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VIBRATION CONTROL OF RECTANGULAR CROSS-PLY FRP PLATES USING PZT MATERIALS

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Abstract

Piezoelectric materials are extensively employed in the field of structures for condition monitoring, smart control and testing applications. The piezoelectric patches are surface bonded to a composite laminate plate and used as vibration actuators. The coupling effects between the mechanical and electric properties of piezoelectric materials have drawn significant attention for their potential applications. In the present work, an analytical solution of the vibration response of a simply supported laminate rectangular plate under time harmonic electrical loading is obtained and a concept is developed for an approximate dynamic model to the vibration response of the simply supported orthotropic rectangular plates excited by a piezoelectric patch of variable rectangular geometry and location. A time harmonic electric voltages with the same magnitude and opposite sign are applied to the two symmetric piezoelectric actuators, which results in the bending moment on the plate. The main objective of the work is to obtain an analytical solution for the vibration amplitude of composite plate predicted from plate theory. The results demonstrate that the vibration modes can be selectively excited and the geometry of the PZT-actuator shape remarkably affects the distribution of the response among modes. Thus according to the desired degree shape control it is possible to tailor the shape, size and properly designed control algorithm of the actuator to either excite or suppress particular modes

Keywords: Cross-ply laminates, Piezoelectric actuators, Harmonic voltages.

1. Introduction

During the past several years, the piezoelectric effect was adopted in many engineering applications, and there has been much research on it. Crawley and de Luis [1] studied bars and beams with surface bonded and embedded piezoelectric

Nomenclatures

$A(x, y)$	Amplitude series
a	Length of the plate
b	Width of the plate
D	Charge density displacement of the plate
D_{ij}	Rigidity of plate
d_{ij}	Piezo-electric constant, m/V
E	Electric field
E_p	Young's Modulus of the isotropic plate
E_{pe}	Young's Modulus of the piezoelectric material
$F(x, y)$	Forcing function
M	Area mass density of the plate
m_x,m_y	Moments produced by PZT plate
S	Strain
T	Piezoelectric material's stress
t_p	Thickness of the plate
t_{pe}	Thickness of the piezoelectric patch
$w(x, y)$	Deflection series

Greek Symbols

ε	Electric permittivity, F/m
ε_{pe}	Strain in piezoelectric actuator
ω_{mn}	Natural frequency
ω	Forcing frequency
ϑ_p	Poisson's ratio of plate
ϑ_{pe}	Poisson's ratio of piezoelectric material

Abbreviations

PZT	Lead Zirconate Titanate

actuators. Her and Liu [2] studied an isotropic plate with embedded piezoelectric actuators to predict the deformed shape. Her and Lin [3] investigated a composite laminate with surface bonded piezoelectric actuators to predict the deformed shape. By using the plate theory, the deflection of the plate can be obtained. Clark et al. [4] observed experimentally the vibration excitation of a simply supported plate with multiple piezoelectric patches bonded on the surface and the results are compared with theoretical predictions. It is shown that the modes can be selectively excited depending on the voltage supplied to the actuator. The location of actuator on the plate is a considerable parameter in plate excitation. Dimitriadis et al. [5] worked on static analysis is used to estimate the loads induced by the two dimensional patches of piezoelectric actuator on the supporting structure. Dynamic analysis is also performed to investigate the vibration response of the plate excited by the piezoelectric patch of variable rectangular geometry. Her and Lin [6] they studied the effects of the size and location of the piezoelectric actuators on the response of the composite laminate are presented through a parametric study. The effect of excitation frequency of actuator on the plate is also studied. The analytical results are validated with the finite element results.

Marinkovic and Zehn [7] a brief review of the developed shell type finite element for smart composite structures is presented. It is a degenerated shell element based on the Reissner-Mindlin kinematical assumptions for modeling thin and moderately thick structures made of multilayered material including piezoelectric active layers polarized in the thickness direction. Wankhade and Bajoria [8] shape control and free vibration analysis of piezolaminated plates subjected to electro mechanical loading are evaluated using finite element method. Chennamsetti [9] carried out the recent research and development work in the field of sensors and instrumentation for SHM applications and damage detection algorithms have been critically reviewed. First order shear deformation theory is employed in the analysis. Kędziora [10] various optimization criteria used for optimal placement of piezoelectric actuators on laminated structures is discussed. Piezoelectric materials are used as layers or fibers that are embedded within or bonded to the surfaces of a structure. Hasanlu et al. [11] considered the vibrations generated by various disturbances, which include free and forced vibrations, a PID control is implemented to damp both the free and forced vibrations. Zhang et al. [12] the effort to reduce vibration and control the mechanical phenomenon has a significant impact on reduced energy waste of a continuous structure (kinematic energy- potential energy).

