

## ADAPTIVE NEURO-FUZZY INFERENCE SYSTEM FOR END MILLING

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### Abstract

Soft computing is commonly used as a modelling method in various technological areas. Methods such as Artificial Neural Networks and Fuzzy Logic have found application in manufacturing technology as well. Neuro-Fuzzy systems, aimed to combine the benefits of both the aforementioned Artificial Intelligence methods, are a subject of research lately as have proven to be superior compared to other methods. In this paper an adaptive neuro-fuzzy inference system for the prediction of surface roughness in end milling is presented. Spindle speed, feed rate, depth of cut and vibrations were used as independent input variables, while roughness parameter Ra as dependent output variable. Several variations are tested and the results of the optimum system are presented. Final results indicate that the proposed model can accurately predict surface roughness, even for input that was not used in training.

Keywords: Artificial Intelligence, modelling, milling, surface roughness

### 1. Introduction

Milling is one of the most commonly used metal removal operations in industry because of the ability to remove material fast and at the same time provide reasonably good surface quality. It is used in a variety of manufacturing industries including aerospace and automotive sectors, where quality is an important factor. Milling is the process of cutting away material by feeding a workpiece past a rotating multiple tooth cutter; the cutting action of teeth provides a fast method of machining. The machined surface may be flat, angular or curved. Milling can be classified into peripheral milling, face milling and end milling. In peripheral or slab milling, the milled surface is generated by teeth located on the periphery of

**Nomenclatures**

|           |  |
|-----------|--|
| $A, B$    | Non-linear parameters                      |
| $O_j$     | Membership function                        |
| $P_i$     | Potential                                  |
| $p, q, r$ | Linear parameters                          |
| $Ra$      | Surface roughness parameter, $\mu\text{m}$ |
| $w_i$     | Weight function                            |

**Greek Symbols**

|          |  |
|----------|--|
| $\alpha$ | Radius parameter, $\alpha = 4/r_a^2$           |
| $\beta$  | Neighbourhood parameter, $\beta = 4/r_\beta^2$ |

**Abbreviations**

|       |                                       |
|-------|---------------------------------------|
| ANFIS | Adaptive Neuro-Fuzzy Inference System |
| CNC   | Computer Numerical Control            |
| FEM   | Finite Elements Method                |
| HSS   | High Speed Steel                      |
| MSE   | Mean Square Error                     |
| NN    | Neural Networks                       |

the cutter body. The axis of cutter rotation is generally in a plane parallel to the workpiece surface to be machined. In face milling the cutter is mounted on a spindle having an axis of rotation perpendicular to the workpiece surface. The milled surface results from the action of cutting edges located on the periphery and face of the cutter. The cutter in end milling generally rotates on an axis vertical to the workpiece. It can be tilted to machine tapered surfaces. Cutting teeth are located on both the end face of the cutter and the periphery of the cutter body.

Surface roughness, which is a key factor in machining, is used to evaluate and determine the quality of a product. It influences several attributes of a part such as fatigue behaviour, wear, corrosion, lubrication and surface friction. Surface roughness refers to deviations from the nominal surface of the third up to the sixth order. First and second order deviations refer to form and waviness respectively. Third and fourth order deviations refer to periodic grooves, cracks and dilapidations, which are connected to the shape and condition of the cutting edges, chip formation and process kinematics. Fifth and sixth order deviations refer to workpiece material structure, which is connected to physical chemical mechanisms acting on a grain and lattice scale. Generally surface roughness can be described as the inherent irregularities of workpiece left by various machining processes. The most common way to describe surface roughness is the average roughness which is often quoted as  $Ra$ . Average roughness is defined as the arithmetic value of the deviation of profile from centreline along a sampling length. It is calculated as:

$$Ra = \frac{1}{l} \int_0^l |y(x)| dx \quad (1)$$

where  $l$  is the sampling length and  $y$  is the ordinate of the profile curve. Surface roughness is influenced by controlled machining parameters, such as feed rate, spindle speed, depth of cut, as well as by non-controlled influences, such as non

homogeneity of workpiece and tool, tool wear, machine motion errors, formation of chips and unpredictable random disturbances. It has been shown that both the controlled and the non controlled parameters cause relative vibrations between the cutting tool and the workpiece.

Modelling and simulation techniques are popular for the analysis of manufacturing processes; especially FEM and NN [1-3]. More specifically, many researchers have made several efforts in order to predict the surface roughness in milling; statistical and empirical models to predict surface roughness have been proposed [4-6]. Soft computing techniques are quite common; NN [7-9], genetic algorithms [10-12] and fuzzy logic [13, 14] have been employed. In this paper a combined method of neural networks and fuzzy logic, namely the Adaptive Neuro-Fuzzy Inference System (ANFIS) is proposed for the prediction of surface roughness in end milling. Several models with different characteristics are built and tested and the optimum is selected. The analysis results indicate that the proposed model can be used to predict surface roughness in end milling with a less than 10% error, even for tests with cutting conditions that were not used in the training of the system.

## 2. ANFIS Modelling

Fuzzy logic systems and neural networks are complementary technologies. Neural networks extract information from a system, while fuzzy logic systems use linguistic information from experts. An ANFIS is an integrated system comprised of neural networks and a fuzzy logic system. It possesses the advantages of the two aforementioned methods, such as learning or optimization ability from neural networks and humanlike if-then rules of thinking from the fuzzy logic system.

