Journal of Engineering Science and Technology
Vol. 12, No. 10 (2017) 2651 - 2663

CONSOLIDATION RELIABILITY ANALYSIS OF STONE COLUMN REINFORCED GROUND

K. S. NG[1,*], Y. M. CHEW[2], S. F. NG[3]

[1]Faculty of Civil Engineering
[2, 3]Department of Computer and Mathematical Sciences
Universiti Teknologi MARA (UiTM Pulau Pinang),
Jalan Permatang Pauh, 13500 Permatang Pauh, Pulau Pinang, Malaysia
*Corresponding Author: ngkokshien@ppinang.uitm.edu.my

Abstract

Stone column is an effective ground improvement method to improve the weak ground. This paper describes the implementation of reliability based analysis on the consolidation behaviour of stone column reinforced ground. Hasofer-Lind reliability index is computed involving non-correlated normal random variables which include stone column diameter, coefficient of volume compressibility, coefficient of consolidation and stress concentration ratio. The sensitivity of these variables on the effect of consolidation settlement is investigated in this study. Results show the importance of considering spatial variability in design and analysis of stone column reinforced ground. The probabilities of failure inferred from reliability indices are compared with Monte Carlo simulation where good agreements are obtained.

Keywords: Stone column, Reliability index, Monte Carlo simulation, Consolidation.

1. Introduction

Stone column is a popular ground improvement method to improve soft soil. The major advantages of reinforcing ground with stone column are the reduction in settlements and the increment of consolidation rate. The improvements are due to the stiff inclusion that increases the average stiffness of the ground and the provision of additional drainage path since the column's material are typically made up of large aggregates with high permeability. Other benefits include the increase of stability and reduce of liquefaction potential. The beneficial of this ground improvement techniques has been reported in many literatures [1 - 3].

Nomenclatures

A_c	Cross-section areas of the column, m^2
A_s	Cross-section areas of the surrounding soil, m^2
$\underline{C}$	Covariance matrix
c_r	Coefficient coefficients of radial consolidation, m^2/day
c_r'	Modified coefficients of radial consolidation, m^2/day
c_v	Coefficient coefficients of vertical consolidation, m^2/day
c_v'	Modified coefficients of vertical consolidation, m^2/day
C_{rv}	Coefficient of consolidation, m^2/day
D	Diameter of column, m
D_e	Diameter of column's influence zone, m
F	Failure region
$g(x)$	Performance function
H	Soil thickness, m
m	Mean values
m_v	Coefficient of volume compressibility, m^2/kN
N	Diameter ratio
n_s	Stress concentration ratio
P_f	Probability of failure
$\underline{R}$	Correlation matrix
S	Total settlement, m
S_a	Admissible settlement, m
t	Consolidation time, days
T_r	Modified time factor in the radial flow
T_v	Modified time factor in the vertical flow
U_a	Allowable degree of consolidation
U_r	Degree of consolidation in radial direction
U_v	Degree of consolidation in vertical direction
U_{rv}	Average degree of consolidation
U_t	Degree of consolidation at specified time
$\underline{x}$	Vector representing the set of random variables

Greek Symbols

σ	Average stress, kN/m^2
σ_c	Stress in the column, kN/m^2
σ_i	Standard deviation
σ_s	Stress in the soil, kN/m^2
α	Area replacement ratio
β	Reliability index
Φ	Cumulative distribution function

Abbreviations

COV	Coefficient of variation
MC	Monte Carlo

However, the accuracy and the reliability of stone column are affected by the insufficiency of data and uncertainty in input parameters during design which may lead to overestimation or underestimation of the stone columns performance. This is because in the design of stone column reinforced ground, deterministic approach is commonly adopted where representative values are used in the analysis while actual soil condition is always subject to variability. Therefore, understanding that the soil parameters are distributed according to their characteristics of variances is important to reasonably predict the performance of stone columns, for instance the analysis to obtain the settlement and the duration of consolidation.

The analysis of stone column improved ground requires the consideration of the time dependent response of two different types of materials (i.e. granular material and surrounding soft soil) which have different stress-strain relationship. Therefore, the complexities of stone column-soil system require some simplification in the analysis to make the problem more tractable. Unit cell concept is a popular simplified approach adopted by many researchers in analysing stone columns behaviour [4 - 6]. It is used to represent a column located on the interior of an infinitely large group of stone columns. The idealization is made to simulate the case of rigid raft or large uniform loaded area as in the case of embankment supported on soft soils with uniformly spaced stone column group.