In the present work dynamic analysis on cross ply composite using Lead Zirconate Titanate (PZT) is carried out. Initially, the loads induced by the piezoelectric actuator on the host plate and the corresponding deflection are to be estimated. Electric voltages with the same magnitude and opposite sign are applied to the two symmetric piezoelectric actuators, which produces forces in the piezoelectric actuators. These forces will induce bending moment on the plate. The analytical solution for the deflection of composite plate computed from plate theory. The results demonstrate that the deflection can be selectively excited and the geometry of the PZT-actuator shape remarkably affects the distribution of the response. Thus according to the desired degree of shape control it is possible to tailor the shape and size of the actuator to either excite or suppress deflection.

Later in the dynamic analysis the vibration response of the structure was investigated. The time harmonic electrical potential is employed on a simply supported plate to develop the excitation by piezoelectric patches. An analytical solution of the free vibration response of simply supported composite plate is obtained.

2. History of PZT

The piezoelectric effect was discovered by Jacques and Pierre Curie in 1880. They discovered that some crystals, which when compressed in particular directions show positive and negative charges on certain positions of their surfaces. The amount of charge produced is proportional to the pressure applied and this charge becomes zero when the pressure is withdrawn. Hankel proposed the name ‘piezoelectricity’. The word ‘piezo’ is a Greek word which means ‘to press’, therefore piezoelectricity means electricity generated from pressure. The polarization in piezoelectric materials is temperature dependent. They always perform better below Curie temperature. When a piezoelectric element is exposed to an alternating electric field, it periodically changes its size according to the frequency of the field. If the applied voltage has the same polarity as the poling voltage is applied to the electrodes of cylinder, then it will lengthen. If a voltage of opposite polarity to the poling voltage is applied to the cylinder, then it will shorten (Fig. 1).

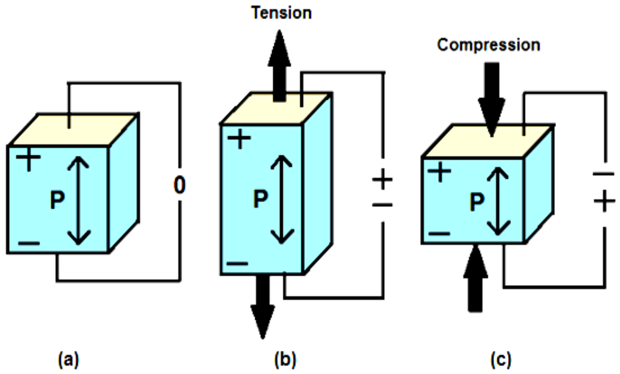

Fig. 1. Direct piezoelectric effect.

Constitutive Relations

A constitutive equation describes how a material strains when it is stressed, or vice-versa. Constitutive equations exist also for electrical problems; they describe how charge moves in a (dielectric) material when it is subjected to a voltage, or vice-versa.

The Hooke's law can be written as

Strain = compliance × Stress

$$S = s.T \quad (1)$$

Since piezoelectric materials are concerned with electrical properties too, it must also consider the constitutive equation for common dielectrics:

Charge density = Permittivity × Electric field

$$D = \varepsilon.E \quad (2)$$

where ε is electric permittivity (F/m)

The constitutive relations describing the piezoelectric property is based on the assumption that the total strain in the piezoelectric material is the sum of mechanical strain induced by mechanical stress and controllable actuation strain caused by the applied electrical voltage [2].

Piezoelectricity is described mathematically within a material's constitutive equation by combining the above two relations, which defines how the piezoelectric material's stress (**T**), strain (**S**), charge-density displacement (**D**), and electric field (**E**) interact.

The piezoelectric constitutive law (in Strain-Charge form) is: [3]

$$S = s_E T + d^t E \quad (3)$$

$$D = d_{ij}\ T + \varepsilon^t E \quad (4)$$

d_{ij} contains piezoelectric coefficients for the material (m/V)

Charge-density displacement, *D* [Refers to what-two mutually perpendicular directions] is defined as the ratio of the strain in j-axis to the electric field applied along i-axis, when all external stresses are held constant. For example d_{31} is the ratio of strain along axis 1 to the electric field applied along axis 3. s_E is compliance coefficients (m^2/N). The typical properties of piezoelectric materials [12] are shown in Table 1.

