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THE EFFICIENCY OF BOND GRAPH APPROACH FOR A FLEXIBLE WIND TURBINE MODELING

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Abstract

Wind turbine became an important source of energy, more advanced in these technology and structures, the complex system modelling with a high degree and flexible bodies by classical methods is a difficult task, the aim of this labour is to apply a graphical method that based on the principle of energy's conservation to develop a mathematical model of a large size wind turbine, that describes his dynamic's behaviour, which includes the flexibility of critical elements of machine as blade, tower and drivetrain. The model performed by decomposing the system on several sub-models and applying the bond graph process for each one; finally we interconnect all of them to build a global model. various simulation are performed to describe the dynamics and vibration behavior of the machine with real conditions and parameters, The efficiency of the bond graph method is approved by comparing these simulation results. With the results simulation of Lagrange's method, the comparison of the model's simulation shows that the results are very close, note that the modelling by bond graph is performed in less time and easier than another analytical method.

Keywords: wind turbine, bond graph, flexibility, energy, blade, dynamics, and non-linear model.

1. Introduction

In recent years, the demand in the wind energy has become very important, due to its competitiveness which is compared to conventional sources. The Wind turbines machines have become more advanced with a complex structure and more flexible. The system's control also became intelligently; but the flexibility of some elements has a large influence on the system's stability, which affects its control and the quality of produced energy. The negligence of this phenomenon during the modelling can be reflected negatively on modelling and system's control

Nomenclatures

a_i	Axial tangential induction factor
Bhs	Damper of high speed shaft, Nmsrad^{-1}
Bls	Damper of low speed shaft, Nmsrad^{-1}
Br	Damper of rotor, Nmsrad^{-1}
C	Compliance element
Cd_i	Drag coefficient of ith section
Cl_i	Lift coefficient of ith section
Ct	Stiffness of tower, N.m^{-1}
e	Effort
f	Flow
F_i	Force applied on ith element, N
Gy	Gyrator
I	Inertia element
It	Inertia of tower, kg.m^2
JG1	Inertia of gearbox, kg.m^2
JG2	Inertia of gearbox, kg.m^2
Jhss	Inertia of high speed shaft, kg.m^2
Jlss	Inertia of low speed shaft, kg.m^2
Khs	Stiffness of high speed shaft, N.m^{-1}
Kls	Stiffness of low speed shaft, N.m^{-1}
mt	Mass of Tower, kg
ng	Gear box ratio
P	Moment
q	Displacement
R	Damper element
Rt	Damper of tower, Nmsrad^{-1}
Se	Effort Source
Sf	Flow Source
TF	Transformer
V_w	Wind speed, m/s

Greek Symbols

ρ	Air density, kg/m^3
Φ	Wind inflow angle, rad
ω_r	Rotor speed, rad/s
α	Angle of attack, rad

Abbreviations

BEM	Blade Elements momentum
BG	Bond graph
FEM	Finite elements method
HWAT	Horizontal axis wind turbine
MGy	Modulate gyrator
Msf	Modulate source flow

conception. For this reason, many researchers aim to develop advanced model of wind turbines that introduce the flexibility of its critical elements, but the most of theme use classical methods that we find in the literature [1-4]. While works have been chosen the bond graph to modelling flexible structures [5-6] or complete structure of the wind turbine are rare [7].

The aim of this work is the modelling of a complete wind turbine, with taking account the flexibility of blades, drivetrain and tower by using a graphical approach called bond graph. This gives a unified way to modelling all the physical systems basing on the conservation energy law; Furthermore, a short introduction is presented in the first part on this paper, more details can be found bellow in [8].

This article is structured as the following explanation. First, it is a short presentation of the BG language elements and the rules that are presented. Second, we applied the BG for the Modeling of several elements for the wind turbine, starting by the flexible blade drivetrain and the tower. These elements are assembled between them for building the complete model. Once it obtained, a simulation of the behaviors of the different element was made by MATLAB software. Hence, the results obtained by this method are compared and approved with a conventional method [9].

2. Introduction to the bond graph

The BG is a graphical approach to describe the dynamics behaviours of all physical systems (mechanical, electrical, thermodynamic) by a unified approach, they are described in the same way, the principle of this method is based on the flow of energy that goes through the several domain of the system and the exchange of energy between them. The energy flow in any physical system is always governed by simultaneous intervention of two independent parameters, in the BG method these two parameters are defined by the general terms of Effort e and Flow f. The power of the instantaneous energy flow is a product of these two factors.

2.1. Bond graph concept

The basic concept in BG method is to specify the flow energy in a system.

The energy flow in all systems is governed by two independent parameters, they are defined in the BG method by general term of effort "e" and flow "f", and this two parameters produce the power of the energy flow.

$$p(t) = e(t).f(t) \tag{1}$$

Momentum (p) and Displacement (q) are defined as:

$$p = \int e(t)dt \tag{2}$$

$$q = \int f(t)dt \tag{3}$$

Therefore Energy (E) from Eq. (1) is

$$E = \int e.fdt = \int f(t)dp = \int e(t)dq \tag{4}$$

In the BG, the exchange of energy is done via energy ports. Should an element exchange energy in a single way that element is called a one-port element. A multi-port element, therefore, it is an element that exchanges energy in different ways.

2.2. Bond graph elements language

2.2.1. The passive elements

The elements R, C and I are named passive elements because they transform received power into dissipated power in heat form "element R" or stocked "element I and C". So the power is transferred to the elements, which are expressing the direction of energy flow by half-arrows, and makes a connection between the different parts of a system and with its environment, Fig. 1 shows the bond graph passive elements.

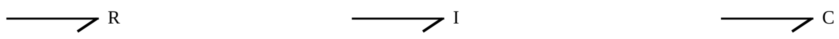


Fig. 1. Bond graph passives components.

2.2.2. The active elements

All systems require an energy source to be operated. In Bond Graph, these sources are expressed by "Se" and "Sf" represent respectively the effort source and the flow source. Taking for example a rotating shaft, the Torque represents the effort source and the rotational speed represents the flow source, Fig. 2 shows the bond graph sources elements.

Fig. 2. Bond graph actives components.

2.2.3. Junctions

A system contains multiple components, in bond graph the connection is made by two types of junction, a common flow junction that represented by "1" and common effort junction that represented by "0"

The 0 junction has the following properties: all bonds impinging upon it have the same effort variable and all flows on attached bonds sum to zero. Similarly the 1 junction has the properties: all bonds impinging upon it have the same flow variable and all effort on attached bonds sum to zero.

In the physical systems the transferred efforts and flows between physical domains can be multiply. This ability must be included in the bond graph approach, for that two means for accomplishing this; the transformer TF and the Gyrator Gy, its presentation and functions are shown in Fig. 3.