Since the stone column is stiffer than the native soil, concentration of stress occurs in stone column with accompanying reduction of stress in the surrounding soil. The stress concentration ratio, n_s is the ratio of the stress in the column, σ_c, to the stress in the soil, σ_s. The stress distribution occurs when the settlement of the column and surrounding soil is roughly equal. Stress concentration ratio is the most important factor in unit cell concept. Aboshi et al. [4] proposed the average stress, σ over the unit cell area corresponding to a given area replacement ratio as:

$$\sigma = \sigma_s \alpha + \sigma_c (1-\alpha) \tag{1}$$

Area replacement ratio $\alpha = A_c/ (A_c + A_s)$, where A_c and A_s are cross-section areas of the column and the surrounding soil respectively. The stresses in the clay and stone column are given as:

$$\sigma_S = \frac{\sigma}{\left[1+(n_S - 1)\alpha\right]} \tag{2}$$

$$\sigma_C = \frac{n_S \sigma}{\left[1+(n_S - 1)\alpha\right]} \tag{3}$$

In the above equations, the most important factors to be considered are n_s and α. The stress concentration ratio is not constant and it depends on various parameters such as the properties of columns and soils, the stress or strain level and the loading type (soft or rigid). Numerous publications have shown that steady stress concentration ratio for stone column reinforced foundations is typically in the range of 2 to 6, with usual values of 3 to 4 [7 - 9]. Stone column diameter is another uncertainty in characterising the stone column properties and it depends on the quality of workmanship, the confining pressure of surrounding soils and the stone column's installation method (wet or dry). In this paper, the significance of the above two random variables (n_s and α.) on the consolidation behaviour i.e. settlement, *S* and the degree of consolidation, *U* of stone column reinforced ground are studied together with the inherent properties of the surrounding soil namely the coefficient of volume compressibility and the coefficient of consolidation which also show large variability in the values. This paper demonstrated how the spatial variability of stone column reinforced ground can be considered in the design using the reliability approach.

2. **Consolidation Settlement**

Assuming one-dimensional (1D) consolidation and equal strain theory, the consolidation settlement, *S* of stone column reinforced ground can be computed as:

$$S = m_v \sigma_s H \tag{4}$$

where m_v is the coefficient of volume compressibility and H is the soil thickness.

The spatial variability of this parameter is about 25-30% [10]. Uncoupled consolidation approach is used in this study where consolidation settlement and consolidation rate are analyzed separately.

Han and Ye [7] presented a simplified and closed form analytical solution for the rate of consolidation of stone column reinforced ground without considering the effect of well resistance and smearing. This solution was developed based on the assumptions of equal strain and 1D deformation of the column and the soil in a unit cell. Modified coefficients of consolidation are introduced to account for the effects of the stone column-soil stiffness ratio:

$$c_r' = c_r\left(1 + n_s \frac{1}{N^2 - 1}\right); \quad c_v' = c_v\left(1 + n_s \frac{1}{N^2 - 1}\right) \tag{5}$$

where c_r = coefficient of consolidation in the radial direction
c_v = coefficient of consolidation in the vertical direction;
N = diameter ratio = D/D_e
D and D_e = diameters of a column and its influence zone, respectively.

Combining the effects of radial and vertical flows, the overall rate of consolidation can be expressed as:

$$U_{rv} = 1 - (1 - U_r)(1 - U_v) \tag{6}$$

where $U_r = 1 - \exp^{-[8/F(N)]T_r}$, the average rate (or degree) of consolidation in the radial direction;
$T_r' = c_r' t / d_e^2$, a modified time factor in the radial flow;
$T_v' = c_v' t / H^2$, a modified time factor in the vertical flow;
$F(N) = [N^2 / -1)]\ln(N) - (3N^2 - 1)/(4N^2)$;
U_v = The degree of consolidation in vertical direction to be determined by Terzaghi 1D solution.

Chang [11] showed that the values of the coefficient of consolidation, C_{rv} can have a substantial degree of variation even in a uniform clay layer. Hong and Shang [12] adopted COV value as high as 0.75 in a sensitivity analysis for the coefficient of consolidation in the radial direction. It is known that the coefficient of consolidation is related to the permeability of soil. Baecher and Christian [10] reported spatial variability of permeability as high as 300%.