Table 1. Typical properties of piezoelectric materials [12].

Property	Piezoelectric layer					
	G1195	PZT 5H	3195HD	3221HD	3203HD	3203STD
Young's Modulus (GPa)	63	63	67	62	62	63
Poisson's ratio	0.3	0.31	0.31	--	0.31	--
Density (kg/m^3)	7600	7600	7800	7870	7870	7700
Piezoelectric constant ($m/V\times10^{-10}$)	1.9	-2.71	-1.9	-3	-3.2	-2.75

3. Methodology

Consider two piezoelectric actuators bonded symmetrically as shown in Fig. 2 [6] over a cross ply composite laminate plate. The strain in the piezoelectric actuator is expressed in terms of piezoelectric constant d_{31}, actuator thickness t_{pe} and applied voltage *V*, as followed in Eq. 5.

$$(\varepsilon_x)_{pe} = (\varepsilon_y)_{pe} = \varepsilon_{pe} = \frac{d_{31}}{t_{pe}} V \tag{5}$$

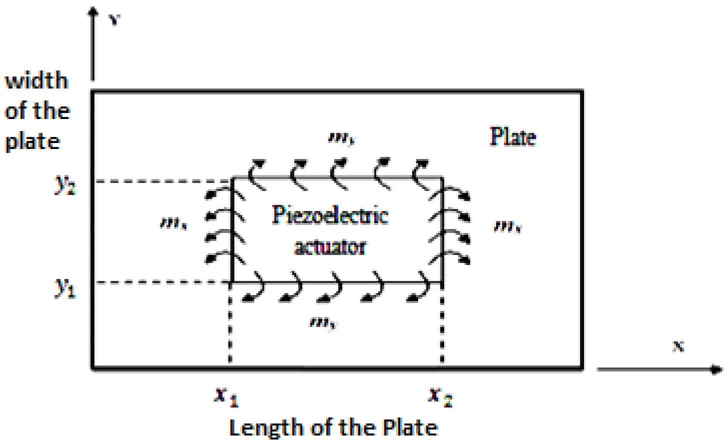

Fig. 2. Bending moment on the isotropic plate induced by the piezoelectric actuator [6].

The governing equation of motion for a cross-ply laminated plate can be expressed in terms of displacement 'A' and induced moments m_x, m_y as follows [1]. The interface between composite plate and actuator modeled is shown in Fig. 3 [6].

$$(D_{11})_p \frac{\partial^4 A}{\partial x^4} + 2H_1 \frac{\partial^4 A}{\partial x^2 \partial y^2} + (D_{22})_p \frac{\partial^4 A}{\partial y^4} + M \frac{\partial^2 A}{\partial t^2} = F(x,y) sin\, \omega t \quad (6a)$$

$$F(x,y) = \frac{\partial^2 m_x}{\partial x^2} + \frac{\partial^2 m_y}{\partial y^2} \quad (6b)$$

where *M* is the area mass density of the composite plate and ω is the excitation frequency. The forcing function *F(x, y)* can be expressed in terms of Fourier series as follows [8]

$$F(x,y) = \sum_{m=1}^{\infty} \sum_{n=1}^{\infty} F_{mn}\, sin \frac{m\pi x}{a} sin \frac{n\pi y}{b} \quad (7)$$

$$F_{mn} = \frac{4}{ab}\left[-\frac{m_y \gamma_m^2 + m_x \gamma_n^2}{\gamma_m \gamma_n}(cos\, \gamma_m x_1 - cos\, \gamma_m x_2)(cos\, \gamma_n y_1 - cos\, \gamma_n y_2)\right] \quad (8)$$

For a simply supported rectangular plate, the displacement '*A*' can be expressed in terms of Fourier series as,

$$A(x,y) = \sum_{m=1}^{\infty} \sum_{n=1}^{\infty} A_{mn}\, sin \frac{m\pi x}{a} sin \frac{n\pi y}{b} sin\, \omega t \quad (9)$$

