# MODELLING AND FAILURE ANALYSIS OF FLEXURE SPRINGS FOR A STIRLING CRYOCOOLER

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#### Abstract

In the range of milliwatt to a few watts cooling capacity, Stirling cycle and pulse tube coolers are most suitable for producing cryogenic temperatures owing to their eco-friendliness, high efficiency, cooling capacity to mass ratio etc. The compressor of a Stirling cooler is powered by a linear motor. The power piston of the cooler is held in position and moves to and fro with the support of so called flexure springs or flexure bearings. Flexures avoid direct contact between moving parts of the compressor of the cooler. Thus, if designed adequately to withstand fatigue, flexure bearings can easily outlast rolling element bearings and slider bearings. In this work, a computational analysis is used to study the performance of flexure spring by varying the geometrical parameters. Three of the most common spring materials namely, SS304, beryllium copper and spring steel are considered for analysis. The analysis was made by varying the parameters like spiral sweep angle, slot width, number of spirals and disc thickness. The influence of each of these parameters on the fatigue life of the spring has been investigated. The results suggest that flexure springs of three spiral arms would be the ideal choice for the selected cryocooler. The variation of stress developed with respect to different design parameters and fatigue damage factor are presented graphically.

Keywords: Stirling cooler, Flexure springs, Parametric study, Finite element analysis.

# 1. Introduction

Cryocoolers have found numerous applications in military and space, particularly for electronics cooling. In space applications, they can replace the stored cryogens on board, thereby reducing the payload. In military applications, infrared imaging is made possible by cooling the sensors to cryogenic temperatures [1]. The energy requirement for the cooler is very less, when compared with the conventional Rankine cycle cooler [2]. The cryocooler can run on reversed Stirling cycle (known as regenerative cryocooler), which uses a small amount of helium gas as the working medium. Helium is an eco-friendly gas, with zero Ozone Depletion Potential (ODP) and it is non-CFC (chlorofluorocarbon) refrigerant too. A closed cycle cryocooler is the need of the hour as the cost of the cryogens is increasing and the Earth's Helium reserve is getting depleted [3].

Stirling cycle coolers are closed cycle coolers, which come under the category of active coolers. Stirling cycle consists of two constant volume and two constant temperature processes. These devices consist of a compressor piston and a displacer piston with a regenerative type heat exchanger. Stirling cycle coolers were the first active coolers to be used successfully in space and have proved to be reliable and efficient [4]. The power piston of the cooler is operated by the motion imparted to it from the linear motor. Linear motors are employed in Stirling coolers, instead of conventional rotary motors. Linear motors generate force in the direction of travel of the piston. They are capable of extreme high speeds, quick acceleration and accurate positioning [3].

The development of long life cryocooler with linear motor is improved by the advent of flexure bearings [5, 6]. Flexure can be defined as a component, which allows motion due to change in shape, i.e., flexing. The present cryocoolers have a linear compressor for compressing the working gas with a small axial displacement. The flexures move by the same amount as that of the piston displacement. Flexure bearings or simply known as flexure springs, minimize the wear and tear due to friction by suspending the piston inside the compressor. The piston moves concentrically with no direct contact with the cylinder. The flexure springs ensure that the clearance seals between the piston and the cylinder are preserved under all operating conditions. As far as cryocoolers are concerned, the concept of flexure suspension systems is first used in the Oxford University Cryocooler narrated by Werret et al. [7].

The complex geometry and the operational requirements such as high fatigue strength and high radial stiffness, preclude the exact analysis in the design of the spring. Therefore, the use of Finite Element Analysis (FEA) is indispensable in this context [8]. Many papers have been published in the area of flexure spring analysis. Gaunekar et al. [9] presented the use of Finite Element Method (FEM) to produce normalized curves for the design of the spiral arm flexure springs. These authors conclude that the finite element method provides a sound basis for analysis and design of so critical a component of long-life miniature cryocoolers as the flexure bearing. In a paper published by Oxford University, Davey et al. [10] investigate the dependency of linearity of motion on clamping conditions of the springs. They made a simple theoretical model with six spiral arms for investigating the linearity. Figure 1 shows the schematic assembly of flexure bearing.