3. Reliability Analysis

The uncertainty in the stone column consolidation analysis can be evaluated with Hasofer-Lind reliability index [13] under the category of First Order Reliability Method (FORM). This approach was proposed by Low [14] using the perspective of an expanding equivalent dispersion ellipsoid centred at the mean in the original space of the random variables. The reliability analysis is easily performed in the Excel spreadsheet platform with cell-object-oriented constrained optimization. The matrix form of the Hasofer-Lind reliability index, β is defined as:

$$\beta = \min_{\underline{x} \in F} \sqrt{(\underline{x} - \underline{m})^T \underline{C}^{-1} (\underline{x} - \underline{m})} \tag{7}$$

or equivalently:

$$\beta = \min_{\underline{x} \in F} \sqrt{\left[\frac{x_i - m_i}{\sigma_i}\right]^T [\underline{R}]^{-1} \left[\frac{x_i - m_i}{\sigma_i}\right]} \tag{8}$$

where $\underline{x}$ is a vector representing the set of random variables, $\underline{m}$ are the mean values, $\underline{C}$ is the covariance matrix, $\underline{R}$ is the correlation matrix, F is the failure region and σ_i is the standard deviation of random variables. In spreadsheet environment, the quadratic form in Eq. 7 is visualized as a tilted multidimensional ellipsoid (centred at the mean) in the original space of the random variables without the need to diagonalise the covariance or correlation matrix. The shortest distance from the transformed failure surface to the origin of reduced variables is the reliability index. For uncorrelated variables, the problem is reduced to:

Minimize: $$\beta = \sqrt{\sum_i \left(\frac{x_i - m_i}{\sigma_i}\right)^2} \tag{9}$$

Subject to: $g(x_i) \leq 0$

where $g(x_i)$ is the performance function. The performance function for the consolidation settlement can be expressed as the condition that the total settlement, S is less than a specified threshold.

$$g(x) \leq S_a - S \tag{10}$$

where S_a is the admissible settlement value. On the other hand, the performance function for the consolidation rate of the improved ground can be expressed as the degree of consolidation after specified time, t is higher than a specified threshold.

$$g(x) \leq U_t - U_a \tag{11}$$

where U_t is the degree of consolidation at specified time and U_a is the allowable degree of consolidation.

The sensitivity information is important in the practical design since the designer can focus on the characterization of variables with stronger influence on the reliability index. The sensitivity factors, α is computed as:

$$\alpha_i = -\frac{(x_i - m_i)}{\beta\sigma_i} \tag{12}$$

The probability of failure, P_f can then be estimated using the following equation:

$$P_f = 1 - \Phi(\beta) \tag{13}$$

where $\Phi()$ is the cumulative distribution function of the standard normal variables. The probability of failure obtained from the reliability index from this study is further compared using Monte Carlo (MC) simulation. MC simulation can be implemented easily using Microsoft Excel's random number generator.

4. Hypothetical Case

A simple stone column reinforced ground case was used to exemplify the use of reliability approach on the consolidation behaviour of the improved ground. In this case, a large water tank with 100 kPa is to be built on a soft ground with 10 m thickness of soft clay layer. The end bearing stone columns are adopted as the ground improvement solution. The properties of the stone column and the treated soil are tabulated in Table 1. The random variables in the table are defined by uncorrelated normal distributions and the range of variability expressed using the coefficient of variation, COV. The values adopted in this study can be regarded as reasonable estimate since the range of variability of the random variables are site dependent. The columns are arranged in square pattern with spacing of 1.5 m. The targeted area replacement ratio from this column configurations is 35%. The coefficients of consolidations for the radial and vertical direction are assumed to be same.

Table 1. Coefficients of variation of the random variables.