The natural frequency of a simply supported plate is:

$$\omega_{mn} = \sqrt{\frac{\gamma_m^4 (D_{11})_p + \gamma_m^2 \gamma_n^2 2H_1 + \gamma_n^4 (D_{22})_p}{M}} \quad (10)$$

$$A_{mn} = \frac{F_{mn}}{M(\omega_{mn}^2 - \omega^2)} \quad (11)$$

whereas the bending moment induced by the piezoelectric actuators on cross-ply laminated plate is derived and can be expressed as follows [3]

$$m_x = C_1 \varepsilon_{pe} \; ; \; m_y = C_2 \varepsilon_{pe} \quad (12)$$

$$C_1 = A_1 (D_{11})_P + A_2 (D_{12})_P \quad (13a)$$

$$C_2 = A_1 (D_{12})_P + A_2 (D_{22})_P \quad (13b)$$

$$A_1 = \frac{2(B_{11})_{pe}(1+\vartheta_{pe})}{(D_{11})_p + 2(D_{11})_{pe}} - \frac{(D_{12})_p + 2(D_{12})_{pe}}{(D_{11})_p + 2(D_{11})_{pe}} A_2 \quad (14a)$$

$$A_2 = \frac{2(B_{11})_{pe}(1+\vartheta_{pe})[((D_{11})_p + 2(D_{11})_{pe}) - ((D_{12})_p + 2(D_{12})_{pe})]}{-((D_{12})_p + 2(D_{12})_{pe})^2 + [(D_{11})_p + 2(D_{11})_{pe}][(D_{22})_p + 2(D_{22})_{pe}]} \quad (14b)$$

$$(D_{11})_{pe} = (D_{22})_{pe} = \frac{1}{3} \frac{E_{pe}}{(1-\vartheta_{pe}^2)}((t+h)^3 - t^3) \quad (15a)$$

$$(D_{12})_{pe} = \frac{1}{3} \frac{\vartheta_{pe} E_{pe}}{(1-\vartheta_{pe}^2)}((t+h)^3 - t^3) \quad (15b)$$

$$(B_{11})_{pe} = \frac{1}{2} \frac{E_{pe}}{(1-\vartheta_{pe}^2)}((t+h)^2 - t^2) \quad (15c)$$

The activated piezoelectric element will induce bending moments on the plate; these moments can be expressed in terms of unit step function with constants C_1 and C_2, i.e., C_1 and C_2 are the constants of the step function. Figure 3 shows the arrangement of host and PZT materials.

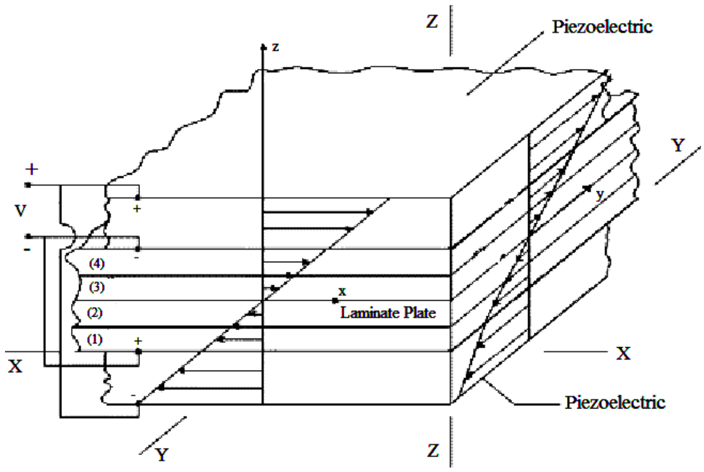

Fig. 3. Interface between composite plate and actuator [6].

4. Numerical Data of the Problem

The orientation for both the Graphite composite and Kevlar composite are considered as [0/90/90/0]. Only to reduce the complexity in the derivation - It is noted that cross-ply symmetric plate eliminates [A] extensional stiffness and [B] bending stiffness terms from the governing equation of the plate so that the derivation converges quickly to yield final solution.

The plate is considered to be made up of Graphite/Epoxy with material properties as shown in Table 2. Table 2 also shows the material properties of Kevlar/Epoxy .

Table. 2 Mechanical properties of Graphite/Epoxy and Kevlar/Epoxy.

Physical Quantity	Magnitude and Measure	
Physical Quantity	**Graphite/Epoxy**	**Kevlar/Epoxy**
Longitudinal Modulus	155 GPa	87 GPa
Transverse Modulus	12.1 GPa	5.5 GPa
Shear Modulus	4.4 GPa	2.2 GPa
Poisson's Ratio	0.248	0.34
Density	1920 kg/m^3	1380 kg/m^3

Piezoelectric Material Properties

Table 3 shows the assumed properties of piezoelectric actuator material (PZT G-1195).

Table 3. Mechanical properties of (PZT G-1195).

