of the sphere to the detector upper face. The measured full-energy peak efficiency $\varepsilon(E)$ at energy E , can be determined by using the following equation [21]:

$$\varepsilon(E) = \frac{N(E)}{T \cdot A_S \cdot P(E) \cdot e^{-\lambda t}} \quad (1)$$

where $N(E)$ is the net peak area in counts, A_S is the activity of source at the time of standardization, $P(E)$ is the absolute γ -ray emission probability, t is the elapsed time since standardization, λ is the decay constant and T is the measuring time (in seconds). The uncertainty of the experimental full energy peak efficiency, σ_ε , is given by [25]:

$$\sigma_\varepsilon = \sqrt{\left(\frac{\partial \varepsilon}{\partial A}\right)^2 \cdot \sigma_A^2 + \left(\frac{\partial \varepsilon}{\partial P}\right)^2 \cdot \sigma_P^2 + \left(\frac{\partial \varepsilon}{\partial N}\right)^2 \cdot \sigma_N^2} \quad (2)$$

where σ_A , σ_P and σ_N are the uncertainties associated with the quantities A_S , $P(E)$ and $N(E)$ respectively. The uncertainty in Eq. (2) is combined with the statistical component from measurement repetitions of the same source, and with the additional systematic component for the positioning of the gas sphere on the

source holder. For the two radionuclides of interest (^{85}Kr and ^{133}Xe), the main contributions to the total uncertainty in the activity measurements are the uncertainties in the emission probability and in the efficiency value.

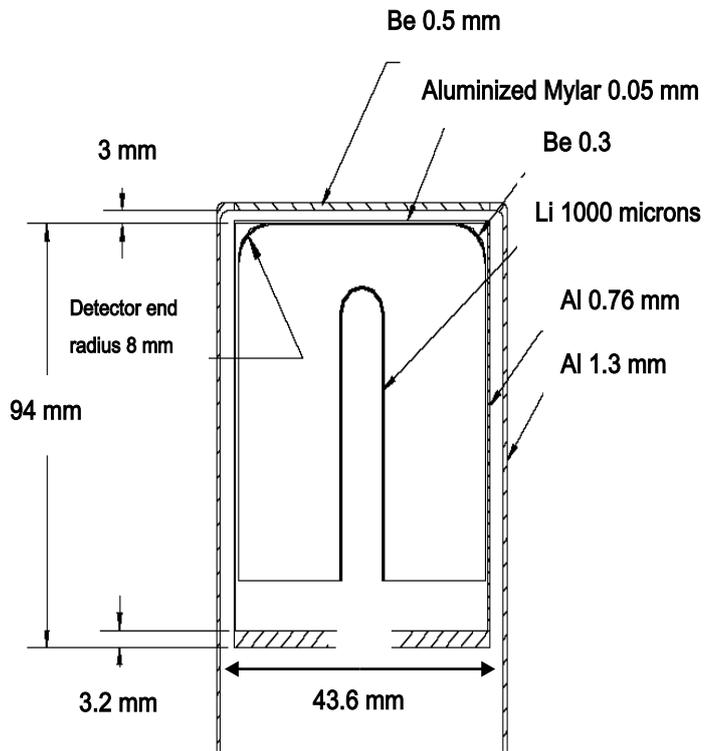


Fig. 1. Schematic diagram of the HPGe detector.



Fig. 2. Schematic diagram of the spherical source.

Table 1. ^{85}Kr and ^{133}Xe Noble gases gamma-ray energies.

Radionuclide	Energy (keV)	Branching Ratio %
^{85}Kr	151.18	0.000023
	362.81	0.0000225
	514.0067	0.434
^{133}Xe	79.6142	0.78
	80.9971	38
	160.613	0.066
	223.2368	0.00019
	301.853	0.0048
	383.851	0.0024
	514.0067	0.434

3. Mathematical viewpoint

As a part of this work, we compared the measured efficiency curves with theoretical calculations using a new simplified method based on the direct mathematical method reported by Abbas [12-20]. As a part of this work, we compared the measured efficiency curves with theoretical calculations using a new simplified method based on the direct mathematical method reported by Abbas [12-20].