Soil Properties/ Geometry configurations	Mean	COV (%)
Stone column		
Diameter, D (m)	1	20
Stress concentration ratio, n_s	4	25
Soil		
Coefficient of volume compressibility, m_v (m^2/kN)	0.00025	30
Coefficient of consolidation, C_{rv} (m^2/day)	0.016	20

The maximum allowable settlement for this example case is 0.25 m and the target duration for 90% degree of consolidation is 25 days. The settlement computed with mean value is 0.12 m which gives the factor of safety of 2 while the reliability analysis yielded Hasofer-Lind reliability index, β of 2.08 and the probability of failure, P_f of 1.90 %. According to guideline by USACE [15], the above design is categorized as "poor" and the review of design is required. The probability of failure by Monte Carlo simulation with 100,000 trials is determined to be 2.19 % which the difference when compared to the reliability index is small and negligible. In addition, the reliability index for consolidation rate analysis is

2.56 and the probability of failure (not achieving target U = 90%) is 0.53%, well comparable to P_f of 0.43% obtained from Monte Carlo simulation. The β and P_f values change with time is shown in Fig. 1. As the consolidation progresses, the probability of failure decreases. Sharp decrease in probability of failure happens in the early construction days which imply the importance of construction lag after the stone columns construction. The longer the construction lag, the higher the structure's reliability. On the other hand, the deterministic approach with mean values suggests that 90% degree of consolidation can be achieved in 2.4 days. The under predicting of required consolidation time will results in large remaining settlement that may lead to the delay of construction progress.

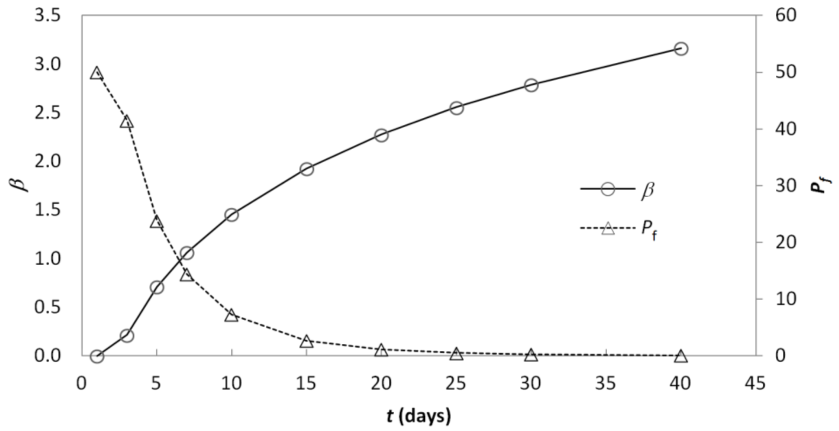
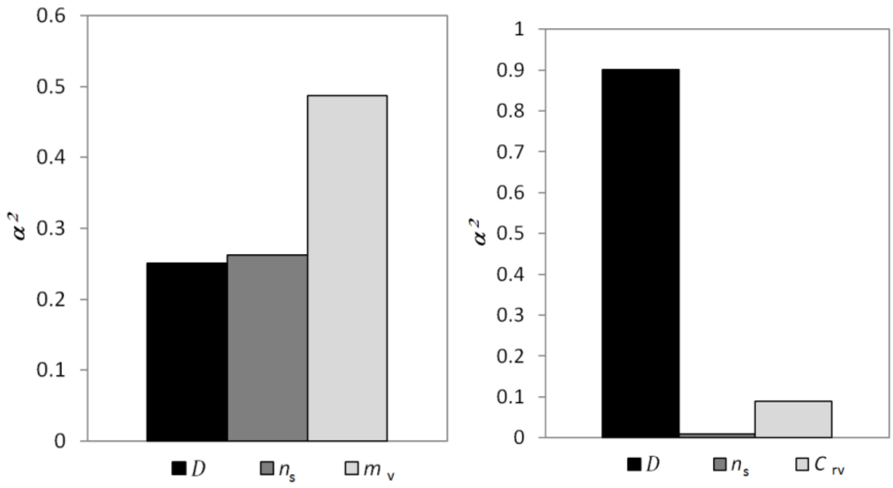


Fig. 1. Reliability index and probability of failure over time.

The relative importance of the random variables is shown in Fig. 2 given by the squares of the corresponding components of the sensitivity factor, α^2. In settlement analysis, the coefficient of volume compressibility has the greatest influence on the reliability result. It is also shown that the effects of the stress concentration ratio and the diameter of columns are substantially large. This result may imply the importance of considering the variability of all these parameters in order to safely design the structure.

