

## ADAPTIVE BEAMFORMING IN SMART ANTENNA USING TCHEBYSCHIEFF DISTRIBUTION AND VARIANTS OF LEAST MEAN SQUARE ALGORITHM

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### Abstract

Tchebyscheff distribution is used for adaptive beam formation for smart antenna combined with different variants of least mean square (LMS) algorithm. Performances for accuracies of generated main beam direction and null direction and also side lobe level (SLL) are obtained using these hybrid algorithms. Tchebyscheff distribution with LMS (TDLMS), Tchebyscheff distribution with normalised LMS (TDNLMS) and Tchebyscheff distribution with sign LMS (TDSLMS) are used for adaptive beam generation and results are compared. Results show that using these hybrid algorithms low side lobe level can be obtained.

Keywords: Adaptive beam, Tchebyscheff distribution, Variants of LMS algorithm, Side lobe level.

### 1. Introduction

Smart antenna uses a number of antenna elements and signal received at each antenna element is adaptively combined to improve the overall performance in communication. Smart antennas can eliminate interference by producing null along the direction of undesired interferer and by producing only radiation beam along the direction of arrival (DOA) of signal [1]. There are various types of algorithms for beam-forming, having their advantages and disadvantages [2-5]. In addition to various methods of DOA estimations, many iterative schemes applicable to adaptive beam-forming have been described in [2]. Recently, a complex quaternion least mean square (LMS) algorithm is used [5] for beam-forming of polarization-sensitive electromagnetic vector-sensor. Side lobes not only consume power but also cause of interference for other users. Therefore attention is also given for the reduction of side lobe levels (SLL) in smart antennas

| <b>Nomenclatures</b> |   |
|----------------------|---|
| $d$                  | Antenna element spacing                                     |
| $T_m$                | Tchebyscheff polynomial                                     |
| <b>Greek Symbols</b> |   |
| $\alpha$             | Progressive phase shift                                     |
| $\theta$             | Direction of beam   |
| $\lambda$            | Wavelength  |
| $\mu$                | Step size parameter   |
| <b>Abbreviations</b> |   |
| DOA                  | Direction of Arrival  |
| LMS                  | Least Mean Square   |
| NLMS                 | Normalized Least Mean Square                                |
| SLL                  | Side Lobe Level   |
| SLMS                 | Sign Least Mean Square                                      |
| TD                   | Tchebyscheff Distribution                                   |
| TDNLMS               | Normalized Least Mean Square with Tchebyscheff Distribution |
| TDSLMS               | Sign Least Mean Square with Tchebyscheff Distribution       |

[6-11]. Side lobe reduction is another issue in adaptive smart antenna. For adaptive antennas when beamforming algorithms are used in addition to array synthesis methods, due to weight updation ( to generate main beam and null in a particular directions), like ordinary array that much of side lobe reduction is not possible. Some of the reports are available where beamforming algorithms are used along with soft computing methods [9-11].

The smart antenna technology can significantly improve wireless system performance by increasing signal quality, network capacity and coverage area. The digital beam forming method using smart antenna is shown in Fig. 1. Signals are processed adaptively in order to exploit the spatial domain of the mobile radio channel. Usually the signals received at the different antennas are multiplied with complex weights. Then adaptively weights are summed up.

In this paper, antenna array synthesis method, like, Tchebyscheff distribution (TD) is used for adaptive beam formation for smart antenna of linear antenna arrays with different variants of least mean square (LMS) algorithm.

Tchebyscheff distribution with LMS (TDLMS), Tchebyscheff distribution with normalized least mean square (TDNLMS) and Tchebyscheff distribution with sign least mean square (TDSLMS) are used for adaptive beam generation.

A comparative study is reported for those hybrid algorithms for accuracy of beam generation and side lobe reduction.