An adaptive neuro-fuzzy system that has a structure similar to that of a neural network and which maps inputs through input membership functions and associated parameters, and then through output membership functions and associated parameters to outputs, can be used to interpret the input/output map. The parameters associated with the membership functions will change through the learning process. The computation of these parameters is facilitated by a gradient vector which provides a measure of how well the fuzzy inference system is modelling the input/output data for a given set of parameters. Once the gradient vector is obtained, any of several optimization routines could be applied in order to adjust the parameters so as, most of the times, to reduce the sum of the squared errors. In the optimization method used in this paper, a combination of least squares estimation and back-propagation is adopted.

### 2.1. ANFIS architecture

The ANFIS architecture and its learning algorithm for the Sugeno fuzzy model are described in this section. For simplicity it is assumed that the fuzzy inference system under consideration has two inputs  $x$  and  $y$ , and one output. For a first order Sugeno fuzzy model, a typical rule set with two if-then rules can be expressed as:

$$\text{Rule 1: IF } x \text{ is } A_1 \text{ and } y \text{ is } B_1 \text{ then: } f_1 = p_1x + q_1y + r_1$$

$$\text{Rule 2: IF } x \text{ is } A_2 \text{ and } y \text{ is } B_2 \text{ then: } f_2 = p_2x + q_2y + r_2$$

with  $p, q$  and  $r$  linear parameters and  $A$  and  $B$  non linear parameters.

Fuzzy reasoning and the corresponding equivalent ANFIS architecture are illustrated in Fig. 1(a) and (b) respectively.

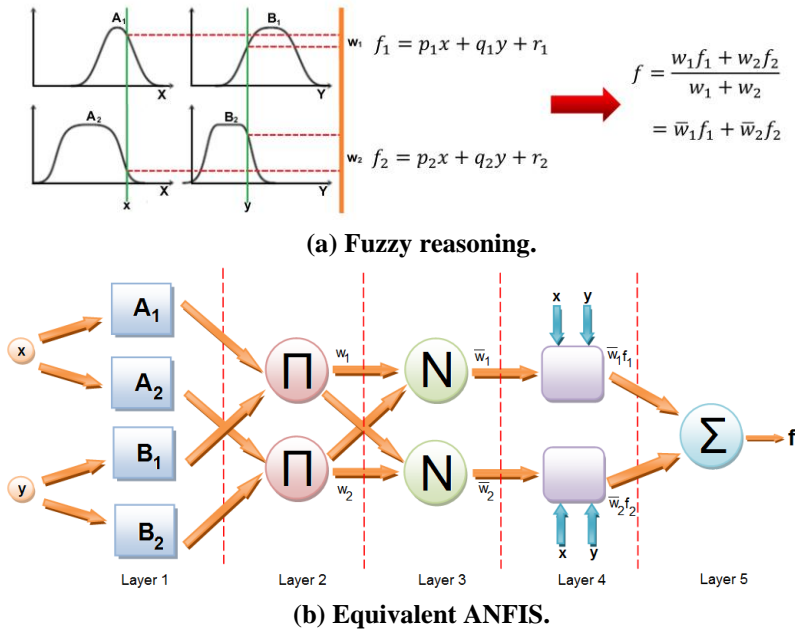


Fig. 1. ANFIS architecture.

As it can be seen from Fig. 1(b), ANFIS consists of five layers. In the first layer every node is a square node with a node function  $O_i^1 = \mu_{A_i}(x)$  (or  $O_i^1 = \mu_{B_i}(y)$ ), where  $x$  (or  $y$ ) is the input to node  $i$ , and  $A_i$  (or  $B_i$ ) is the linguistic label associated with this node function. In other words,  $O_i^1$ , is the membership function of  $A_i$  and it specifies the degree to which the given  $x$  satisfies the quantifier  $A_i$ . In the at hand paper the chosen membership function was the Gaussian one:

$$\mu(x) = \exp \left[ - \left( \frac{x - c_i^1}{\sigma_i^1} \right)^2 \right] \tag{2}$$

where  $c$  is the centre and  $\sigma$  is the spreading.

In layer 2, the product layer, every node is a circle node labeled  $\Pi$ . The number of nodes in this layer equals to the number of the system's rules; for the case examined there should be two nodes. The output  $w_1$  and  $w_2$  are the weight functions of the next layer. The output of this layer is the product of the input signals which is defined as:

$$w_i = \mu_{A_i}(x) \cdot \mu_{B_i}(y), \text{ for } i=1,2 \tag{3}$$

In layer 3, the normalized layer, every node is a circle node labelled  $N$ . The  $i$ -th node calculates the ratio of the  $i$ -th rule's firing strength to the sum of all rules firing strengths:

$$\bar{w}_i = \frac{w_i}{w_1 + w_2}, \text{ for } i=1,2 \quad (4)$$

In layer 4, the de-fuzzy layer, every node is adaptive and is represented as a square. The relationship between the input and the output of this layer can be defined as:

$$O_i^4 = \bar{w}_i \cdot f_i = \bar{w}_i (p_i \cdot x + q_i \cdot y + r_i), \text{ for } i=1, 2 \quad (5)$$

where  $p$ ,  $q$  and  $r$  denote the linear parameters or so called consequent parameters of the node.