In this paper, the performance parameters of the flexure spring such as maximum stress developed, number of operating hours before failure and fatigue damage are analyzed by varying a combination of geometrical parameters viz. disc thickness, spiral sweep angle, number of spiral arms, and spiral slot width.

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Three different materials are identified and compared on the basis of above mentioned parameters.

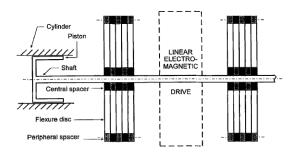


Fig. 1. Schematic assembly of flexure bearing [9].

### 2. Design Aspects

The life of the linearly driven cryocoolers has been increased by incorporating clearance seal technology. This has been made possible by employing a non-conventional suspension system, as described above, known as flexure bearing or flexure spring, instead of helical spring [11]. Flexure springs combine the function of both spring and bearing in a single part and have a high radial to axial stiffness ratio. As the flexure springs do not have any rubbing parts, the life of the entire system increases tremendously in comparison with the conventional helical springs.

A typical unit of a flexure suspension system used in linear compressor is having each unit in the form of a thin flat metal disc [12]. A number of spiral arms are cut on the disc for linear movement normal to the plane of the disc. A simple flexure spring with three spiral arms is shown in Fig. 2. Twelve circular holes are made on the periphery to clamp the disc rigidly onto a support. The piston rod is fixed to the flexure through the central hole. A linear electromagnetic drive provides the required force for motion.

The design of flexure springs is critical in the reliable performance of the cooler over its entire life time. Flexure supports the piston and restricts the movement of piston only in the axial direction [13]. The design criteria are to have less axial stiffness and high radial stiffness [5]. The minimal axial stiffness will provide the sufficient displacement in the axial direction, while high radial stiffness restricts the radial movement to the minimum to maintain the component clearance. The lower axial stiffness compared to gas spring stiffness above the piston keeps the moving mass to a minimum, which controls the vibrations in the system. Each spiral arm of the flexure is subjected to alternating stresses at the frequency of operation of the cooler. For a limited axial displacement, the magnitude and location of the maximum stress developed are dependent on the spiral profile, disc thickness and spiral cut width. At the same time, the spring should have high fatigue strength to withstand the repeated cyclic loading [14].

In the present work, the involute of a circle is adopted to form the spiral profile. With regard to spatial distribution of the spiral arms, the eccentric type

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design is selected for analysis. Eccentric type is the one with the spiral arm involute having different starting points as depicted in Fig. 2. Considering the slot width of the spiral profile, there can be two different types of springs; one with constant slot width and the other with varying slot width. The former one is adopted for simplicity.

The design variables of the disc are the spiral angle, number of spirals, spiral width and the disc thickness. The outer diameter of the spring is automatically fixed by the size of the cooler. The number of holes drilled on the outer periphery for clamping the spring to the support affects the stress developed. The spiral slots themselves are Archimedean spirals.

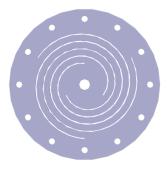


Fig. 2. Flexure spring geometry.

# 3. Method of Analysis

As the exact analysis is not possible to carry out, the FEA was chosen. A nonlinear static structural analysis was conducted using analysis software ANSYS 14.0. The modelling of the spring is carried out using CATIA and Solid Edge. The holes on the outer periphery of the disc were fixed and an axial displacement was given to the centre hole. The displacement was given to the hole at the centre since the piston rod is connected to it for support. The mesh is successively refined to reach at a convergence point and to decide on the adequacy of the mesh. The criterion for refinement of meshing is to have a fine mesh especially at the ends of the spiral and curling.

It is observed that a high value of stresses occur at the ends of the spiral arm in the basic configuration. This makes the stress concentration around the ends of the spiral slots. In order to reduce the stress concentration, spiral arms are provided with a curling end. The inner end is curled into the centre and outer end is curled out of the centre to give a better solution to these stress concentrations. A number of simulations have been made in order to arrive at the best curling of the ends. The curling was initially provided centrally at the ends of the spiral slots (not shown), later modified to the one as shown in Fig. 3(a). The curling results in significant reduction in the stress concentration as can be observed from the results.