Physical Quantity	Magnitude and Measure
Young's modulus	63 GPa
Poisson's ratio	0.3
Density	7600 kg/m^3
The piezoelectric constant	1.9×10^{-10} m/V
Thickness	0.15876 mm

The composite material Graphite/Epoxy and Kevlar/Epoxy with stacking sequence [0/90/90/0] consider for the present study. The voltage of $\pm 1V$ is applied to the upper and lower actuators respectively, and this voltage results in bending moment acting on the host composite plate.

5. Results Discussion

Figures 4 [10] and [6] show the size and location of PZT patches on the FRP plate of selected geometry. Table 4 gives the exact values of physical arrangement of PZT patches [10].

Table 4. Position of PZT patch.

Size of the PZT Patch (mm)	Position of patch (m)			
	x_1	x_2	y_1	y_2
60×40	0.16	0.22	0.13	0.17
80×60	0.15	0.23	0.12	0.18
100×80	0.14	0.24	0.11	0.19

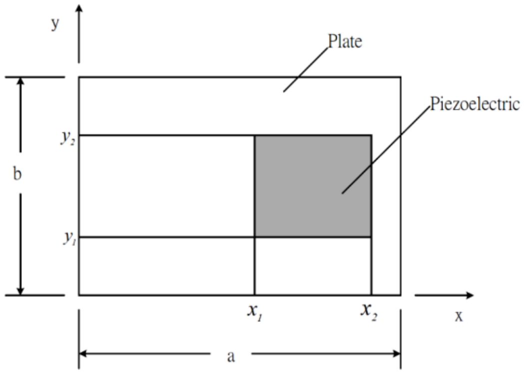

Fig. 4. Actuator on composite plate [10].

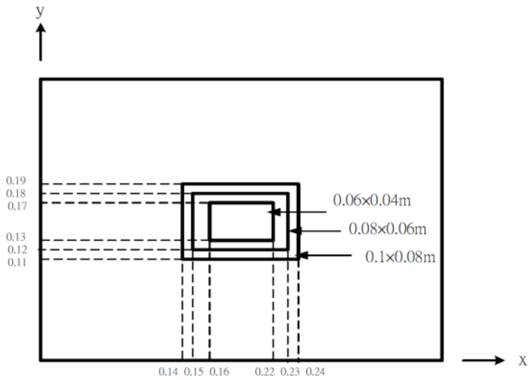

Fig. 5. Three different sizes of PZT actuators [6].

The excitation frequency can be calculated from Eq. (10) to excite desired mode selectively. Three different excitation frequencies (300, 510 and 740 rad/s) were tested. Mode 1 Frequency 300 rad/s, Mode 2 frequency 740 rad/s and in between any value say 510 rad/s is consider to observe the modal pattern, where (*m*, *n*) are the mode values. The modal frequencies of the graphite/epoxy and kevlar/epoxy plates correspond to first 3-modes are listed in Tables 5 and 6 respectively.

The vibration profile of the plate excited by the piezoelectric actuator with excitation frequencies 300, 510 and 740 rad/s are shown in Figs. 6, 7 and 8 respectively. The frequencies 300 and 740 rad/s are close to resonant frequencies of mode (1, 1) and mode (1, 2). The frequency 510 rad/s is far from resonant frequencies of modes (1, 1) and (1, 2). The maximum amplitude excited by three different sizes of actuators embedded on the center of the plate at three different excitation frequencies are listed in Table 7. The positions of the patches in Fig. 4 are corresponding to Table 4.

From Fig. 9, it can be observed that mode (1, 2) can be obtained by placing the actuator of size 60 × 40 at the top region of the plate and the excitation frequency should be 740 rad/s which is nearer to the natural frequency of mode (1, 2).

The excitation frequency can be calculated from the equation (10) to excite a mode selectively. Three different excitation frequencies (310, 530 and 760 rad/s) were tested. The vibration profile of the plate excited by the piezoelectric actuator with excitation frequencies 310, 530 and 760 rad/s are shown in Figs. 10, 11 and 12 respectively. The frequencies 310 and 760 rad/s are close to resonant frequencies of mode (1,1) and mode (1,2). The frequency 530 rad/s is far from resonant frequencies of modes (1,1) and (1,2).