Finally, layer 5, the total output layer, computes the overall output as the summation of all incoming signals:

$$O_i^5 = \sum_i \bar{w}_i f_i = \frac{\sum_i w_i f_i}{\sum_i w_i} \quad (6)$$

## 2.2. Subtractive clustering

In order to obtain a set of  $m$  fuzzy conditional rules capable of representing the system under study, clustering algorithms are particularly suited, since they permit a scatter partitioning of the input-output space, which results in finding only the relevant rules. Comparing to grid-based partitioning methods, clustering algorithms have the advantage of avoiding the explosion of the rule base, a problem known as the curse of dimensionality. In this work Chiu's subtractive clustering was applied. The subtractive clustering method to initialize the membership functions and to reduce the number of fuzzy rules before they got trained by the neuro-fuzzy network was used. Subtractive clustering is an unsupervised algorithm and it is based on a measure of the density of data points in the feature space. A set of points are defined as possible group centres, each of them being interpreted as an energy source. The centre candidates are the data samples themselves.

Let  $X_N$  be a set of  $N$  data samples  $x_1, x_2, \dots, x_N$  defined in an  $m+n$  space, where  $m$  denotes the number of inputs and  $n$  the number of outputs. In order to make the range of values in each dimension identical, the data samples are normalized, so that they are limited by a hypercube.

The potential associated to  $x_i$  is:

$$P_i = \sum_{j=1}^N \exp\left(-\alpha \|x_i - x_j\|^2\right) \quad (7)$$

with  $\alpha = \frac{4}{r_a^2}$ ,  $r_a$  being the radius parameter, a constant which defines the neighbourhood radius of each point and  $x_i, x_j$  are the input and the output vectors respectively.

Points  $x_j$  located out of the radius of  $x_i$  will have a smaller influence on its potential. On the other hand, the effects of points close to  $x_i$  will grow with the proximity. Radius parameter is directly related to the number of clusters found. Thus, a small radius will lead to a high number of rules, which if excessive, may result in over fitting. On the other hand, a higher radius will lead to a smaller number of clusters, which may originate under fitting and models with reduced representation accuracy. Therefore in practice it is necessary to test several values for radii and select the most adequate according to the results obtained.

After the potential value of each data point has been calculated, the data point with the highest potential value is selected as the first cluster centre. Let  $x_1^*$  be the first cluster centre and its potential  $P_1^*$ . The potential of all the data points is changed as:

$$P_i \leftarrow P_i - P_1^* \cdot e^{-\beta \|x - x_1^*\|^2} \quad (8)$$

where  $\beta = \frac{4}{r_b^2}$ ,  $r_b$  defining the neighbourhood radius with sensitive reductions in its potential.

Therefore, the data points near the first cluster centre will have significantly reduced potential value, thereby making the point unlikely to be selected as the next cluster centre. The process of acquiring new centre and revising potentials repeats until the remaining potential of all data points are below some fraction of the potential of the first cluster centre  $P_1^*$ . Another advantage of subtractive clustering is that the algorithm is noise robust, since outliers do not significantly influence the choice of centres, due to their low potentials.

### 2.3. Application of the method

For the application of the method, experimental results from the relevant literature were exploited [15]. The experiments pertain to the CNC end milling 6061 aluminium alloy blocks. The tool used was a four-flute 3/4 inch diameter milling cutter of HSS. During the machining an accelerometer sensor was used to measure the vibrations. In order to get a vibration voltage average value per revolution, a proximity sensor was utilized to count the rotations of spindle. Vibration voltage values and rotation signals were collected and converted into digital data by A/D converter which was connected with a personal computer. Spindle speed, feed rate, depth of cut and vibrations were selected as independent variables in this study. Vibrations depend partly on the other three independent variables and thus they could be treated as a dependent variable. However, due to the complex structural system consisting of workpiece, fixture, cutting tool and machine tool the vibrations and consequently the roughness parameter  $Ra$  cannot be described quite accurately by the limited set of independent variables. Therefore, vibrations are treated as an independent variable, as well.

Two sets of experimental data were obtained: training data set and testing data set. The training data set was obtained on the basis of four levels of spindle speed (750, 1000, 1250, 1500rpm), six levels of feed rate (152.4, 228.6, 304.8, 457.2,

533.4, 609.6 mm/min) and three levels of depth of cut (0.254, 0.762, 1.27mm). For each combination of spindle speed, feed rate and depth of cut, the corresponding vibration data (in  $\mu\text{V}$ ) were recorded. The corresponding value of the roughness average  $Ra$  (in  $\mu\text{m}$ ), the dependent output, was collected for each measurement. The training data used for the analysis are presented in Table 1.