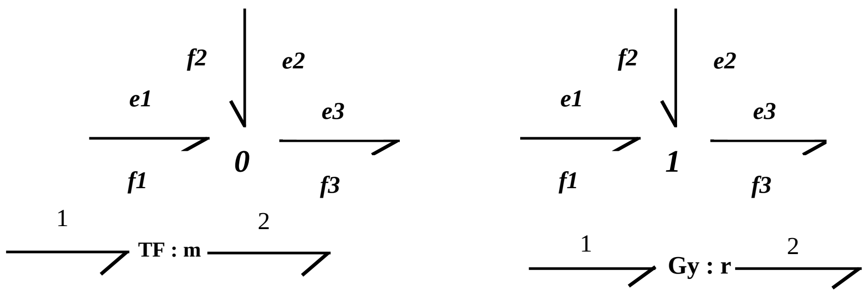


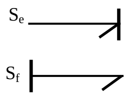
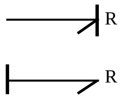
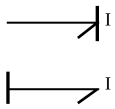
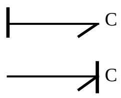
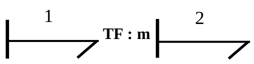
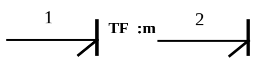
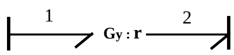
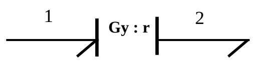
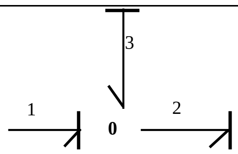
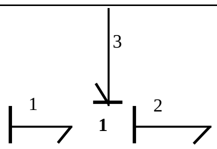
Fig. 3. Illustration of junctions.

2.2.3. The causality assignment rules

Bond graphs have a notion of causality, indicating which side of a bond determines the instantaneous effort and which determines the instantaneous flow. In formulating the dynamic equations that describe the system, causality defines, for each modelling element, which variable is dependent and which is independent.

Table 1 shows the permitted causality permutations for components, junctions and transformers respectively [10].

Table 1. Causality stroke and Assignments for bond graph.

Causal form	Causal relation	Type
	$e = S_e$ $f = S_f$	Fixed Causality
	$f = \frac{e}{R}$ $e = R * f$	Resistor Conductivity
	$f = \frac{1}{I}\int e\,dt$ $e = I\frac{df}{dt}$	Integral Derived
	$e = \frac{1}{C}\int f\,dt$ $f = I\frac{de}{dt}$	Integral Derived
	$e_1 = m * e_2$; $f_2 = m * f_1$	Symmetric
	$e_2 = \frac{e_1}{m}$; $f_1 = \frac{f_2}{m}$	Symmetric
	$e_1 = r * f_2$; $e_2 = r * f_1$	Antisymmetric
	$f_2 = \frac{e_1}{r}$; $f_2 = \frac{e_2}{r}$	Antisymmetric
	$e_2 = e_3 = e_1$ $f_1 = f_2 + f_3$	One effort is imposed on the junction 0
	$f_2 = f_3 = f_1$ $e_1 = e_2 + e_3$	One flow is imposed on the junction 1

3. Flexible Wind turbine Modelling

This labour focus on the modelling of a large size of horizontal axis wind turbine, many components of this machine have a flexible structure, for example the blades have high flexibility, the choice of flexible materials in its construction increase the life time of them, but it can affect some other performance like stability of control system, the including of this phenomena in the modelling phase is very important to describe the real behaviour of the wind turbine. The proposed model have a reduced degree of freedom, it includes the flexibility of the critical components of the machine, the flap wise of blades, tower vibration and the torsion of the drivetrain, The wind turbine is clearly a complex multibody assembly of bodies that can be assumed to be rigid while others must be considered flexible [11], as shown in Fig. 4, the modelling of wind turbine requires some simplifying assumptions which are:

- The model is assumed as multibodys system, with one dimensional beam,
- The system is subdivided into subsystems, which are modeled independently according to their appropriate approach, in order to group them in one model.
- The number of degrees of freedom is reduced to the extent not affect the system behavior and not neglects the flexibility of different elements
- The way rotation is neglected.
- The number of degrees of freedom is reduced to the extent that not affected system behavior and not neglect the flexibility of different elements.
- The aerodynamic model is independently treated by the adaptation of the BEM method with the bond graph method in order to couple it by the structural model.
- The incident wind is assumed to be uniform with the rotor plane.

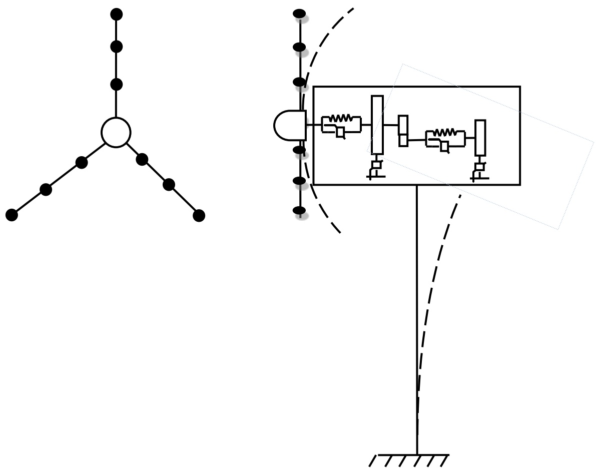


Fig. 4. Structural model of wind turbine.

The first step in the BG modelling process is building of a simplified BG model, called word BG, this first level of BG describes the general view of the system and the energetic flow that goes through different components and the interaction between them. The word BG of our HWAT is presented by Fig. 5.

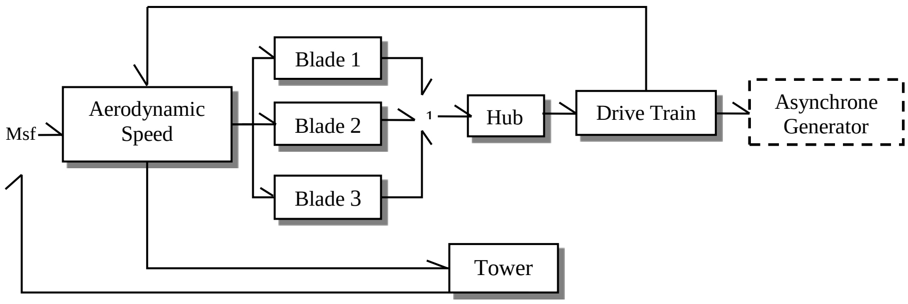


Fig. 5. Word BG model of wind turbine.