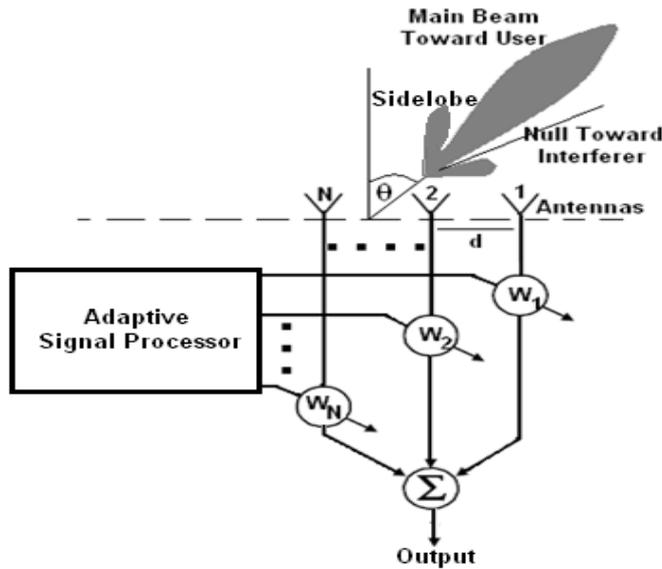


Fig. 1. Adaptive antenna array.

### 2. Tchebyscheff Distribution

Tchebyscheff distribution (TD) for current amplitude, fed in antenna, is used for antenna array synthesis [12, 13] to achieve narrowest beam for a given side lobe level. Also it is used to obtain lowest side lobe level for a given beamwidth. Tchebyscheff distribution, expressed in polynomial form as [13]

$$T_m(x) = \cos(m \cos^{-1} x); -1 < x < +1$$

$$= \cos(m \cosh^{-1} x); |x| > +1 \tag{1}$$

$$T_m(x_o) = b \tag{2}$$

It can be calculated by notifying that if  $b = \cosh \rho$  then

$$x_o = \cosh\left(\frac{\rho}{m}\right) \tag{3}$$

Beamwidth is directly related to ‘ $b$ ’ and ‘ $x_o$ ’ is related to the position of main beam.

### 3. Beamforming Algorithms

To minimize error  $e(n)$  between desired signal  $d(n)$  and array output  $y(n)$  adaptive algorithm is used as [14, 15]

$$e(n) = d(n) - y(n) \tag{4}$$

In least mean square (LMS) algorithm iterative procedure is used making successive corrections to the weight vector in the direction of the negative of the

gradient vector. This eventually leads to the minimum mean square error. The weight vector updating equation for LMS algorithm is [14, 15]

$$w(n+1) = w(n) + \mu x(n)e^*(n) \quad (5)$$

where  $\mu$  is the step size parameter and  $e(n)$  is the error between output and the desired signal.

Normalized least mean square (NLMS) is used to achieve faster convergence and stability of the algorithm. Here, step size is divided by the norm of the input signal to avoid gradient noise amplification due to  $x(n)$ . For normalized least mean square (NLMS) algorithm, the weight vector updation equation is [14, 15].

$$w(n+1) = w(n) + \frac{\mu}{\|x(n)\|^2} x(n)e^*(n) \quad (6)$$

In sign least mean square (SLMS) algorithm only the polarity information of the error signal or data signal or both data and error signal are used for the error updation. In SLMS, adaptation equation is [14, 15]

$$w(n+1) = w(n) + \mu x(n)e^*(n) \text{sgn}[x(n)] \quad (7)$$

$$\begin{aligned} \text{sgn}[x(n)] &= 1; & x(n) > 0 \\ &= 0; & x(n) = 0 \\ &= -1; & x(n) < 0 \end{aligned}$$

#### 4. Adaptive Beamforming

Adaptive beam formation is done for a linear array of uniform element spacing of  $0.5\lambda$  and antenna arrays of 10 elements and 21 elements are considered. Array factor (AF) for a linear array is [12, 13]

$$AF_L = \sum_{n=0}^{N-1} A_n e^{jn\left(\frac{2\pi d}{\lambda} \cos\theta + \alpha\right)} \quad (8)$$

where to generate the main beam at wavelength  $\lambda$  toward the desired beam direction  $\theta_0$  degree from the broadside direction, the progressive phase shift is

$$\alpha = -\frac{2\pi d}{\lambda} \cos\theta_0 \quad (9)$$

and final array factor for Tchebyscheff array is

$$AF = AF_L T_m(x) \quad (10)$$

Normalized array factor is

$$AF_{nom} = \frac{AF}{AF_{max}} \quad (11)$$

In Eq. (11),  $AF_{max}$  is the maximum value of array factor AF, given by Eq. (10).