The efficiency, ε_{point} , of a right circular cylindrical ($2R \times L$) detector for an arbitrarily positioned radiating point source (defined by (ρ, h) where, ρ is the lateral displacement between the source and the detector axis, whereas h is the source-to-detector distance) is defined as:

$$\varepsilon_{point} = \varepsilon_g \times \varepsilon_i \quad (3)$$

where ε_g is the geometrical efficiency ($\varepsilon_g = \Omega/4\pi$; Ω is the solid angle subtended by the detector at the source point and represented by equation (4)) while ε_i is the intrinsic efficiency and is given by equation (5).

$$\Omega = \int \int_{\theta \phi} \sin \theta \, d\phi \, d\theta \quad (4)$$

$$\varepsilon_i = f_{att} (1 - e^{-\mu \cdot d}) \quad (5)$$

where θ and ϕ are the polar and the azimuthal angles, respectively, and are declared in Fig. 3. The factor f_{att} determining the photon attenuation by the source container and the detector end cap materials, and is expressed as:

$$f_{att} = e^{-\sum_i \mu_i \cdot \delta_i} \quad (6)$$

where, μ_i is the attenuation coefficient of the i^{th} absorber for a gamma-ray photon with energy E_γ [26] and δ_i is the gamma photon path length through the i^{th} absorber. This factor f_{att} is applicable to the full-energy peak efficiency, since any early interaction with any absorbing medium will remove the count from the full-

energy peak, but not form the probability the total efficiency (where Compton scattered photons still contributes).

Finally, d is the path length travelled by a photon through the detector active volume for an isotropic emission. Calculating the total efficiency can be obtained by replacing μ with the total attenuation coefficient of the detector's material for gamma-ray energy E_γ , excluding the coherent scattering part. Whereas, for the full-energy peak efficiency computation, μ is replaced by the peak attenuation coefficient for the detector's material μ_p , which represents the only part contributing to the full-energy peak (photoelectric coefficient + the fractions of the incoherent and pair production coefficients leading to the full-energy peak).

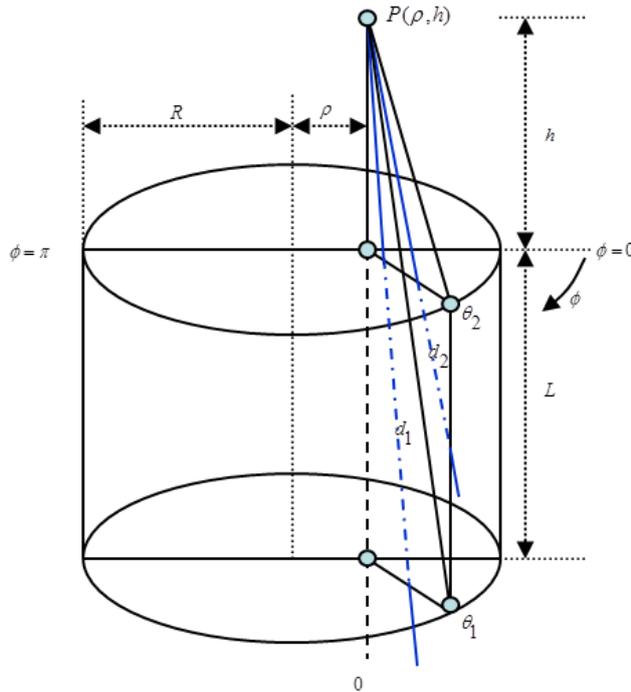


Fig. 3. The possible cases of the photon path lengths through the detector active volume.

In the case of a located point source at a lateral distance less than the radius, i.e., $\rho < R$; the probability of striking the point where the photon actually enters the detector active volume must be known to calculate the detection efficiencies. To calculate the photon path lengths travelled through the detector active volume and consequently the detection efficiencies, there are two main cases to be considered (see Fig. 3):

$$d_1 = \frac{L}{\cos \theta} \tag{7}$$

$$d_2 = \frac{M(\phi)}{\sin \theta} - \frac{h}{\cos \theta} \tag{8}$$

$$M(\phi) = -\rho \cos \phi + \sqrt{R^2 - \rho^2 \sin^2 \phi} \quad (9)$$

Finally, combining Eqs. (3) to (9) results in a direct mathematical expression for the absolute efficiency ε_{point} of a cylindrical detector for an arbitrarily positioned radiating point source which is derived as:

$$\varepsilon_{point} = \frac{1}{2\pi} \int_0^\pi \left(\int_0^{\theta_1} f_1 d\theta + \int_{\theta_1}^{\theta_2} f_2 d\theta \right) d\phi \quad (10)$$

where,

$$\theta_1 = \tan^{-1} \left(\frac{M(\phi)}{h+L} \right) \quad (11)$$

$$\theta_2 = \tan^{-1} \left(\frac{M(\phi)}{h} \right) \quad (12)$$

$$f_i = f_{att} \cdot (1 - e^{-\mu_s d_i}) \sin \theta \quad (13)$$

The gas sphere can be considered as a volumetric source (with volume V) that consists of a group of point sources that are uniformly distributed; each point has an efficiency ε_{point} , consequently the volumetric source efficiency is given by:

$$\varepsilon_{volume} = \frac{1}{V} \int_V \varepsilon_{point} dV \quad (14)$$

where, V is the volume of the source. But, in the volumetric source, not all the emitted photons from the source nuclei exit the source volume, but part of them is absorbed in the source, affecting the efficiency calculations. The factor concerning this effect is called the self-absorption factor f_{self} , which is given by:

$$f_{self} = e^{-\mu_s d_s} \quad (15)$$

where μ_s is the source attenuation coefficient and d_s is the distance travelled inside the source material.