4. Wind turbine rotor modelling

4.1. Blade modelling

wind turbines blades are classified among the high flexible structures, in this work, it introduces the aero-elasticity phenomenon in our model by coupling two models, an aerodynamic model that calculates the aerodynamics forces applied to the blades by using the blade elements momentum BEM method, and a structural model that describes the dynamic behavior, it's developed by the BG method and it's based on the beam model of Euler Bernoulli. In the structural modelling some assumptions are made:

The blade segments are subject to the assumption of Euler-Bernoulli. Euler Bernoulli method used to modelled beams with small deformation, but the blades has a height deformations, for this raison , the flexible blade is partitioned into three sections with different cross sections and lengths , the first represent the blade hub, the second is the body section and the third part is the blade tip.

- All beams elements are considered as uniform and one dimensional beam.
- The aerodynamic model is subject to the BEM method assumptions.
- There is no interaction between each blade element
- The forces applied on the blade elements by the flow stream are determined locally by the two-dimensional lift and drag characteristics of the airfoil shape.

Figure 6 shows the coupling schema of two models.

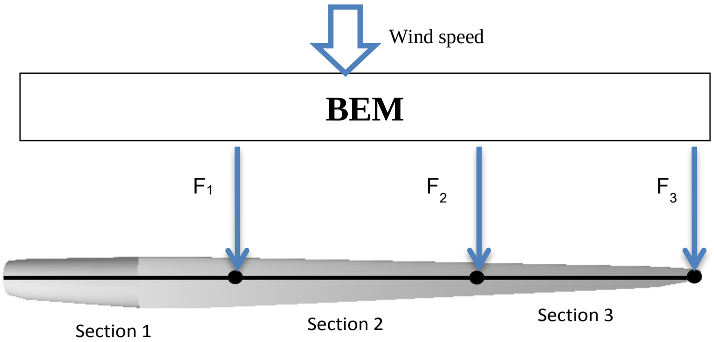


Fig. 6. Coupled model of flexible blade.

4.2. Dynamic model

The structural model describes the dynamic behaviour and the vibration of the blade; different methods are used for the modelling of flexible blades [12, 13]. the finites elements method is among the most used methods in the reformulation of flexible bodies, but the adaptation of the BG method with this approach will be difficult because of the high numbers of degrees of freedom, the multibody approaches will be adequate to our needs, by using a lumped parameter approach [5] it can approximate a flexible body using a series of rigid bodies connected by springs and dampers, The material properties and the cross section of the flexible body determine the spring stiffness and damping coefficients, and beams of sections are presented by Euler-Bernoulli beam, and the reformulation of dynamic model by the BG process [8], Fig. 7 shows the proposed discretization of the blade.

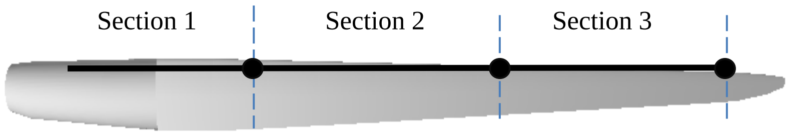


Fig. 7. Discretization of the blade structure.

By the application of the BG process of one dimension, and consideration the blade model proposed previously, which represents by three elements of different sections, one can draw the BG model of the blade that presented in Fig. 8.

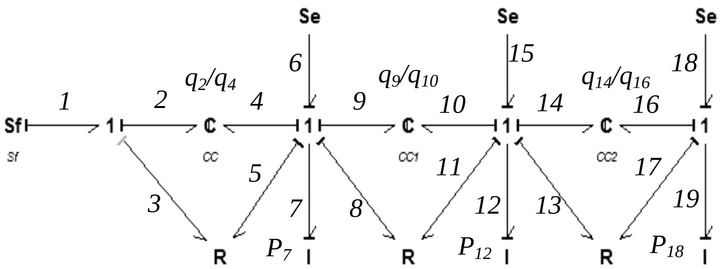


Fig. 8. BG model of flexible blade.

The connection between the hub and the blade is assumed rigid, so the boundary condition will be zero, this means S_f = 0, the aerodynamics forces are represented by "S_{e1}", "S_{e2}", and, "C_i" "R_i"and "I_i" represents respectively the stiffness, damping and the masses of each elements of blade.

Dynamic equations

To get the dynamic equations of blade from the causal BG model, it has followed the BG process [8]; firstly, it determines the equation corresponding to each junction and each element as presented in the Table 2.

Table 2. Equations associated to junctions of blade model.

Junction "1"	Elements Junction
$"1"\begin{cases} f_1 = f_2 = f_3 \\ e_1 - e_2 - e_3 = 0 \end{cases}$	$S_f : f_1 = S_f$ $S_{e1} : e_6 = S_{e1}$ $S_{e2} : e_{15} = S_{e2}$ $S_{e3} : e_{19} = Se_3$
$"1"\begin{cases} f_4 = f_5 = f_6 = f_7 = f_8 = f_9 \\ e_6 - e_4 - e_5 - e_7 - e_8 - e_9 = 0 \end{cases}$	$R_1 : \begin{cases} e_3 = R_1 f_3 \\ e_5 = R_1 f_5 \end{cases}$ $R_2 : \begin{cases} e_8 = R_2 f_8 \\ e_{11} = R_2 f_{11} \end{cases}$ $R_3 : \begin{cases} e_{13} = R_3 f_{13} \\ e_7 = R_3 f_{17} \end{cases}$
$"1"\begin{cases} f_{10} = f_{11} = f_{12} = f_{13} = f_{14} = f_{15} \\ e_{15} - e_{10} - e_{11} - e_{12} - e_{13} - e_{14} - e_{15} = 0 \end{cases}$	$I_1 : f_7 = \frac{1}{I_1} P_7$ $I_2 : f_{12} = \frac{1}{I_2} P_{12}$ $I_3 : f_{18} = \frac{1}{I_3} P_{18}$
$"1"\begin{cases} f_{16} = f_{17} = f_{18} = f_{19} \\ e_{19} - e_{16} - e_{17} - e_{18} = 0 \end{cases}$	$C_1 : \begin{cases} e_2 = \frac{1}{C_1} q_2 \\ e_4 = \frac{1}{C_1} q_4 \end{cases}$ $C_2 : \begin{cases} e_9 = \frac{1}{C_2} q_9 \\ e_{10} = \frac{1}{C_2} q_{10} \end{cases}$ $C_3 : \begin{cases} e_{14} = \frac{1}{C_3} q_{14} \\ e_{16} = \frac{1}{C_3} q_{16} \end{cases}$

The equations of generalized coordinate can be formulated from the BG model and the junction equations:

$\left[\dot P_7, \dot P_{12}, \dot P_{18}, \dot q_2, \dot q_4, \dot q_9, \dot q_{16}, \dot q_{14}\right]$