Table 5. Natural frequencies (rad/s) of graphite/epoxy laminate plate obtained by Eq. (10).

		n		
		1	**2**	**3**
	1	302.6421	743.7968	1563.3188
m	**2**	938.2993	1210.5683	1887.3055
	3	2046.6759	2231.5167	2723.7787

Table 6. Natural frequencies (rad/s) of Kevlar/Epoxy laminate plate obtained by Eq. (10).

		n		
		1	**2**	**3**
	1	313.4949	758.3954	1589.6438
m	**2**	983.1316	1253.9798	1933.7332
	3	2148.1036	2331.3777	2821.4545

Table 7. Maximum vibration amplitude (mm) excited by three different sizes of actuators embedded on the center of the plate with different excitation frequencies.

Size of the patch [mm]	**Graphite**			**Kevlar**		
	Natural Frequency (rad/s)			**Natural Frequency (rad/s)**		
	300	**510**	**740**	**310**	**530**	**760**
60 × 40	0.033	0.00018	0.000165	0.0362	0.00023	0.00024
80 × 60	0.065	0.00035	0.000278	0.0711	0.00045	0.00040
100 × 80	0.106	0.00058	0.000355	0.1157	0.00074	0.00052

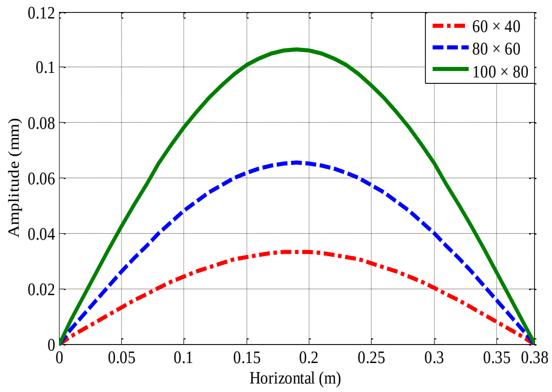

Fig. 6. Vibration amplitude of the Graphite/Epoxy plate obtained from the equation (9) along the horizontal (*y*=*b*/2) for three different sizes of actuator on the center of the plate for excitation frequency 300rad/s.

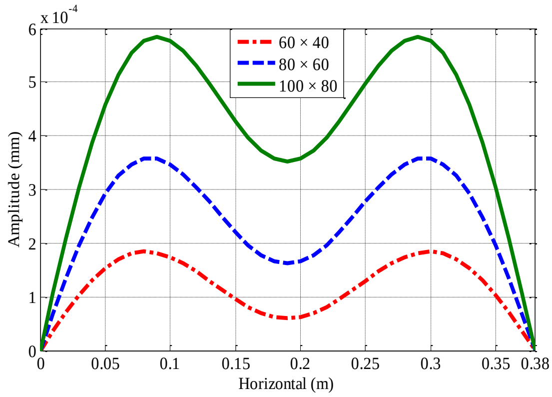

Fig. 7. Vibration amplitude of the Graphite/Epoxy plate obtained from the equation (9) along the horizontal (*y*=*b*/2) for three different sizes of actuator on the center of the plate for excitation frequency 510 rad/s.

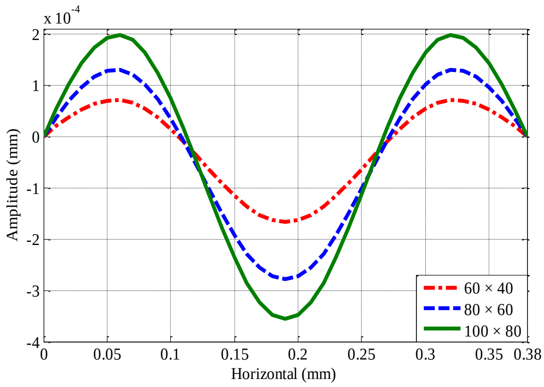

Fig. 8. Vibration amplitude of the Graphite/Epoxy plate obtained from the equation (9) along the horizontal (y=b/2) for three different sizes of actuator on the center of the plate for excitation frequency 740 rad/s.

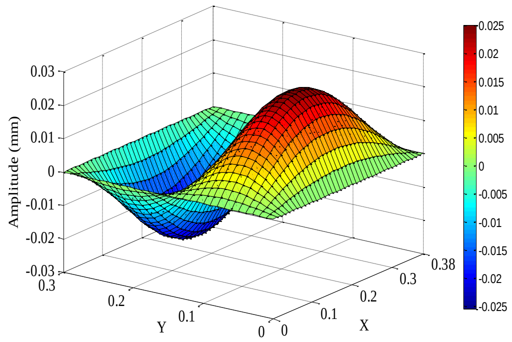

Fig. 9. Vibration profile of Graphite/Epoxy plate excited by the piezoelectric actuator at top of the plate with excitation frequency 740 rad/s.