The step-wise flow chart for the application of Tchebyscheff distribution with LMS algorithms (TDLMS) is shown in Fig. 2.

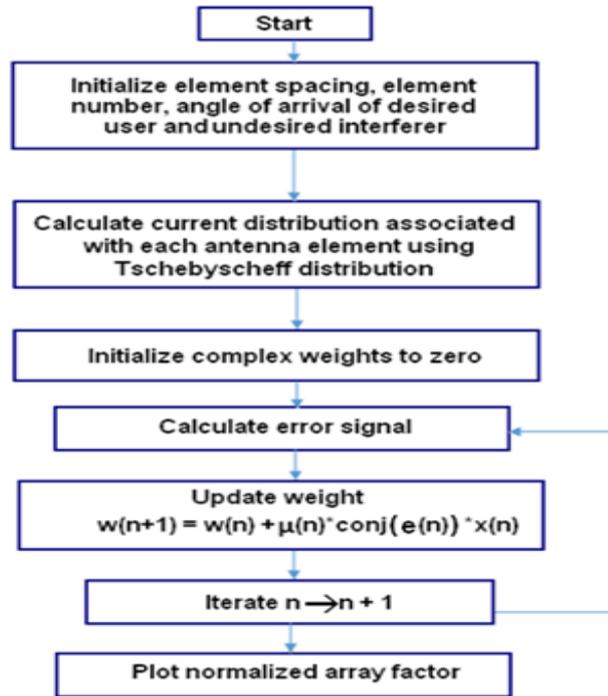


Fig. 2. Flow chart for TDLMS algorithm.

## 5. Results and Discussion

For TDNLMS and TDSLMS same procedure, as mentioned in Fig. 2, is followed with different weight updation equations mentioned in Eq. (6) and in Eq. (7) respectively. Uniform linear antenna arrays of 10 elements and 21 elements are considered here. Comparisons of results using TDLMS, TDSLMS and TDNLMS for 10 element array (Figs. 3 and 4) are shown below where in Tchebyscheff distribution SLL is fixed at -25 dB. Comparisons of results of TDLMS, TDSLMS and TDNLMS for 21 element array are shown in Figs. 5 and 6. Here, inter-element spacing  $d=0.5\lambda$ , desired angle of arrival  $30^\circ$  and angle of interferer (null)  $60^\circ$ . Number of iterations of the program for each algorithm is 100.

Performances of different algorithms for main beam direction, null direction and maximum side lobe level ( $SLL_{max}$ ), used for adaptive beam formation, are compared in Table 1.

Side lobe level (SLL) is fixed at -25 dB in Tchebyscheff distribution. In all the cases for hybrid algorithms, side lobe levels are reduced appreciably but performances of the algorithms are not same for beam and null formations. In Figs. 3 and 4, for  $N=10$ , first-null-beamwidths are  $42^\circ$  (for  $\mu = 0.02$ ) and  $40^\circ$  (for  $\mu = 0.04$ ) respectively. In Figs. 5 and 6, for  $N=21$ , first-null-beamwidths are  $14^\circ$  (for  $\mu = 0.02$ ) and  $13^\circ$  (for  $\mu = 0.04$ ) respectively.

Square error plots for 10 antenna elements for LMS, NLMS and SLMS algorithms are shown in Fig. 7. Error for TDLMS is more than the others but convergence of TDLMS is better.