In the consolidation rate analysis, the column's diameter was found to have the highest influence on the reliability results and the least influence is for the stress concentration ratio. For comparison, the work by Alonso and Jimenez [16] on stone column reinforced ground is discussed. In their study, similar reliability approach but using coupled consolidation theory suggested that the coefficient of radial consolidation has the highest influence on the reliability results and that column diameter and soil deformability are also important parameters. The influence of stress concentration ratio and coefficient of volume compressibility were not studied in their research and only the radial consolidation was considered which may lead to conservative design. Another study by Deb and Majee [17] has produced a probabilistic design chart for stone column reinforced ground. However, their work only focus on the bearing capacity and the consolidation rate, but not on the consolidation settlement. Besides, they also have not investigated the relative importance of the random variables. Therefore no comparison can be made from this study.



(a) (b)

Fig. 2. Sensitivity of random variables in analysis of (a) settlement, and (b) consolidation rate.

The results of this reliability analysis only valid to the discussed example. In order to obtain a better insight to the reliability of stone column reinforced ground, the uncertainty of the random variables are investigated using the parametric study. The possible variability values (range of variability) of four random variables are shown in Table 2 from low to high COV. In each case, only one variable is changed while other variables remain the same as in the above example.

Table 2. Different variances of the random variables.

Soil Properties/ Geometry configurations	COV (%)
Stone column	
Diameter, *D* (m)	10, **20**, 30
Stress concentration ratio, n_s	10, **25**, 40
Soil	
Coefficient of volume compressibility, m_v (m^2/kN)	10, 20, **30**
Coefficient of consolidation, C_{rv} (m^2/day)	**20**, 40, 60

Figure 3 shows the influence of the COV on the reliability index for the settlement analysis. Since the settlement computation is a simple linear equation and the random variables are non-correlated and normal distributed, therefore it is expected the reliability index reduces as COV increases due to the fact that the larger the value of COV, the more chances of large values to exist in settlement computations. The highest reliability index is obtained when the COV of the m_v is 10% and it decreases more rapidly compared to other variables as the COV increases. It can be concluded that for the range of COV between 10-30%, the coefficient of volume compressibility has the highest influence on the reliability results.

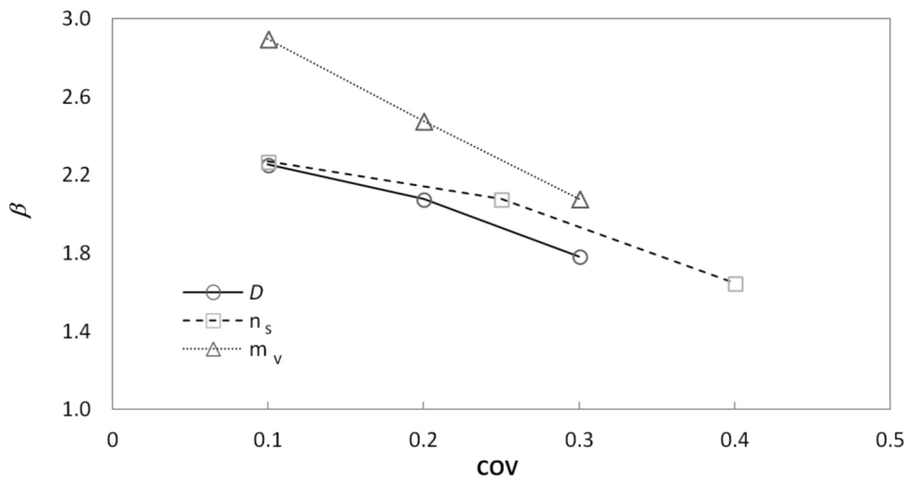
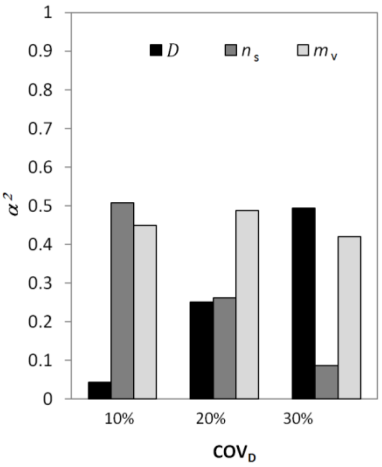


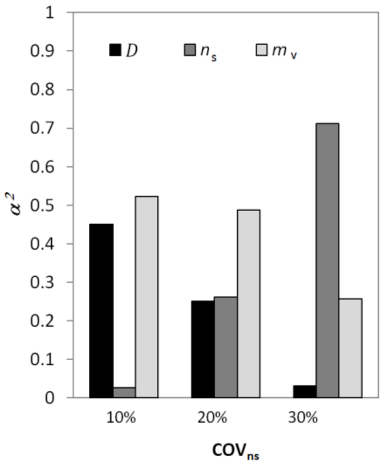
Fig. 3. Influence of COV range on reliability index in settlement analysis.