With q and p present respectively the displacement and the moment.

$$\dot P_7 = Se_1 - \frac{1}{C_1} q_4 - R_1 \frac{1}{I_1} P_7 - R_2 \frac{1}{I_1} P - \frac{1}{C_2} q_9 \text{ With } q_4 = q_9 \tag{4}$$

$$\dot P_{12} = Se_2 - \frac{1}{C_2} q_{10} - R_2 \frac{1}{I_2} P_{12} - R_3 \frac{1}{I_2} P_{12} - \frac{1}{C_3} q_{14} \tag{5}$$

$$\dot P_{18} = Se_3 - \frac{1}{C_3} q_{16} - R_3 \frac{1}{I_3} P_{18} \tag{6}$$

$$\dot{q}_4 = f_4 = f_7 = \frac{1}{I_1} P_7 \tag{7}$$

$$\dot{q}_2 = f_2 = S_f \tag{8}$$

$$\dot{q}_9 = f_9 = f_7 = \frac{1}{I_1} P_7 \tag{9}$$

$$q_{10} = f_{10} = f_{12} = \frac{1}{I_2} P_{12} \tag{10}$$

$$q_{14} = e_{14} = f_{14} = f_{12} = \frac{1}{I_2} P_{12} \tag{11}$$

$$\dot{q}_{16} = f_{16} = f_{18} = \frac{1}{I_3} P_{18} \tag{12}$$

From this equations it find the state equation of the blade:

$$\begin{bmatrix} \dot{p}_7 \\ \dot{p}_{12} \\ \dot{p}_{18} \\ \dot{q}_9 \\ \dot{q}_{14} \\ \dot{q}_{16} \end{bmatrix} = \begin{bmatrix} -\frac{R_1}{I_1} & 0 & 0 & -\left(\frac{1}{c_1}+\frac{1}{c_2}\right) & 0 & 0 \\ 0 & -\left(\frac{R_2}{I_2}+\frac{R_3}{I_3}\right) & 0 & 0 & -\left(\frac{1}{c_2}+\frac{1}{c_3}\right) & 0 \\ 0 & 0 & -\frac{R_3}{I_3} & 0 & 0 & -\frac{1}{c_3} \\ \frac{1}{I_1} & 0 & 0 & 0 & 0 & 0 \\ 0 & \frac{1}{I_2} & 0 & 0 & 0 & 0 \\ 0 & 0 & \frac{1}{I_3} & 0 & 0 & 0 \end{bmatrix} \begin{bmatrix} P_7 \\ P_{12} \\ P_{18} \\ q_9 \\ q_{14} \\ q_{16} \end{bmatrix} + \begin{bmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 1 \\ 0 & 0 & 0 \\ 0 & 0 & 0 \\ 0 & 0 & 0 \end{bmatrix} \begin{bmatrix} S_{e1} \\ S_{e2} \\ S_{e3} \end{bmatrix} \tag{13}$$

4.3. Aerodynamic model

The aerodynamic model is responsible to define the aerodynamic forces applied on the surfaces of blades. it predicts the loads from the wind speed and rotational speed of rotor thus the blade position and its air foils geometry, several methods can be found in bibliography that can be used to build an aerodynamic model [14], in our work the loads are calculated by using the blade Elements momentum method (BEM), that it formulates by the BG language, to be compatible for coupling with the structural model.

By applying this method it can find the aerodynamics loads applied for each section, the Eq. (14) present the forces applied on the surfaces of the i[th] section.

$$dF_i = 2\rho u(V - V_w)dA = 4\pi\rho V_w^2(1-a_i)rdr \tag{14}$$

It can present also as the following expression:

$$dF_i = \sigma\pi\rho \frac{V_w^2(1-a_i)^2}{\sin^2(\phi)}(c_{li}\cos\phi_i - c_{di}\sin\phi_i)rdr \tag{15}$$

where "V_w"represents the wind velocity, "φ" the air density, "Φ_i"the wind inflow angle represented by Eq. (16), "C_{li}"and "C_{di}"are the lift and drag coefficients, it depends of the geometry of the blades airfoil, for this study, it uses the S809 airfoil.

$$\Phi_I = \tan^{-1}\left(\frac{V_w\left(1-a_i\right)}{\omega_r r_i\left(1+a_i^{'}\right)}\right) \tag{16}$$

"a_i" and "a_i'"Represent respectively the axial and the tangential induction factor.

By equalled the Eqs. (14) and (15), it can obtain the following equation;

$$\frac{a}{1-a} = \sigma c_l \frac{\cos\phi}{4\sin^2\phi}\left(1+\frac{c_l}{c_d}\tan\phi\right) \tag{17}$$

By solving of the Eq. (16) it obtains:

$$a_i = \left(1+\frac{4\sin^2\Phi_i}{\sigma_{ii}^{'}\left(C_{li}\cos\Phi_i + C_{di}\sin\Phi_i\right)}\right)^{-1} \tag{16}$$

By rearranging the Eqs. (15) and (16) it obtains:

$$a_i^{'} = \left(-1+\frac{4\sin^2\Phi_i}{\sigma_i^{'}(C_{li}\cos\Phi_i - C_{di}\sin\Phi_i}\right)^{-1} \tag{17}$$

The parameters of equations are presented graphically in Fig. 9

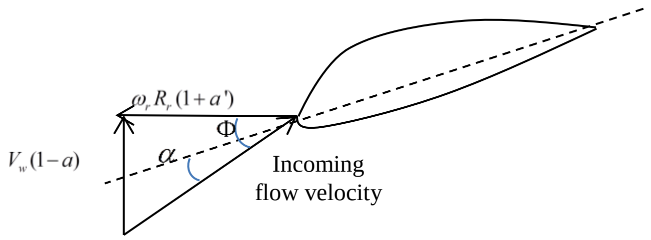


Fig. 9. Airfoil section and parameters.

Figure 10 represents the BG reformulation of the aerodynamic model, a modulated gyrator element (MGY) is used to implement Eq. (11), since wind source (MSf) is transformed into a Se source (aerodynamic force).

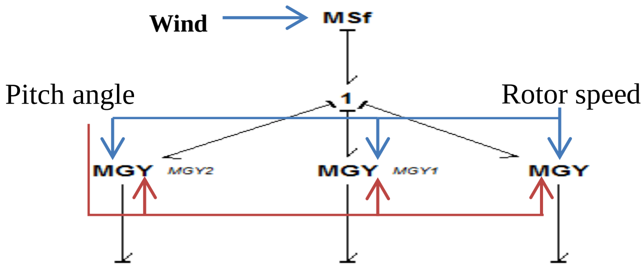


Fig. 10. BG of aerodynamic model.