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The analysis has been carried out by fully constraining the motion at the 12 holes at the periphery of the disc. An axial displacement as required by the moving component (piston) of the cooler is given to the spring perpendicular to the disc plane. The meshing is refined, Fig. 3(b), for each simulation especially in the region closer to the ends of the spiral slots. Many angles of revolution for spiral are investigated beginning from 180° through 720° with an increment of 90°. Three different materials have been identified for spring; SS 304, beryllium copper and spring steel. Of the three materials, spring steel is having higher fatigue Strength with light weight and beryllium copper has the least fatigue strength.

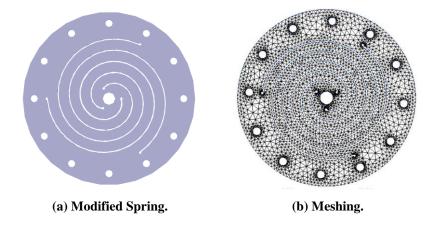


Fig. 3 Modified spring geometry.

#### 4. Results and Discussion

3-D modelling and finite element analysis of the flexure spring is conducted using software. The performance parameters of the flexure spring are determined for various materials by simulation method and the results are tabulated and presented in the form of curves as discussed below.

The analysis is extended by varying a combination of the geometrical parameters such as the disc thickness, the sweep angle, the spiral slot width and number of spirals. The results show that the increase in the disc thickness increases the maximum stress developed. A similar result is obtained with the spiral slot width. Some good compromise, without affecting the design constraints of the cooler, should be made to get the optimised geometry.

For a constant set of parameters such as disc thickness, spiral slot width and number of spirals, the effect of spiral angle on maximum stress developed is determined and plotted as shown in Fig. 4.

The maximum stress is noted down for the given axial displacement in each case. It is observed that the stress developed decreases with the increase in spiral angle. It is attributed to the flexibility the spring gets with the increase in spiral angle. As the spiral angle increases, the flexure can move more freely, so less stress is developed. From the results, it is observed that 720° spiral angle has the least stress developed in it for all the materials under consideration.

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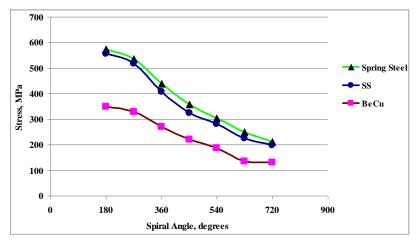


Fig. 4. Variation of maximum stress with spiral angle.

Figure 5 shows the variation of maximum stress developed for different spiral slot width, other parameters being fixed. Spiral slot width is varied from 0.5 mm to 0.75 mm in steps of 0.05 mm. Maximum Stress is noted down for the given axial displacement. It is observed that the stress developed decreases slightly with the increase in spiral slot width. As the incremental increase is only 0.05 mm, the maximum stress developed is nearly constant towards the end. It is recommended to adopt 0.5 mm thickness considering the fabrication simplicity and since there is little deviation of the stress developed for higher thickness.

When the analysis is conducted by varying the thickness of the spiral disc for a spiral angle of  $720^{\circ}$ , Fig. 6, it is observed that the maximum stress developed increases steadily with the thickness. It is attributed to the fact that the stress is directly proportional to the force applied. More the thickness of the disc, more the force applied to produce an equal axial displacement. Hence, from the results obtained, it is concluded that the 0.5 mm disc is the optimal one.

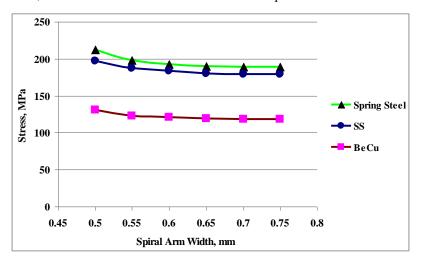


Fig. 5. Variation of maximum stress with spiral slot width.

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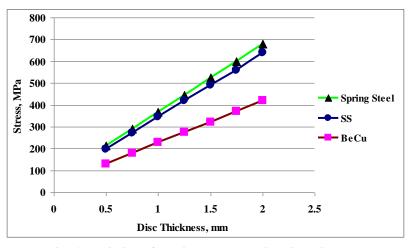


Fig. 6. Variation of maximum stress with disc thickness.