The effect of different variances in column's diameter, COV_D is shown in Fig. 4(a). As the COV_D increases from 10% to 30%, the influence of column's diameter on the reliability results also increases. However, the reverse trend is shown for the stress concentration ratio where n_s reduces as COV_D increases. Meanwhile, the coefficient of volume compressibility still plays an important role in affecting the settlement reliability result since the sensitivity factor has not changed much as the COV_D increases. The different value of COV_{ns} for the stress concentration ratio affects the settlement reliability results as indicated in Fig. 4(b). The influence of column's diameter drops significantly as the COV_{ns} increases but the opposite trend is observed for the stress concentration ratio. Even though the coefficient of volume compressibility still remain important but the influence drop as the COV_{ns} increases. When the COV_{mv} for the coefficient of volume compressibility is 10%, the influence of the stress concentration is the highest and it reduces significantly when COV_{mv} increases as shown in Fig. 4(c). On the contrary, the influence of column's diameter increases considerably when COV_{mv} changes from 10% to 20%.

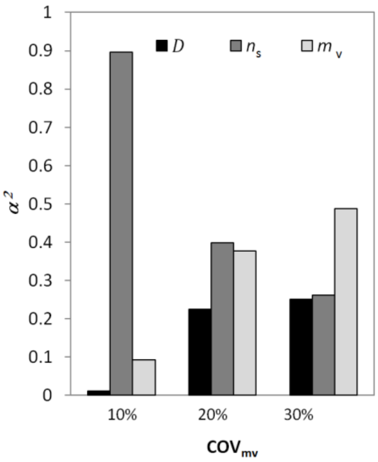
Subsequently, the results of reliability index on the consolidation rate is shown in Fig. 5 for different values of COV. The diameter of column has the greatest effect on the reliability index stemmed from the fact that the value decreases in higher rate (β drops from 4.16 to 1.75 when COV reduces from 10% to 30%) compared to other random variables. On the other hand, the stress concentration ratio seems to have negligible influence on the reliability index, unlike the results obtained for the settlement analysis, and this would also means that the consolidation rate is not sensitive to the variances of stress consolidation ratio. Result also shows that the coefficient of consolidation has considerable effect on the result where the reliability index reduces as the COV increases. In other words, the probability of not achieving 90% target degree of consolidation increases with the increase of coefficient of consolidation variance.



(a)



(b)



(c)

Fig. 4. Influence of COV for (a) column's diameter, (b) stress concentration ratio, and (c) coefficient of volume compressibility in settlement analysis.

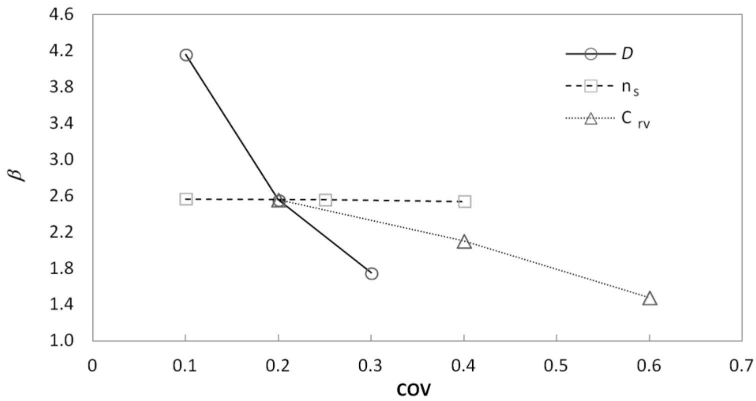


Fig. 5. Influence of COV range on reliability index in consolidation rate analysis.