5. Drive Train Model

This part of system is responsible for the transmission of energy captured by the rotor to the generator machine; it consists of rotary shafts and a speed multiplier. Shafts begets vibration frequency during operation, it can excite some resonances frequencies, which affects the control system, the taking account of this phenomenon is necessary in the modelling, the transmission part is represented by a simplified drivetrain that assumed with three masses, the system have two principals shift, this masses represent respectively the low speed shaft inertia noted "J_{lss}", the gear box inertias noted "J_{G1}"and "J_{G2}", and "J_{hss}" the inertia of the height speed shaft, the flexibility of shafts is represented by a spring-damper couple noted respectively by (B_{ls} , K_{ls}) and (B_{hs} , K_{hs}), "B_r", "R_g"and "B_g"present the friction coefficients of the rotational shifts, gearbox and the generator.

The driveline presented in Fig. 11 was translated to the bond graph as illustrated in Fig. 12, the inputs are the aerodynamic torque captured by the rotor and the electromagnetic torque of the generator, it presented respectively by a source effort "Se1" and "Se2", the inertia and damping elements are connected to the constant flow "1" junction, and the stiffness were attached to the constant effort "0" junction, the gearbox ratio were represented by transformer "TF".

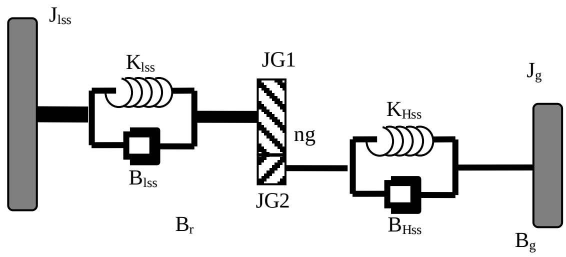


Fig. 11. BG model of drivetrain.

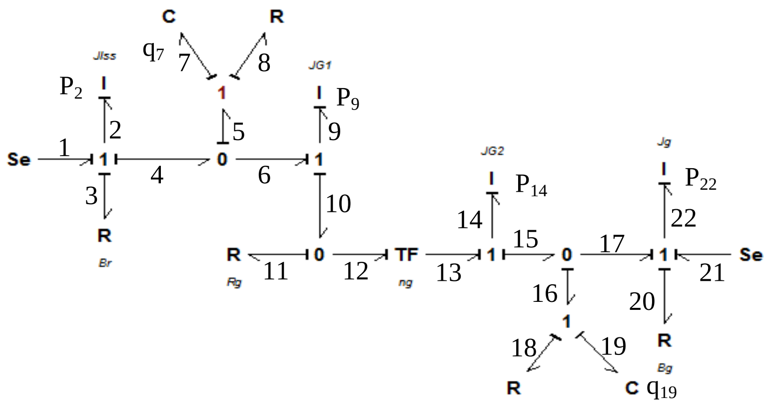


Fig. 12. BG model of drivetrain.

Dynamic equations:

Table 3 gives the equations of each junction and element of drivetrain Bond graph

Table 3. Equation associated to junctions.

Junction "1"	Junction "0"	Transformer	Elements Junction
$"1"\begin{cases} f_1 = f_2 = f_3 = f_4 \\ e_1 - e_2 - e_3 - e_4 = 0 \end{cases}$	$"0"\begin{cases} e_4 = e_5 = e_6 \\ f_4 - f_5 - f_6 = 0 \end{cases}$	$"TF : n_g"\begin{cases} e_{13} = \frac{1}{n_g} e_{12} \\ f_{12} = \frac{1}{n_g} f_{13} \end{cases}$	$"S_e":\begin{cases} T_a : e_1 = S_{e1} \\ T_{em} : e_{21} = S_{e2} \end{cases}$
$"1"\begin{cases} f_5 = f_7 = f_8 \\ e_5 - e_7 - e_8 = 0 \end{cases}$	$"0"\begin{cases} e_{10} = e_{12} = e_{12} \\ f_{10} - f_{11} - f_{12} = 0 \end{cases}$		$"I":\begin{cases} J_{ss} : f_2 = \frac{1}{Jss} P_2 \\ J_{G1} : f_9 = \frac{1}{J_{G1}} P_9 \\ J_{G2} = f_{14} = \frac{1}{J_{G2}} P_{14} \\ J_g : f_{22} = \frac{1}{J_g} P_{22} \end{cases}$
$"1"\begin{cases} f_{13} = f_{14} = f_{15} \\ e_{13} - e_{14} - e_{15} = 0 \end{cases}$	$"0"\begin{cases} e_{15} = e_{16} = e_{17} \\ f_{15} - f_{16} - f_{17} = 0 \end{cases}$		$"I":\begin{cases} J_{ss} : f_2 = \frac{1}{Jss} P_2 \\ J_{G1} : f_9 = \frac{1}{J_{G1}} P_9 \\ J_{G2} = f_{14} = \frac{1}{J_{G2}} P_{14} \\ J_g : f_{22} = \frac{1}{J_g} P_{22} \end{cases}$
$"1"\begin{cases} f_{16} = f_{18} = f_{19} \\ e_{16} - e_{18} - e_{19} = 0 \end{cases}$			$"R":\begin{cases} Br : e_3 = B_r f_3 \\ B_g : e_{20} B_g f_{20} \\ B_{ls} : e_8 = R_1 f_8 \\ B_{hs} : e_{18} = R_2 f_{18} \\ R_g : e_{11} = R_g f11 \end{cases}$
$"1"\begin{cases} f_{17} = f_{20} = f_{21} = f_{22} \\ e_{17} - e_{20} - e_{21} - e_{22} = 0 \end{cases}$			$"C":\begin{cases} k_{ls} : e_7 = \frac{1}{k_{ls}} q_7 \\ k_{hs} : e_{19} = \frac{1}{k_{hs}} q_{19} \end{cases}$
$"1"\begin{cases} f_6 = f_9 = f_{10} \\ e_6 - e_9 - e_{10} = 0 \end{cases}$			

Expressions of variables stats equations:

By applying the BG process, it defines the states variables that are presented by the following equation:

$$\dot{P}_2 = T_a - \left[+ \frac{B_r + B_{ls}}{J_{ls}} \right] P_2 - \frac{1}{C_1} q_7 + \frac{k_{ls}}{J_{G1}} P_9 \tag{19}$$

$$\dot{P}_9 = \frac{1}{C_1} q_7 + \frac{B_{ls}}{J_{ls}} P_2 - \frac{B_{ls}}{J_{G1}} P_9 - \frac{B_g}{J_{G1}} P_9 + \frac{B_g}{n_g J_{G2}} P_{14} \tag{20}$$

$$\dot{P}_{22} = \left(\frac{R_2}{JG_2} P_{14} - \frac{R_2}{J_g} P_{22} + \frac{1}{C_2} q_{19} \right) + T_{em} - \frac{B_g}{J_g} P_{22} \tag{21}$$

$$\dot{q}_{14} = \frac{R_g}{n_g J_{G1}} P_9 - \frac{R_g}{n_g J_{G2}} P_{14} - \frac{1}{C_2} q_{19} - \frac{R_2}{J_g} P_{22} \tag{22}$$

$$\dot{q}_7 = \frac{1}{J_{ls}} P_{14} + \frac{R_2}{J_{G1}} P_9 \tag{23}$$

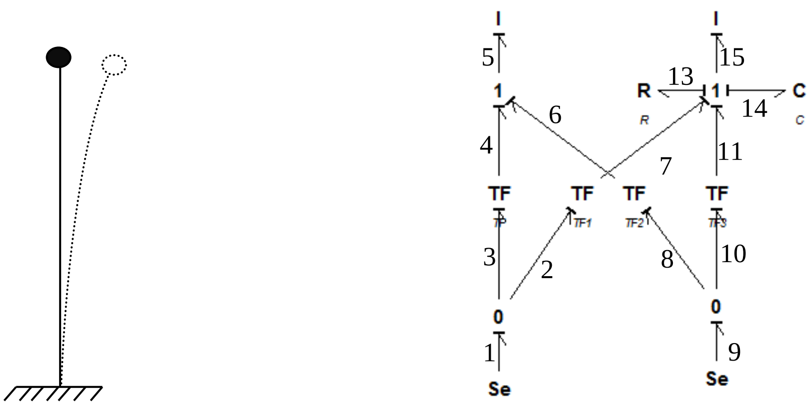
$$\dot{q}_{12} = \frac{1}{J_{JG2}} P_{14} - \frac{1}{J_g} P_{22} \tag{24}$$

Finally, the state equation becomes:

$$\begin{bmatrix} \dot{p}_2 \\ \dot{p}_9 \\ \dot{p}_{22} \\ \dot{P}_{14} \\ \dot{q}_7 \\ \dot{q}_{19} \end{bmatrix} = \begin{bmatrix} -\frac{B_r - R_1}{J_{ls}} & \frac{R_1}{JG_1} & 0 & 0 & -\frac{1}{c_1} & 0 \\ \frac{R_1}{J_{ls}} & -\frac{(R_1 + R_g)}{JG_1} & 0 & \frac{R_g}{n_g J_{G2}} & \frac{1}{c_1} & 0 \\ 0 & 0 & -\frac{R_2 - B_g}{J_g} & \frac{R_2}{JG_2} & 0 & \frac{1}{c_2} \\ 0 & \frac{R_g}{n_g J_{G1}} & \frac{R_2}{J_g} & -\frac{R_g}{n_g{}^2 J_{G2}} - \frac{R_2}{JG_2} & 0 & -\frac{1}{c_2} \\ \frac{1}{J_{ls}} & \frac{1}{J_{G1}} & 0 & 0 & 0 & 0 \\ 0 & 0 & -\frac{1}{J_g} & \frac{1}{J_{G2}} & 0 & 0 \end{bmatrix} \begin{bmatrix} P_2 \\ P_9 \\ P_{22} \\ P_{14} \\ q_7 \\ q_{19} \end{bmatrix} + \begin{bmatrix} 1 & 0 \\ 0 & 1 \\ 0 & 0 \\ 0 & 0 \\ 0 & 0 \\ 0 & 0 \end{bmatrix} \begin{bmatrix} T_a \\ T_{em} \end{bmatrix} \tag{25}$$

6. Tower motion model

The HWAT tower is submitted to a low frequency vibration compared with blades vibration; this vibration does not have an important influence on mechanical system, but it can affect the aerodynamics load by affecting the input parameters. In our model presented in Fig. 13, the tower is presented as a uniform beam and the deformation is presented as the first mode of oscillations.



(a) Representation of the tower **(b) BG model**

Fig. 13. Tower model.

Dynamic equations:

The same way, in Table 4 it determines the equations associated with junctions and elements.

Table 4. Equation associated to junctions for tower model.

Junction “1”	Junction “0”	Transformer	Elements Junction
$"1"\begin{cases} f_4 = f_5 = f_6 \\ e_4 + e_6 - e_5 = 0 \end{cases}$	$"0"\begin{cases} e_1 = e_{12} = e_3 \\ f_1 - f_2 - f_3 = 0 \end{cases}$	$"TF_1 : m_1"\begin{cases} e_4 = \frac{1}{m_1} e_3 \\ f_3 = \frac{1}{m_1} f_4 \end{cases}$	$"S_e":\begin{cases} T_a : e_1 = S_{e1} \\ T_a : e_2 = S_{e2} \end{cases}$
$"1"\begin{cases} f_{11} = f_{12} = f_{13} = f_{14} \\ e_{11} - e_{12} - e_{13} - e_{14} = 0 \end{cases}$	$"0"\begin{cases} e_8 = e_9 = e_{10} \\ f_9 - f_8 - f_{10} = 0 \end{cases}$	$"TF_2 : m_2"\begin{cases} e_7 = \frac{1}{m_2} e_2 \\ f_2 = \frac{1}{m_2} f_7 \end{cases}$	$"I":\begin{cases} I_1 : f_5 = \frac{1}{I_1} P_5 \\ I_1 : f_{14} = \frac{1}{I_2} P_{14} \end{cases}$
			$"R": e_{13} = R.f_{13}$
			$"R": e_{12} = \frac{1}{c} q_{12}$

From the previous equations it determines the equations of generalized tower condones it obtains;

$$\dot{P}_5 = e_5 = e_4 + e_6 = \frac{1}{m_1} Se_1 + \frac{1}{m_3} Se_2 \tag{26}$$

$$\dot{P}_{14} = \frac{1}{m_4} Se_2 - \frac{1}{C} q_{12} - \frac{R}{I_2} P_{14} \tag{27}$$

$$\dot{q}_{12} = f_{12} = f_{14} = \frac{1}{I_2} P_{14} \tag{28}$$

From these equations it can build the state equation of the tower model;

$$\begin{bmatrix} \dot{p}_5 \\ \dot{p}_{14} \\ \dot{q}_{12} \end{bmatrix} = \begin{bmatrix} 0 & 0 & 0 \\ 0 & -\frac{R}{I_2} & -\frac{1}{C} \\ 0 & \frac{1}{I_2} & 0 \end{bmatrix} \begin{bmatrix} p_5 \\ p_{14} \\ q_{12} \end{bmatrix} + \begin{bmatrix} \frac{1}{m_1} & \frac{1}{m_2} \\ 0 & \frac{1}{m_4} \\ 0 & 0 \end{bmatrix} \begin{bmatrix} Se_1 \\ Se_2 \end{bmatrix} \tag{29}$$

7. Assemblages Model

The behavior of each element is influenced by the other’s behaviour, to have a complete model description of the wind turbine; the assembly of all elements models is necessary.