The analysis is extended by altering the number of spiral arms keeping the optimized angle obtained from the analysis. The number of spiral arms is increased from one to four and the simulation is run for each pass as shown in Fig. 7. It is observed that, when a single spiral arm is used the stress developed is the maximum. As the spring finds it difficult to move with a single spiral, the stress developed is at its peak. The stress developed decreases drastically in the beginning, and then slowly. It can be observed that when the number of spirals increased from three to four, the stress developed is nearly constant. The optimum number of spirals taken is three, based on the observations.

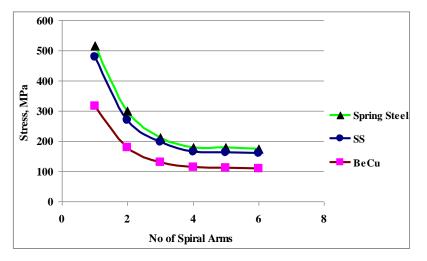


Fig. 7. Variation of maximum stress with number of spiral arms.

Fatigue damage is defined as the design life divided by the available life. The cryocooler was designed to operate for a period of 10 years, and the same is taken as the design life of the flexure spring. A value of fatigue damage greater than one denotes the failure of the spring before design life. A contour plot of the fatigue

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damage at a given design life was obtained using ANSYS for different conditions as explained before. The variation of fatigue damage with respect to spiral angle and number of spirals are then plotted as shown in Figs. 8 and 9 respectively.

As it can be seen from Fig. 8, fatigue damage decreases with increase in spiral sweep angle. Also it has been observed that the fatigue damage is least and less than one for spring with beryllium copper material.

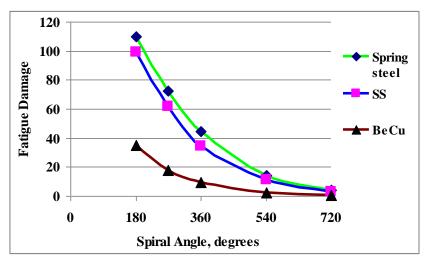


Fig. 8. Fatigue damage vs. spiral angle.

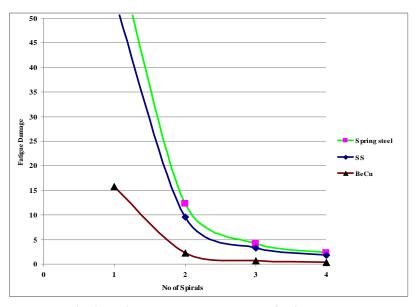


Fig. 9. Fatigue damage vs. number of spiral arms.

Figure 9 indicates that the fatigue damage decreases as the number of spirals increases. Again, in this case also the fatigue damage is least for beryllium copper. When we compare Figs. 7 and 9, we can observe that with the increase in

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the number of spiral arms, the stress developed and thereby fatigue damage decrease. But, as the number of spirals increases, the spring becomes weaker and the chance to fail is more as can be observed from Fig. 9. In Fig. 9, the fatigue damage stabilises at a minimum value once the number of spirals are increased to three. Therefore, it is recommended to adopt three spiral arm spring for the use in cryocooler based on the results.

## 5. Conclusions

This paper focuses on parametric study and finite element analysis of flexure springs used in the cryocoolers. The flexure spring is designed for the use in a Stirling cryocooler. The simulations are run by varying the geometrical parameters such as spiral sweep angle, spiral slot width, number of spirals and disc thickness. Various materials are identified and compared with for the design criteria of high fatigue life. The fatigue damage factor is also calculated from the analysis and plotted. The optimized geometry meeting the design constraints has been obtained by the analysis conducted. Some concluding observations from the investigation are given below.

- It is observed that the maximum stress is developed at both ends of the spiral arms. Curling is introduced at the ends to minimize the stress concentration and the results obtained are satisfactory.
- The flexure spring with three spiral arms of 0.5 mm slot width on a 0.5 mm thick circular disc and with 720° spiral angle has been identified as the best possible configuration for the designed cryocooler.
- The work can be extended by considering the concentric type spiral profile and/or the spiral slots with varying width.

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