Increasing the value of COV_D for the column's diameter increases the sensitivity of this random variable as shown in Fig. 6(a). Moreover, the decrease in COV_D reduces the sensitivity of the C_{rv} drastically from α^2 = 0.64 to 0.09 thus imply the loss of influence for this random variable on the reliability result. The influence of stress concentration variability is ignorable as shown in Fig. 6(b). Undoubtedly, the variability of column diameter plays a very important role in controlling the consolidation rate of stone column reinforced ground. However, the influence of column's diameter reduces profoundly as the COV_C increases from 20% to 60% as shown in Fig. 6(c). Hence, it confirms the study by Alonso and Jimenez [16] where C_{rv} has the greatest influence on reliability result if high COV is used. This result strongly suggest the importance of ascertain the variability of C_{rv} in the design as high COV can increase the risk that the ground consolidation will not be achieved within the specified time frame.

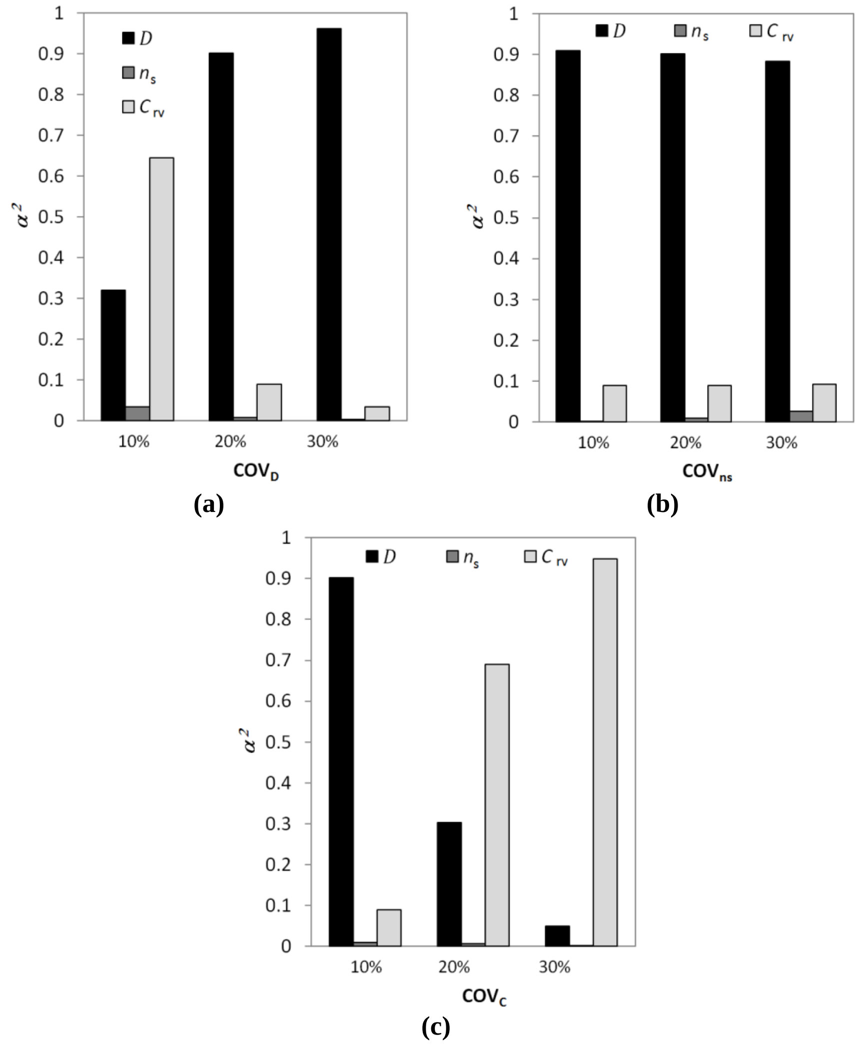


Fig. 6. Influence of COV for (a) column's diameter, (b) stress concentration ratio, and (c) coefficient of consolidation in consolidation rate analysis.