The elements models presented in previous sections are assembled as it’s shown in Fig. 14.

The BG causal model is considered as a structural model and we synthesis this last one by taking into account the control criteria (controllability and observability).

In terms of BGs, the notions of input reachable and output reachable are expressed as the existence of causal paths between dynamical elements (I, C in integral causality) and sources and detectors

Theorem 1: A system is structurally state controllable if all dynamical elements (I, C) in integral causality are causally connected with a source (Se, Sf).

Theorem 2: A system is structurally state observable if all dynamical elements (I, C) in integral causality are causally connected with a detector (De, Df).

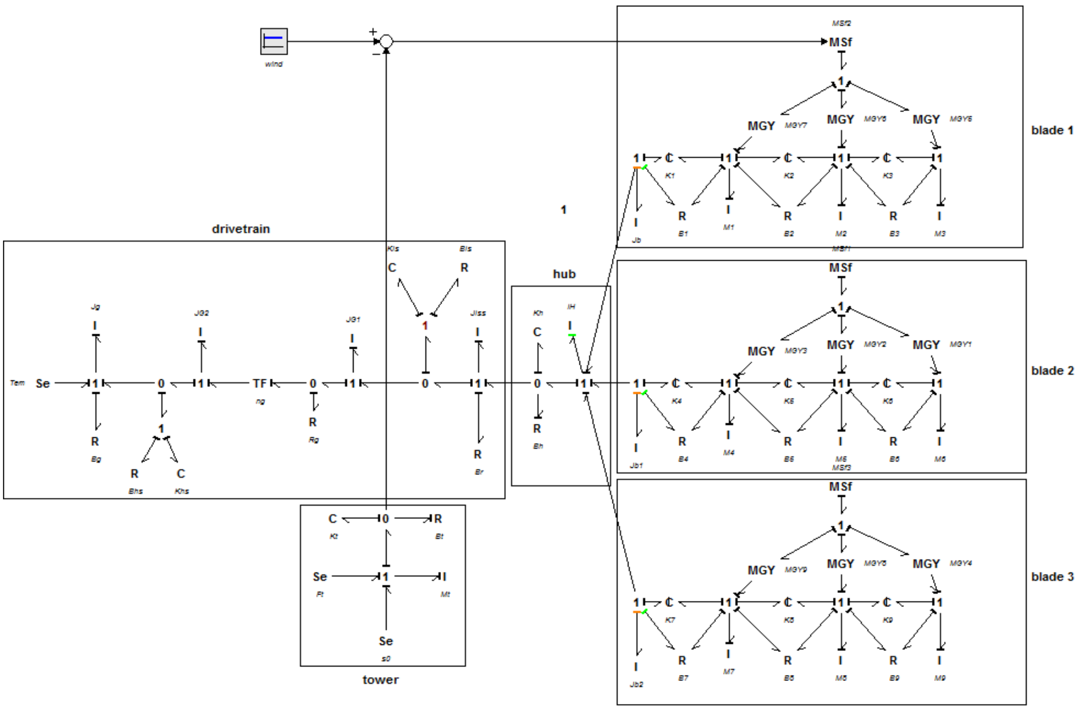


Fig. 14. Assemblage BGs of wind turbine.

8. Simulation Results and Discussion

The results presented in this section are obtained by the simulation of dynamic equations of both models by Matlab software. The two models are simulated in the same conditions and the same parameters. Table 5 present the parameters and conditions of simulation.

8.1. Blade behavior

Figures 15, 16 and 17 present the responses of the three blade elements, when applying the pulse on these items. Note that the three elements behave as a uniform beam in a free vibration.

The simulation of the movement equation of blade obtained by the BG method is similar to the behavior of the simulation of equation obtained by other methods in several studies in the literature.

Table 5. Simulation parameters of wind turbine.

Description	Value
Blade parameters	
Inertia of the blade	7.5.107Kg.m^2
Mass of the blade	5250Kg
Stiffness of the blade	1.2738.109N.m^{-1}
Damping of the blade	25.69.105 Nmsrad^{-1}
Length of the nacelle	3.3m
Tower parameters	
Mass of the tower	1.6547e10^{6}Kg
Inertia of the tower	8.10^{9}Kg.m²
Stiffness of the tower	2.55.10^{10} N.m^{-1}
Damping of the tower	66.710^{5}Nmsrad^{-1}
Drivetrain parameters	
Rotor Inertia	55.10^{6}K.m²
Generator Inertia	390Kg.m²
Stiffness of the shaft	2.710^{9}N.m^{-1}
Damping of the shaft	945.10^3 Nmsrad^{-1}
Rotor external damping	34600 Nmsrad^{-1}
Generator external damping	3.034 Nmsrad^{-1}
Gearbox ratio	857

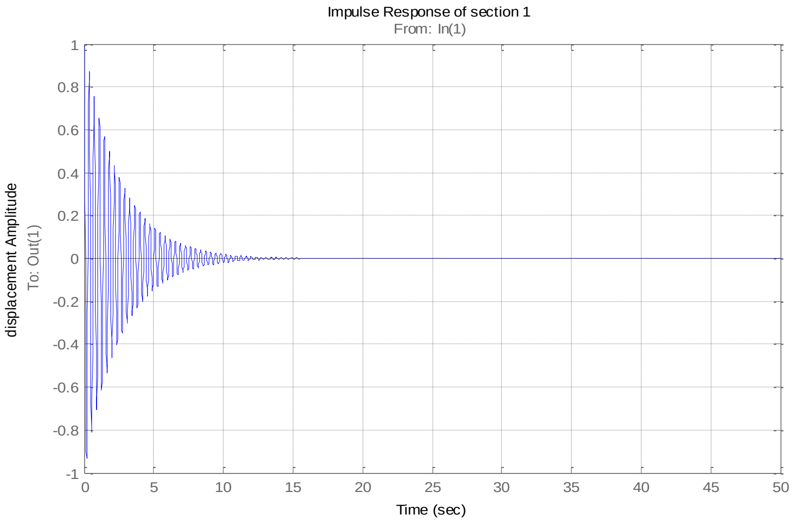


Fig. 15. Vibration of element 1 of blade.