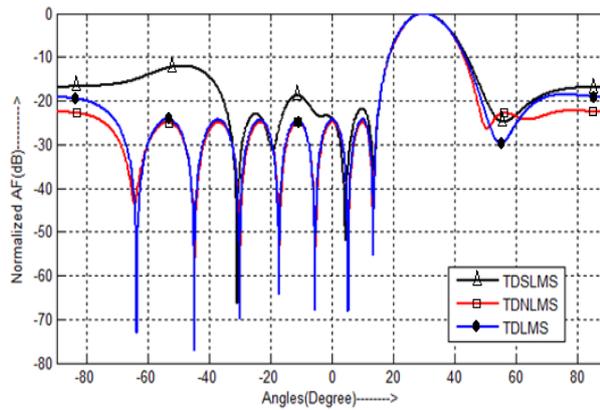


Fig. 3. Normalized array factor for  $N=10$  with step size=0.02.

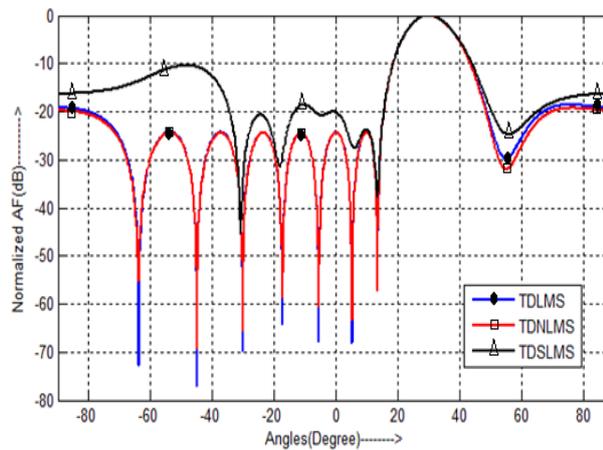


Fig. 4. Normalized array factor for  $N=10$  with step size=0.04.

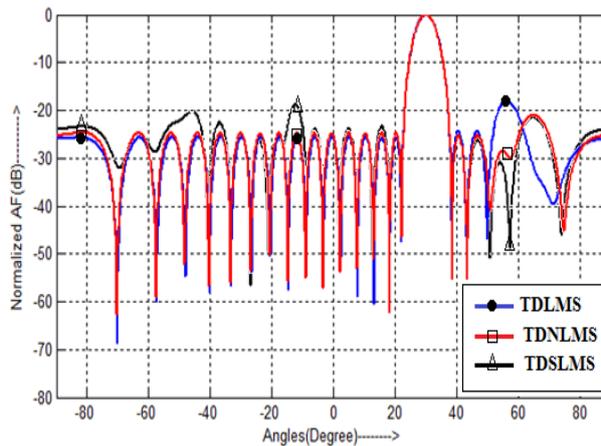


Fig. 5. Normalized array factor for  $N=21$  with step size=0.02.

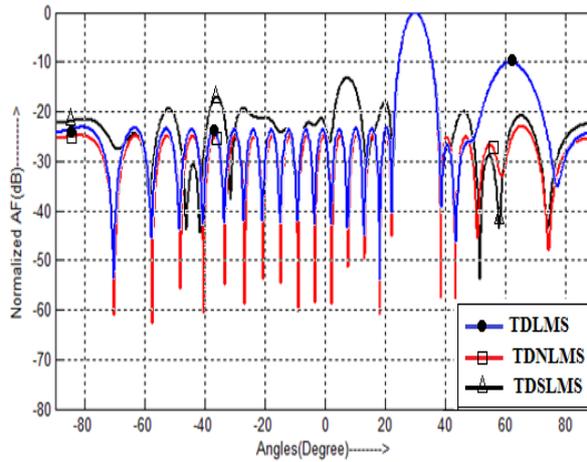


Fig. 6. Normalized array factor for  $N=21$  with step size=0.04.