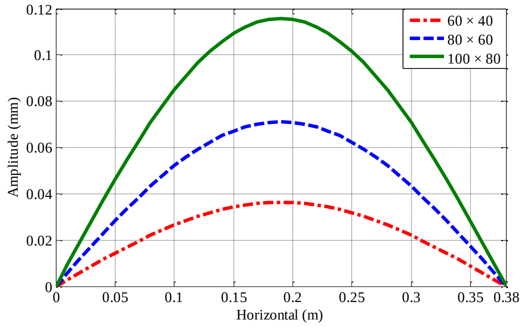

Fig. 10. Vibration amplitude of the Kevlar/Epoxy plate obtained from the equation (9) along the horizontal (y=b/2) for three different sizes of actuator on the center of the plate for excitation frequency 310 rad/s.

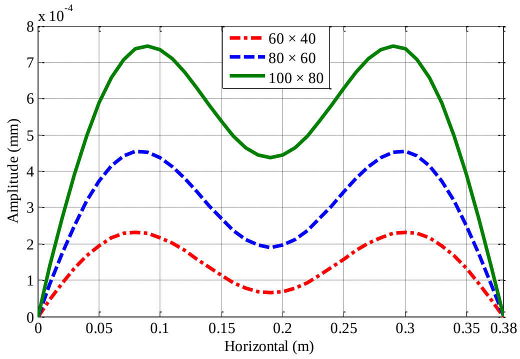

Fig. 11. Vibration amplitude of the Kevlar/Epoxy plate obtained from the equation (9) along the horizontal (y=b/2) for three different sizes of actuator on the center of the plate for excitation frequency 530 rad/s.

The maximum amplitude excited by three different sizes of actuators embedded on the center of the plate at three different excitation frequencies are listed in Table 7. From Fig. 12, it can be observed that mode (1, 2) can be obtained by placing the actuator of size 60 × 40 at the top region of the plate and the excitation frequency should be 760 rad/s which is nearer to the natural frequency of mode (1, 2) is shown in Fig. 13.

Mode (2, 1) can be excited by placing an actuator of size 60 × 40 at the right region of the plate and the excitation frequency should be 980 rad/s. The profile of vibration for mode (2, 1) is shown in Fig. 14.

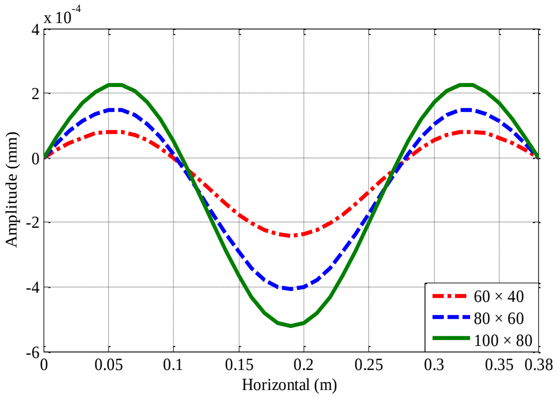

Fig. 12. Vibration amplitude of the Kevlar/Epoxy plate obtained from the equation (9) along the horizontal (*y*=*b*/2) for three different sizes of actuator on the center of the plate for excitation frequency 760 rad/s.

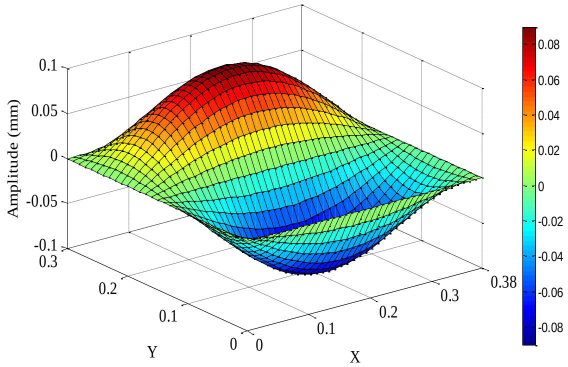

Fig. 13. Vibration profile of Kevlar/Epoxy plate excited by the piezo-electric actuator at top of the plate with excitation frequency 760 rad/s.