In this work, training data comprised 21 measurements selected randomly out of the 400 measurements originally presented in [15]. The test data set was obtained on the basis of four levels of spindle speed (750, 1000, 1250, 1500rpm), seven levels of feed rate (152.4, 228.6, 304.8, 381, 457.2, 533.4, 609.6mm/min) and three levels of depth of cut (0.254, 0.762, 1.27mm). Also for the test data set the data on vibrations and surface were recorded. The test data set comprised 10 measurements that are shown in Table 2. Note that in the test data set a value for the feed rate, namely 381mm/min that has not been used in the training data set was also considered. This was chosen in order to check whether the constructed system could predict correctly the value of the roughness parameter  $Ra$  when it has as input values that it has not been trained for. This is an ability that some systems have and it is called interpolation. The aim of this work was to create a system that could predict the roughness parameter  $Ra$  quite accurately; it is quantified as a small value of Mean Squared Error (MSE) of training and test data respectively.

**Table 1. Training set data.**

| No of Training data | Speed ( $\text{min}^{-1}$ ) | Feed (mm/min) | Depth of cut (mm) | Vibrations ( $\mu\text{V}$ ) | Surface roughness ( $\mu\text{m}$ ) |
|---------------------|-----------------------------|---------------|-------------------|------------------------------|-------------------------------------|
| 1                   | 1500                        | 152.4         | 1.27              | 0.10168                      | 1.4224                              |
| 2                   | 1500                        | 457.2         | 0.254             | 0.13581                      | 3.048                               |
| 3                   | 1500                        | 609.6         | 0.762             | 0.19091                      | 2.6162                              |
| 4                   | 1500                        | 304.8         | 0.254             | 0.11231                      | 2.2352                              |
| 5                   | 1250                        | 304.8         | 0.254             | 0.1448                       | 2.54                                |
| 6                   | 1250                        | 609.6         | 1.27              | 0.18291                      | 3.0734                              |
| 7                   | 1250                        | 152.4         | 1.27              | 0.096899                     | 1.8034                              |
| 8                   | 1000                        | 609.6         | 1.27              | 0.18417                      | 3.6068                              |
| 9                   | 1000                        | 152.4         | 0.762             | 0.10976                      | 1.9812                              |
| 10                  | 1000                        | 304.8         | 1.27              | 0.18001                      | 2.3368                              |
| 11                  | 1000                        | 457.2         | 0.762             | 0.16149                      | 3.1496                              |
| 12                  | 750                         | 457.2         | 0.762             | 0.14068                      | 3.7338                              |
| 13                  | 750                         | 304.8         | 0.762             | 0.12654                      | 2.5908                              |
| 14                  | 750                         | 152.4         | 1.27              | 0.089752                     | 1.8288                              |
| 15                  | 750                         | 609.6         | 0.762             | 0.17928                      | 4.3434                              |
| 16                  | 1500                        | 228.6         | 0.254             | 0.08833                      | 1.3462                              |
| 17                  | 1250                        | 228.6         | 0.762             | 0.13814                      | 2.0828                              |
| 18                  | 1000                        | 533.4         | 0.254             | 0.10338                      | 3.7846                              |
| 19                  | 750                         | 228.6         | 0.254             | 0.093096                     | 2.7686                              |
| 20                  | 750                         | 533.4         | 0.254             | 0.11352                      | 4.5212                              |
| 21                  | 750                         | 533.4         | 1.27              | 0.16586                      | 3.81                                |

**Table 2. Test set data.**

| No of Test data | Speed ( $\text{min}^{-1}$ ) | Feed mm/min | Depth of cut (mm) | Vibrations ( $\mu\text{V}$ ) | Surface roughness ( $\mu\text{m}$ ) |
|-----------------|-----------------------------|-------------|-------------------|------------------------------|-------------------------------------|
| 1               | 1500                        | 609.6       | 1.27              | 0.17874                      | 2.794                               |

|    |      |       |       |          |        |
|----|------|-------|-------|----------|--------|
| 2  | 1250 | 457.2 | 0.254 | 0.14558  | 2.921  |
| 3  | 1250 | 381   | 0.254 | 0.13378  | 2.7178 |
| 4  | 1000 | 533.4 | 0.762 | 0.16794  | 3.683  |
| 5  | 1500 | 381   | 0.254 | 0.14637  | 2.794  |
| 6  | 1250 | 533.4 | 0.254 | 0.13001  | 3.2766 |
| 7  | 1000 | 228.6 | 0.254 | 0.091113 | 2.3368 |
| 8  | 1000 | 381   | 0.762 | 0.14862  | 2.7432 |
| 9  | 750  | 533.4 | 0.762 | 0.16241  | 4.1402 |
| 10 | 750  | 381   | 1.27  | 0.15298  | 2.6416 |