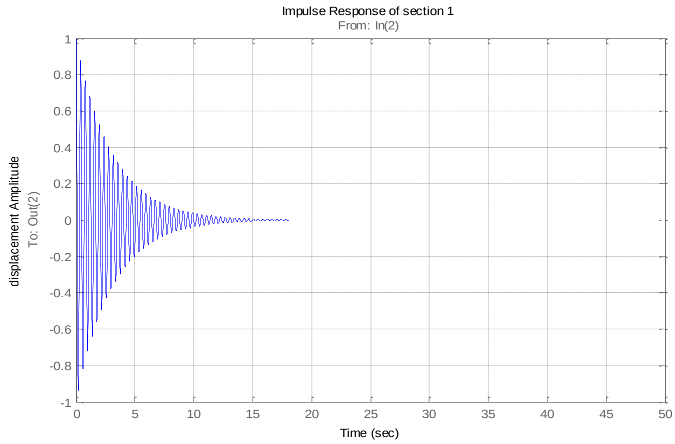


Fig. 16. Vibration of element 2 of blade.

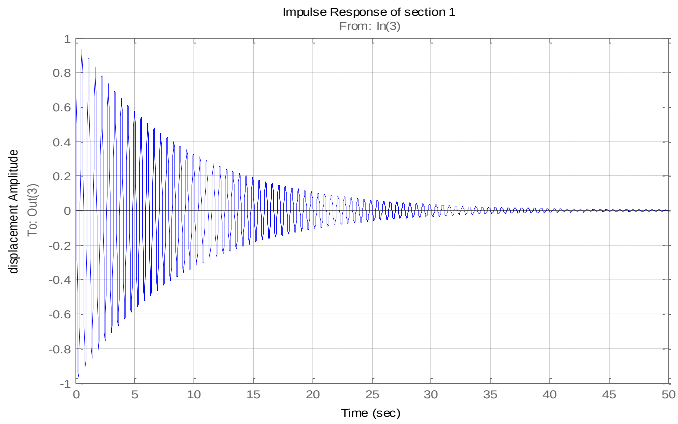


Fig. 17. Vibration of element 3 of blade.

8.2. Drivetrain simulation

Figure 18 shows a comparison between two generator speeds curves obtained by two different methods; it notes that the BG gives a very close result to the result obtained by the Lagrange method applied in previous work [9]. The same thing for the other variables in the shaft transmission, Fig. 19 shows the evolution of torque of the shaft slow speed obtained by the two methods.

8.3. Tower simulation

The tower is also a flexible element which acts also as a non-uniform beam, but with less influence on the systems. Figure 20 shows the vibration of tower, as a result, it notices that the frequency vibration is lower than the blades. It returns to the geometric shape and nature of the materials with which it is built. Note that the model found with the BG method is the same as the model developed by Lagrange's method in the previous work [9].

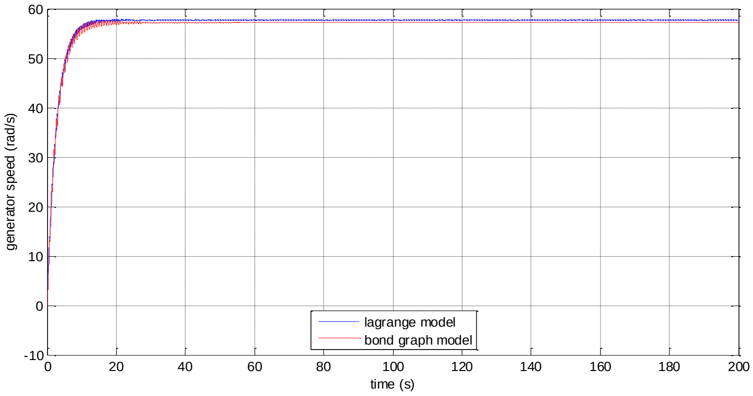


Fig. 18. Generator speed Comparison of BG and Lagrange methods.

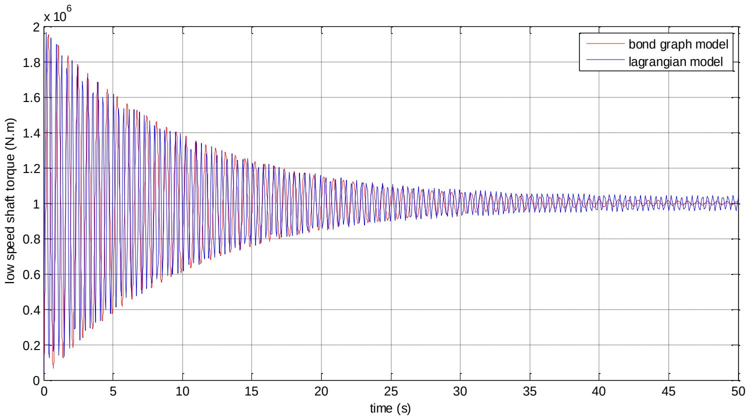


Fig. 19. Low speed shaft torque of BG and Lagrange method.

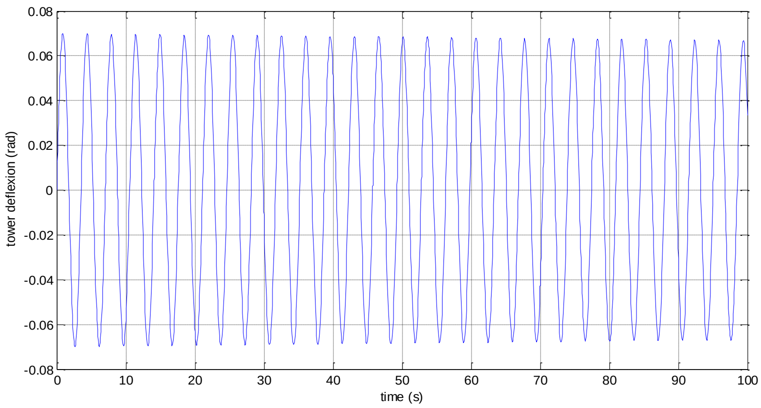


Fig. 20. After plan bending of tower.

9. Conclusions

This work shown the efficiency of the BG approach to modelling the flexible wind turbine, the proposed model 0includes the flexibility of critical elements of wind turbine, it introduced the flexibility of, shafts, blades , and the tower, the elements of system are modelled independent that it finally got an assembled model that represent the behavior of wholes. The system behavior is simulated under nominal condition. Every time it adds more degrees of freedom in our system, the complexity of system increases, then its modeling becomes more difficult if it uses a conventional method.

BG is an efficient method for modeling of rigid and flexible multibodys systems, by simulation; it compares the behavior of our model with another model obtained by classical method (Lagrange's method). It notes that the results obtained are largely similar. And this method provides modeling ease in less time.

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