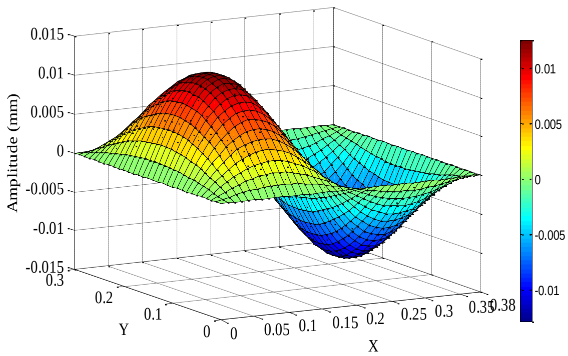

Fig. 14. Vibration profile of Kevlar/Epoxy plate excited by the piezoelectric actuator at right region of the plate with excitation frequency 980 rad/s.

6. Conclusions

This investigation has revealed that piezoelectric actuators can effectively use for shape control in the flexible structures. With the help of theory of elasticity and plate theory, the amplitude of vibration of a simply supported plate subjected to electrical voltages are obtained. This is done using analytical methods. Some concluding observations from the investigation are given below.

- The effect of size and location of the piezoelectric actuators on the amplitude of the plate and the effect of excitation frequency on vibration displacement, modal shapes are presented through a parametric study.
- It is observed that as the size of the actuator increases, the actuator is capable of controlling larger amplitudes, i.e., maximum amplitude controlled by the actuator increases.
- When the input frequency to the actuator is close to the resonant frequency, a mode can be excited provided that the actuator is properly located.
- It is concluded that the PZT-actuator with reverse polarity of tested size can be used as sensor to control the range of amplitudes predicted in the study.
- The PZT materials in the form of patch can be used to control the vibrations of thin walled structures. The size and the location of the patch can be decided based on the vibration characteristics obtain from the present study.
- The results obtained in the present study is very much helpful to design and develop a suitable control algorithms for a given PZT patch size and its location to control predetermined vibration of thin walled members (aerospace and automotive structures).
- Parametric study shows that the magnitude of input frequency and location significantly affect the ability of the piezoelectric actuators to excite certain modes.
- When the input frequency is close to the resonant frequency of a mode, the subsequent mode can be excited.
- Present study demonstrates the potential of controlling vibration in a plate structure using two dimensional patch type actuators.

References

1. Crawley, E.F.; and de Luis, C. (1987). Use of piezoelectric actuators as elements of intelligent structures. *AIAA Journal*, 25(10), 1373-1385.
2. Her, S.-C.; and Liu, C.Y. (2007). The deflection of a simply supported plate induced by piezoelectric actuators. *Journal of Mechanical Science and Technology*, 21, 1745-1751.
3. Her, S.-C.; and Lin, C.-S. (2013). Vibration analysis of composite laminate plate excited by piezoelectric actuators. *Journal of Sensors*, 13(3), 2997-3013.
4. Clark, R.L.; Flemming, M.R.; and Fuller, C.R. (1993). Piezoelectric actuators for distributed vibration excitation of thin plates: A comparison between theory and experiment. *ASME Journal*, 115(3), 332-339.

5. Dimitriadis, E.K.; Fuller, C.R.; and Rogers, C.A. (1991). Piezoelectric actuators for distributed vibration excitation of thin plates. *ASME Journal*. 113(1), 100-107.
6. Her, S.-C.; and Lin, C.-S. (2010). Deflection of cross-ply composite laminates induced by piezoelectric actuators. *Journal of Sensors*. 10(1), 719-733.
7. Marinkovic, D.; and Zehn, M. (2015). Finite element formulation for active composite laminates. *American Journal of Engineering and Applied Sciences*, 8(3), 328-335.
8. Wankhade, R.L.; and Bajoria, K.M. (2016). Shape control and vibration analysis of piezolaminated plates subjected to electro-mechanical loading. *Open Journal of Civil Engineering*, 6(3), 335-345.
9. Chennamsetti, R. (2015). Structural health monitoring of composite structures. *Journal of the Indian Institute of Science*, 95(3), 267-274.
10. Kędziora, P. (2013). Optimal design of PZT actuators and sensors in composite structural elements. *Key Engineering Materials,* 542, 59-73.
11. Hasanlu, M.; Bagheri, A.; and Najafi, F. (2016). Optimal placement of piezoelectric S/A for active vibration control of engineering structures by using controller design. *Research & Reviews: Journal of Engineering and Technology*, 5(4), 22-44.
12. Zhang, S.; Schmidt, R.; and Qin, X. (2015). Active vibration control of piezoelectric bonded smart structures using PID algorithm. *Chinese Journal of Aeronautics*, 28(1), 305-313.