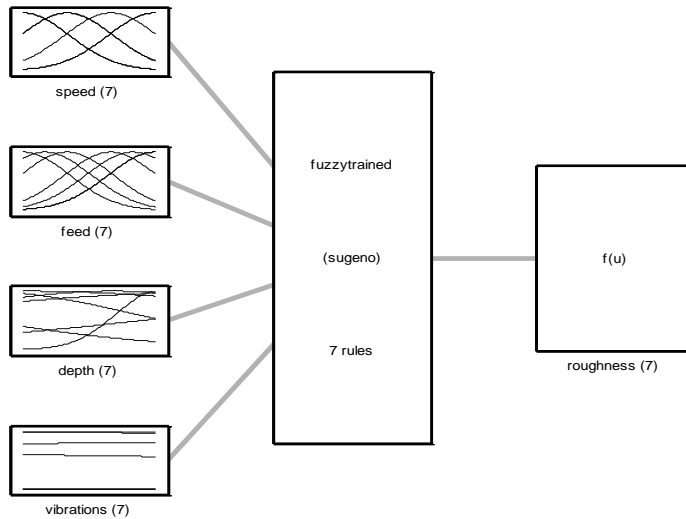
### 3. Results and Discussion

The analysis was realized with Matlab. Subtractive clustering algorithm was implemented to the training data set. In order to find the value of the radius parameter which would give the best results, all possible models with value from 0.1 till 1.2 with changing step 0.1, were tested. The membership functions and the fuzzy if-then rules which were estimated by the subtractive clustering algorithm were used as initial membership functions and if-then fuzzy rules in the neuro-fuzzy system. After the completion of each training process the final MSE of training and test data respectively was recorded. In the training procedure the final MSE error of training data was chosen to be equal to zero. For the termination of the analysis, the maximum repetitions made by the program before it stopped, the so-called epochs, were chosen to be 600; this value was decided after performing some test runs in Matlab. Furthermore, the initial step size of training was adjusted. This value has a severe effect in the training process. The default value chosen by the program was equal to 0.01; for initial step size smaller than 0.01 the final MSE values were prohibitively large. In the analysis described in this paper, values of initial step size greater than 0.01 were examined. In particular, values of the initial step size from 0.01 till 1.2 by changing step of 0.01 were considered. All these tests were held for every value of the radius parameter. The training process of ANFIS stopped whenever the designated epoch number was reached or the training error goal was achieved.

By comparing all the models with the characteristics described above, it was concluded that the ANFIS system that produced smaller training and test mean squared errors, was the one that had been created by using an initial training step size of 1.2 while the radius parameter was equal to 1.0. For the described system, the MSE of training data was equal to  $1.81 \cdot 10^{-8}$  while the respective MSE of test data was 0.0136. As one can notice, the values of both the mean squared errors of training and test data are significantly small.

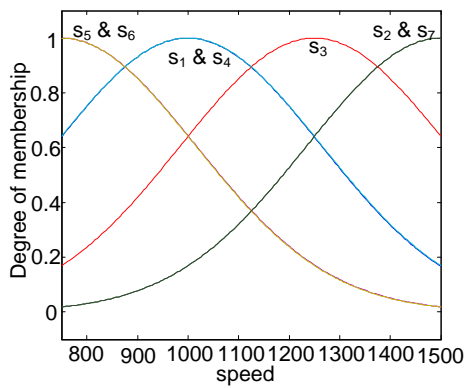
In Fig. 2, a high level diagram of the fuzzy inference system, is shown. Inputs and their membership functions appear to the left of the FIS structural characteristics, while the output appears on the right. All the membership functions used in the chosen neuro-fuzzy system were Gaussians ones. The membership functions of the four inputs of the system are shown in Fig. 3, as they were calculated after the training process.



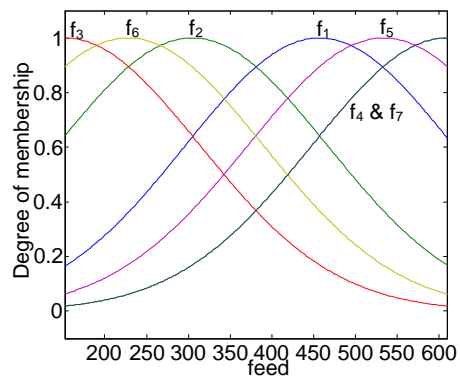


System fuzzytrained: 4 inputs, 1 outputs, 7 rules

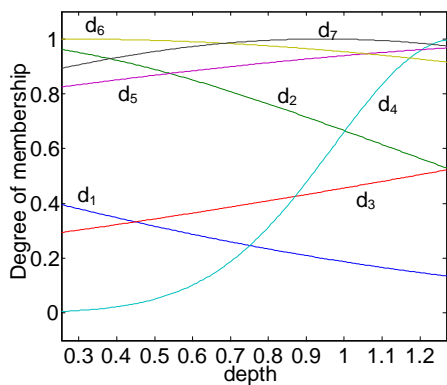
**Fig. 2. Fuzzy rule architecture.**



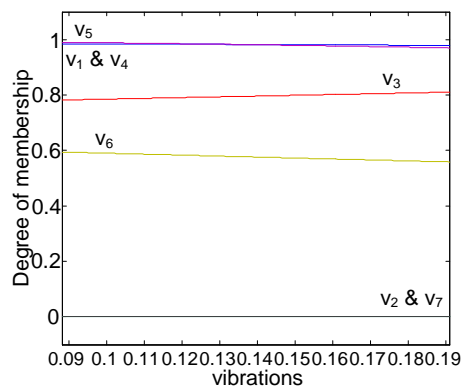
**(a) Spindle speed.**



**(b) Feed rate.**



**(c) Depth of cut.**



**(d) Vibrations.**

**Fig. 3. Membership functions of inputs.**

The program evaluated that from the training data set only 7 independent fuzzy rules could be derived. The 7 fuzzy rules which correspond to the previously mentioned membership functions are:

1. If (speed is  $s1$ ) and (feed is  $f1$ ) and (depth is  $d1$ ) and (vibrations is  $v1$ ) then (roughness is  $r1$ )
2. If (speed is  $s2$ ) and (feed is  $f2$ ) and (depth is  $d2$ ) and (vibrations is  $v2$ ) then (roughness is  $r2$ )
3. If (speed is  $s3$ ) and (feed is  $f3$ ) and (depth is  $d3$ ) and (vibrations is  $v3$ ) then (roughness is  $r3$ )
4. If (speed is  $s4$ ) and (feed is  $f4$ ) and (depth is  $d4$ ) and (vibrations is  $v4$ ) then (roughness is  $r4$ )
5. If (speed is  $s5$ ) and (feed is  $f5$ ) and (depth is  $d5$ ) and (vibrations is  $v5$ ) then (roughness is  $r5$ )
6. If (speed is  $s6$ ) and (feed is  $f6$ ) and (depth is  $d6$ ) and (vibrations is  $v6$ ) then (roughness is  $r6$ )
7. If (speed is  $s7$ ) and (feed is  $f7$ ) and (depth is  $d7$ ) and (vibrations is  $v7$ ) then (roughness is  $r7$ )

As mentioned, the system used was a first order Sugeno type system. The linear equations of the output of the system that can be seen in the fuzzy rules are the following:

$$r1=0.003517\text{speed}-0.0036\text{feed}-2.502\text{depth}+0.7122\text{vibrations}-0.8406$$

$$r2=-8.367\times 10^{-106}\text{speed}-1.275\times 10^{-106}\text{feed}-1.428\times 10^{-106}\text{depth}-4.93\times 10^{-110}\text{vibrations}-5.58\times 10^{-109}$$

$$r3=-0.002005\text{speed}+0.01103\text{feed}+1.318\text{depth}+3.299\text{vibrations}+0.7446$$

$$r4=1.687\times 10^{-17}\text{speed}-3.749\times 10^{-18}\text{feed}-3.386\times 10^{-20}\text{depth}-2.267\times 10^{-21}\text{vibrations}-2.541\times 10^{-20}$$

$$r5=0.0002813\text{speed}-0.000596\text{feed}-1.341\text{depth}-5.178\text{vibrations}-7.452$$

$$r6=-0.0004338\text{speed}-0.005371\text{feed}-1.279\text{depth}-1.489\text{vibrations}+4.24$$

$$r7=1.679\times 10^{-26}\text{speed}+6.793\times 10^{-27}\text{feed}+8.238\times 10^{-30}\text{depth}+2.131\times 10^{-30}\text{vibrations}+1.114\times 10^{-29}$$

The entire implication process from the beginning to the end can be seen in Fig. 4, when the vector (speed = 1125, feed = 381, depth = 0.762, vibrations = 0.1392), is used as input to the system.

The structure of the described neuro-fuzzy system is shown in Fig. 5. There are 4 input nodes while there are 7 nodes connecting to each of the input nodes, in the second layer of the system, which is equal to the total number of the fuzzy rules, as described.

The alteration of the value of mean squared error of training data versus the epochs can be seen in Fig. 6. In Figs. 7(a) and (b), the experimental values of

surface roughness and the corresponding calculated value of surface roughness by the neuro-fuzzy system for the training data and the test data are shown, respectively. Figure 8 shows the percentage error in the computation of surface roughness of the test data. All the test data have error less than 10%.

In Fig. 9, the total surfaces which describe the input-output space of the neuro-fuzzy system, when only two of the input variables are altered each time, are shown. The input vector used was (speed = 1125, feed = 381, depth = 0.762, vibrations = 0.1392). The two input variables that were not changed each time, took their values from the above vector. The two input variables that are altered each time take all the possible values between their width of rate.

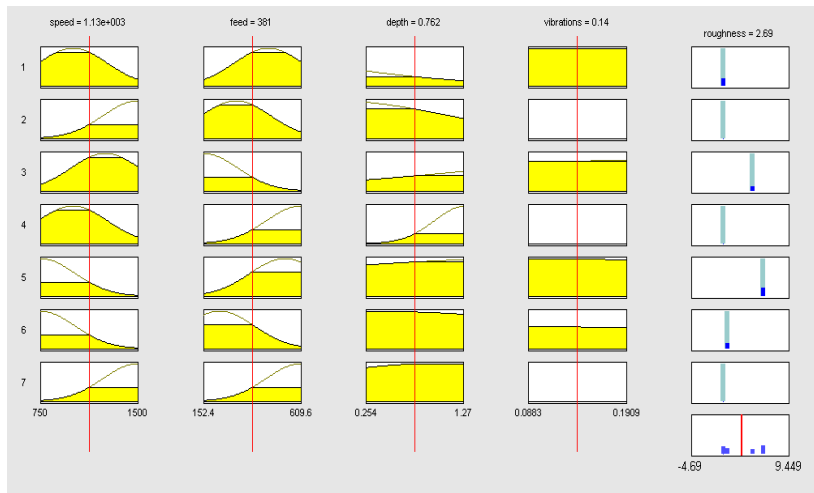


Fig. 4. Implication method.