5. Conclusion

This paper has used the reliability approach to investigate the influence of spatial variability of key parameters on consolidation of stone column reinforced ground. The key parameters include the column's diameter, the stress concentration ratio, the coefficient of volume compressibility and the coefficient of consolidation. Results of the study show that the coefficient of volume compressibility has the greatest influence on the results of settlement reliability. The reliability results are also significantly influenced by the other two random variables, namely the stress concentration ratio and the column's diameter. On the other hand, in consolidation rate analysis, the stress concentration ratio appear to have the least influence on the reliability index. The column's diameter has profound effect on the sensitivity of consolidation rate especially when the variances of coefficient of consolidation is small. When the input variance is getting larger (C_{rv} > 20%), the consolidation rate is greatly affected and the risk increases. Other findings of this study include the failure probability of design decreases as the consolidation time increases and the increase of reliability when the COV increases. The Monte Carlo simulation provides comparison to the reliability index where good agreements are obtained in this study. Deterministic approach with mean values tends to suggest the design is safe (small settlement and fast consolidation rate) but the reliability analysis has suggest otherwise. Therefore, this research urges the use of reliability approach in routine design practice to supplement the traditional (deterministic) approach.

References

1. Babu, M.D.; Nayak, S.; and Shivashankar, R. (2013). A critical review of construction, analysis and behaviour of stone columns. *Geotechnical and Geological Engineering*, 31(1), 1-22.
2. Van Impe, W. (1983). Improvement of settlement behaviour of soft layers by means of stone columns. *Proceedings, 8th European Conference on Soil Mechanics and Foundation Engineering: Improvement of Ground*, 1, 309-312.
3. Goughnour, R.R.; and Bayuk, A.A. (1979, March). A field study of long term settlements of loads supported by stone columns in soft ground. *Proceedings of the International Conference on Soil Reinforcement*. Paris, 279-285.
4. Aboshi, H.; Ichimoto, E.; Enoki, M.; and Harada, K. (1979). The “Compozer”-a method to improve characteristics of soft clays by inclusion of large diameter sand columns. *Proceedings of International Conference on Soil Reinforcement*. Paris, 211-216.
5. Balaam, N.P.; and Booker, J.R. (1981). Analysis of rigid rafts supported by granular piles. *International Journal for Numerical and Analytical Methods in Geomechanics*, 5(4), 379-403.
6. Ng, K.S.; and Tan, S.A. (2014). Floating stone column design and analyses. *Soils and Foundations*, 54(3), 478-487.
7. Han, J.; and Ye, S.L. (2001). Simplified method for consolidation rate of stone column reinforced foundations. *Journal of Geotechnical and Geoenvironmental Engineering*, 127(7), 597-603.
8. Kirsch, F.; and Sondermann, W. (2003). Field measurements and numerical analysis of the stress distribution below stone column supported

embankments and their Stability. *International Workshop on Geotechnics of Soft Soils-Theory and Practice*. Essen, 595-600.

9. Han, J. (2012). Recent advances in columns technologies to improve soft foundations. *Ground Improvement and Ground Control: Transport Infrastructure Development and Natural Hazards Mitigation (Indraratna B, Rujikiatkamjorn C and Vinod J (eds))*. Research Publishing Services, Singapore, 1, 99-113.
10. Baecher, G.B.; and Christian, J.T. (2003). *Reliability and statistics in geotechnical engineering*. Chichester, U.K: Wiley.
11. Chang, C.S. (1985). Uncertainty of one-dimensional consolidation analysis. *Journal of Geotechnical. Engineering*, 111(12), 1411-1424.
12. Hong, H.P.; and Shang, J.Q. (1998). Probabilistic analysis of consolidation with prefabricated vertical drains for soil improvement. *Canadian Geotechnical Journal*, 35, 666-677.
13. Hasofer, A.M.; and Lind, N.C. (1974). Exact and invariant second-moment code format. *Journal of the Engineering Mechanics Division*, 100(1), 111-121.
14. Low, B.K. (1996). Practical probabilistic approach using spreadsheet. In *Uncertainty in the Geologic Environment: From Theory to Practice*, ASCE, 1284-1302.
15. USACE (1997). *Engineering and design: introduction to probability and reliability methods for use in geotechnical engineering*. Department of the Army, Washington, D.C. Engineering Circular No. 1110-2-547.
16. Alonso, J.A.; and Jimenez, R. (2011). Reliability analysis of stone columns for ground improvement. In *Geo-Risk 2011: Risk Assessment and Management*, ASCE, 493-500.
17. Deb, K.; and Majee, A. (2014). Probability-based design charts for stone column-improved ground. *Geomechanics and Engineering*, 7(5), 539-552.