4. Results

The absolute full-energy peak efficiency values have been calculated for a closed end HPGe-detector by using the direct mathematical method and compared with our experimental measurements for different window thicknesses 0.5, 1 and 1.5 mm, as shown in Figs. 4-6, respectively. As seen from these figures, increasing the window thickness is nearly insignificant in the energy range higher than 110 keV which is the inversion point of the efficiency-energy curve, whereas, it decreases the efficiency values slightly in the range less than the inversion point. The difference percentage $\Delta\%$ between the calculated values and the experimentally measured ones is given by:

$$\Delta\% = \frac{\varepsilon_{theo} - \varepsilon_{expt}}{\varepsilon_{expt}} \times 100\% \quad (16)$$

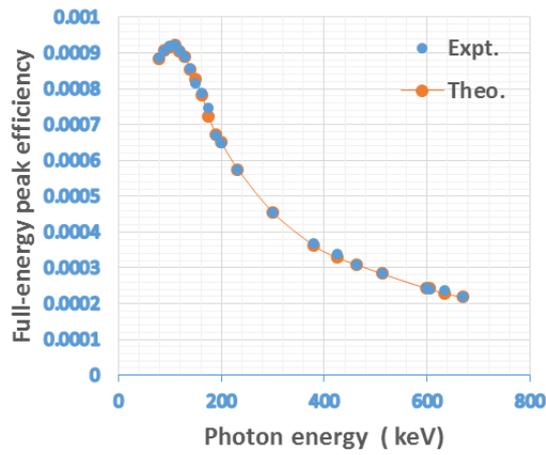


Fig. 4. Efficiency curves for the detector at window Be thickness = 1.5 mm.

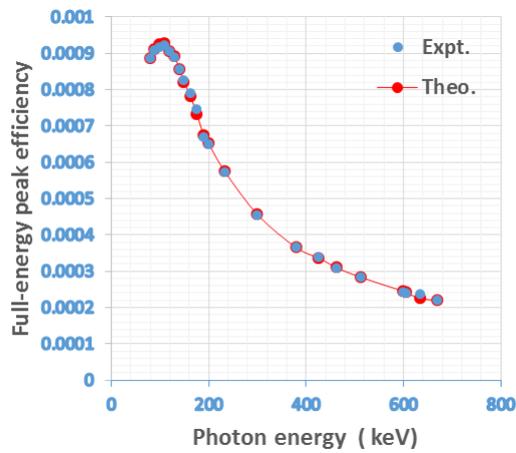


Fig. 5. Efficiency curves for the detector at window Be thickness = 1.0 mm.

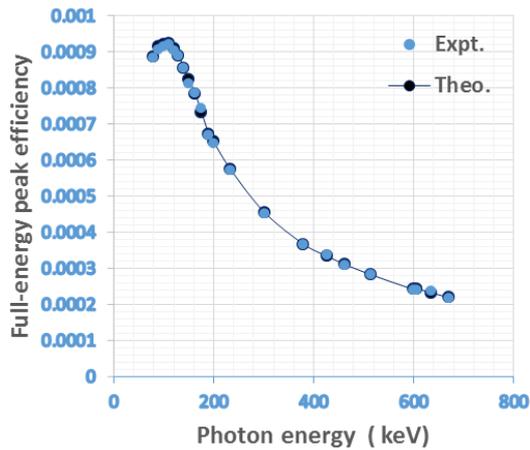


Fig. 6. Efficiency curves for the detector at window Be thickness = 0.5 mm.

5. Conclusions

The absolute full-energy peak efficiency for a closed end HPGe detector in the case of a gas sphere source filled with noble gases ^{85}Kr and ^{133}Xe have been measured. In addition, the attenuation of the photons by the source container, the window and the end cap materials are also presented in a simple straightforward mathematical expression. In the comparisons of the efficiency values in Figs. 4 to 6 common features can be seen. Firstly, the variation of the window thickness has no significance for the efficiency values in the gamma ray energy higher than the inversion point of the efficiency-energy curve. Secondly, the percentage errors between the measured efficiency values and the calculated ones is less than 4%. This means that the direct mathematical approach (Selim and Abbas Method) is efficient and sufficiently powerful compared to the other methods to allow its use in the evaluation of the total and the full-energy peak efficiencies. It can be applied to build a calibration curve for cylindrical detectors for these sources without any need to optimize the detector parameters. Contrary to Monte Carlo simulations which generally require long computing times, from some tens of minutes to several hours (depending on the CPU used) as mentioned by Lépy et al. [27], the present computer program needs only short computing times (typical running time for each point is about one minute).

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