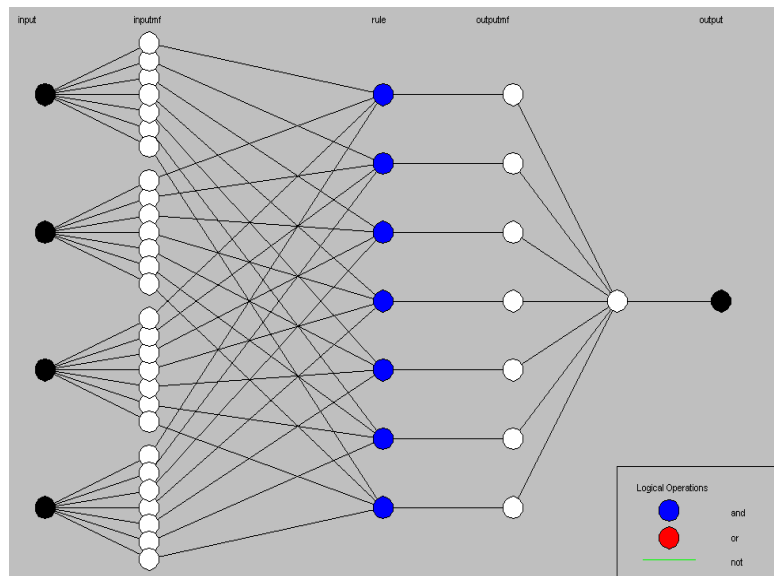
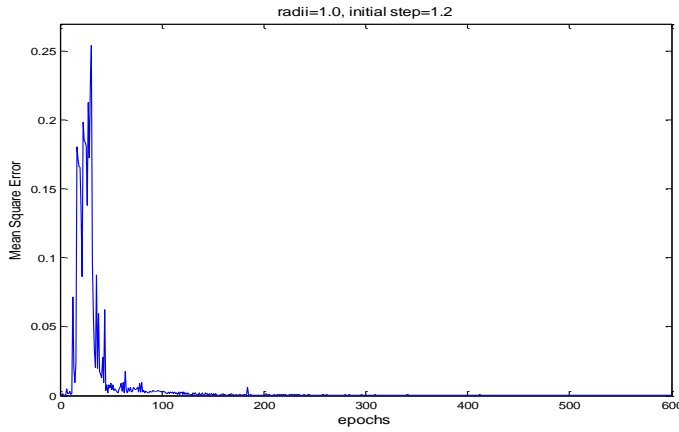
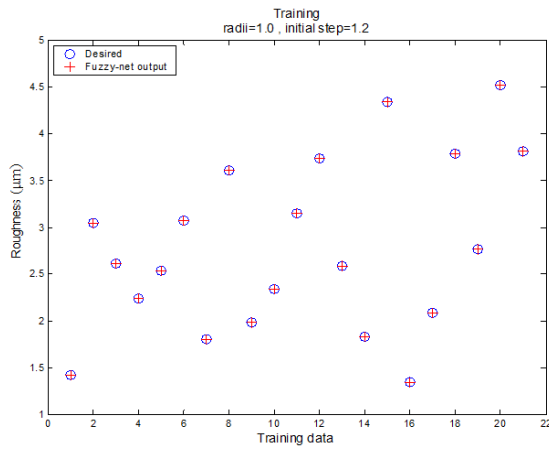


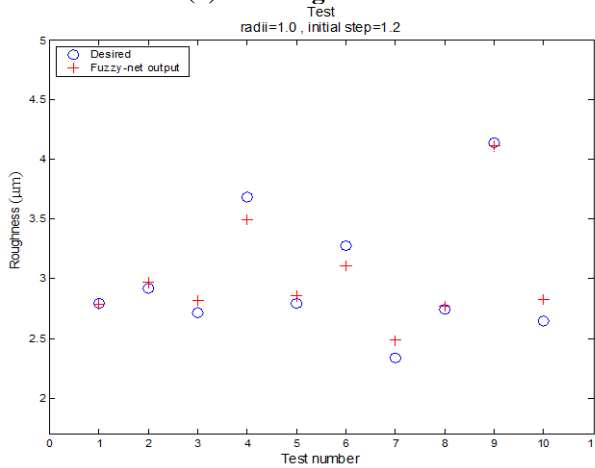
Fig. 5. Structure of neuro-fuzzy system.