Table 1. Comparison of results.

| Algorithms | Number of Element in the Array $N=10$ |             |                         | Number of Element in the Array $N=21$ |                        |                         |
|------------|---------------------------------------|-------------|-------------------------|---------------------------------------|------------------------|-------------------------|
|            | Main Beam (Deg.)                      | Null (Deg.) | SLL <sub>max</sub> (dB) | Main Beam (Deg.)                      | Null (Deg.)            | SLL <sub>max</sub> (dB) |
| TDLMS      | 30                                    | 57.5        | -19                     | Not working properly                  | Not working properly   | -18.5                   |
| TDNLMS     | 30                                    | 57.5        | -20                     | 30                                    | No precise null at 60° | -20.5                   |
| TDSLMS     | 30                                    | 57.5        | -14                     | 30                                    | 59.8                   | -19                     |

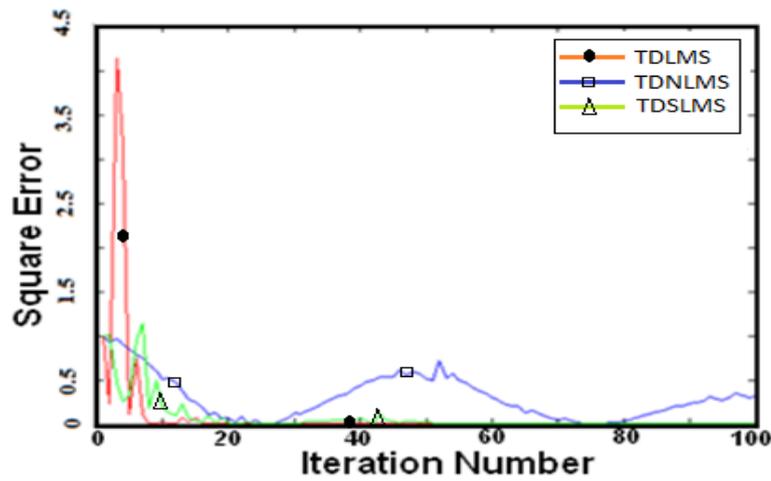


Fig. 7. Square error plot for  $N=10$  with step size = 0.02.

Performances of hybrid beamforming algorithms are compared in this paper. Remarkable side lobe level reduction is achieved using TDNLMS. For all the cases step size of  $\mu = 0.02$  and  $0.04$  are taken because extensive work on beam formation of adaptive antennas using LMS, NLMS and SLMS shows that for these values of  $\mu$ , performances of all these algorithms are simultaneously good [16], where Tchebyscheff distribution is not used. The performances of algorithms, tabulated in Table 1, show that using TDLMS, TDSLMS and TDNLMS side lobe reduction is achieved but accuracies for interferer directions (nulls) have small deviation from the expected direction. Side lobe reduction using TDNLMS is better as compared to TDLMS and TDSLMS. Here, TDLMS, TDNLMS and TDSLMS are used for beam generation with Tchebyscheff distribution. Weight updating methods are different for LMS, NLMS and SLMS [16] algorithms and therefore results deviate from each other. But performance of beamforming depends on this array spacing. Like, phased array antenna with the angle of arrival grating lobe may appear, even after maintaining the maximum inter-element spacing [17]. In Tchebyscheff distribution, the value of SLL is chosen -25 dB which is fairly a good approximation to obtain overall required SLL of -10 dB to -20 dB for adaptive antenna application [18].

## 6. Conclusions

Beam formation in smart antenna along with array synthesis is a difficult task. Here, array synthesis using Tchebyscheff distribution is used along with variants of LMS algorithm. Lower SLLs are obtained using TDLMS, TDNLMS and TDSLMS than using LMS, NLMS or SLMS. In some cases, precise values of nulls are not obtained. Improvement may be possible by choosing lower values of side lobe level in Tchebyscheff distribution instead of -25 dB. Antenna arrays with various inter-element spacing are used for adaptive beam generation, all of which are not included here. It is found that performances of SLMS and TDSLMS are better for large antenna arrays. For large number of elements proper step size parameter is required for beamforming. For large antenna array TDLMS and TDNLMS, proper adaptive beamforming is not achieved for step size of 0.02. Therefore, step size is a vital parameter for good performance of adaptive array. Use of Taylor distribution, Schelkunoff polynomial method, Woodward-Lawson method for beamforming with variants of LMS algorithm may be extension of this work. The research findings, presented in this paper, may be useful for adaptive beam generation in mobile communication, satellite communication and radio detection.

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