**Fig. 6. MSE error variation versus epochs.**

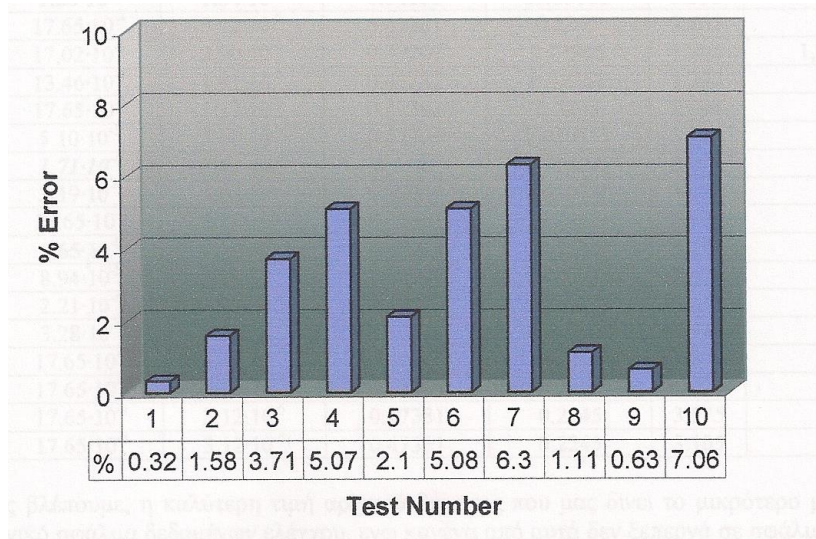


**(a) Training data set.**

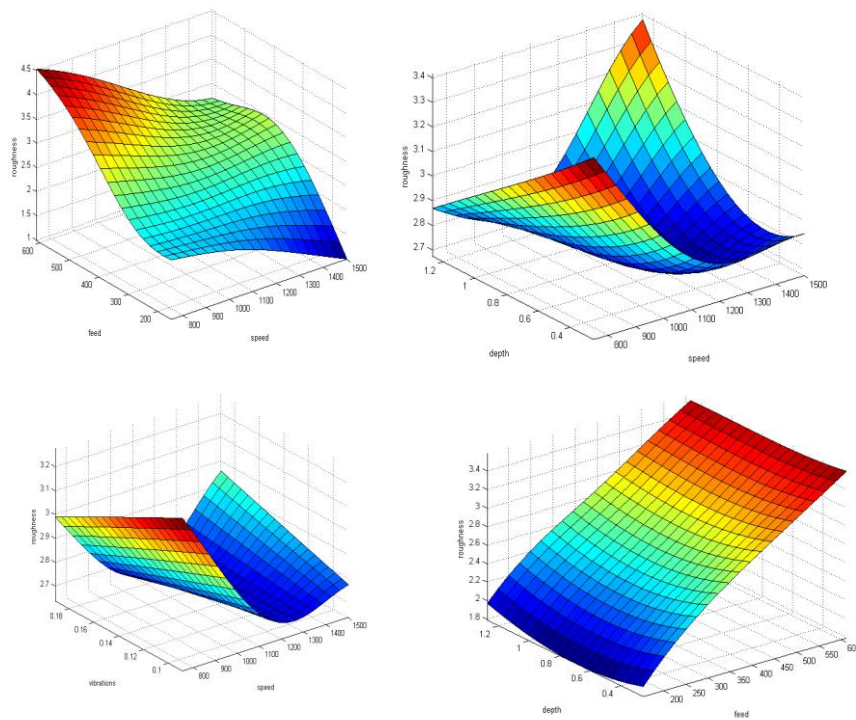


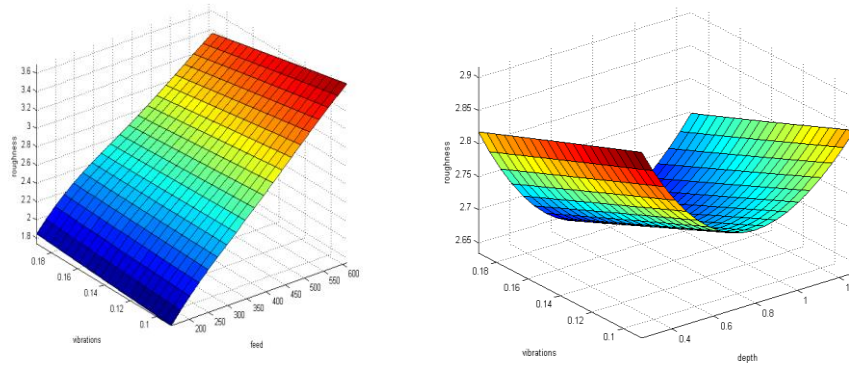
**(b) Test data set.**

**Fig. 7. Experimental values and ANFIS predicted results for training and test data sets.**



**Fig. 8. Discrepancies between experimental values and ANFIS predicted values for each value of the test data set, in percentage.**





**Fig. 9. Input-output surface of neuro-fuzzy system.**

From the figures of the input membership functions and the input-output surfaces it can be noticed that the two input variables that are the most significant ones, the ones that influence more the value of the output, are the spindle speed and the feed rate. The membership functions of the depth of cut and the vibrations are unchangeable in the whole width of rate. The same conclusion could be derived from the linear equations of the output, in which the factors of spindle speed and feed rate are significantly higher than those of depth of cut and vibrations. It is worth noticing that neuro-fuzzy systems are very stable systems; if the initial parameters are not changed, they will give the same results for all the runs of the program.

#### 4. Conclusions

In this work, a neuro-fuzzy system was implemented in order to predict the surface roughness in end-milling. Four independent variables were used as inputs, namely spindle speed, feed rate, depth of cut and vibrations. The only output of the system was corresponding to the roughness parameter  $Ra$ . By applying subtractive clustering with a value of radius parameter equal to 1.0 in order to find the initial membership functions of the variables and the fuzzy rules, and then train the neuro-fuzzy system by using as initial step size 1.2, the MSE of training data was equal to  $1.81 \cdot 10^{-8}$  while the MSE of test data was equal to 0.0136. The results were quite satisfying. The neuro-fuzzy systems are well suited for all the problems, since they combine all the advantages of neural networks and fuzzy logic.

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