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ANALYSIS OF MONTE CARLO SIMULATION SAMPLING TECHNIQUES ON SMALL SIGNAL STABILITY OF WIND GENERATOR- CONNECTED POWER SYSTEM

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Abstract

Monte Carlo simulation using Simple Random Sampling (SRS) technique is popularly known for its ability to handle complex uncertainty problems. However, to produce a reasonable result, it requires huge sample size. This makes it to be computationally expensive, time consuming and unfit for online power system applications. In this article, the performance of Latin Hypercube Sampling (LHS) technique is explored and compared with SRS in term of accuracy, robustness and speed for small signal stability application in a wind generator-connected power system. The analysis is performed using probabilistic techniques via eigenvalue analysis on two standard networks (Single Machine Infinite Bus and IEEE 16–machine 68 bus test system). The accuracy of the two sampling techniques is determined by comparing their different sample sizes with the IDEAL (conventional). The robustness is determined based on a significant variance reduction when the experiment is repeated 100 times with different sample sizes using the two sampling techniques in turn. Some of the results show that sample sizes generated from LHS for small signal stability application produces the same result as that of the IDEAL values starting from 100 sample size. This shows that about 100 sample size of random variable generated using LHS method is good enough to produce reasonable results for practical purpose in small signal stability application. It is also revealed that LHS has the least variance when the experiment is repeated 100 times compared to SRS techniques. This signifies the robustness of LHS over that of SRS techniques. 100 sample size of LHS produces the same result as that of the conventional method consisting of 50000 sample size. The reduced sample size required by LHS gives it computational speed advantage (about six times) over the conventional method.

Keywords: Monte Carlo simulation, Latin hypercube sampling, Simple random sampling, Small signal stability, Power system.

Nomenclatures

h	Function of the model
T	Transpose of matrix
x	Vectors of input variables
y	Vector output variables
D	Known probability distribution
N	Number of samples
y_i	Model output
$\bar{y}_{(srs)}$	Mean of the output using SRS
$var\left(\bar{y}_{srs}\right)$	Variance of the output mean using SRS
$\bar{y}_{(lhs)}$	Mean of output using LHS
$var\left(y_{(lhs)}\right)$	Variance of the sample using LHS
$f_W(v)$	Probability of observing wind speed, v,
k	Weibull shape parameter
c	Weibull scale parameter
P_e	Electrical power generated from wind turbine
C_p	Coefficient of performance of wind turbine
η_m	Mechanical transmission efficiency
η_g	Generator efficiency
v_i	Vector of wind speeds, m/s
v_{ci}	Cut-in wind speed, m/s
v_r	Rated wind speed, m/s
v_{CO}	Cut-out wind speed, m/s
P_r	Wind turbine rated power, Watt
U	Terminal voltage, volt
X_1	Stator reactance, pu
X_2	Rotor reactance, pu
X_m	Magnetising reactance, pu
r_2	Rotor resistance, pu
s_i	Vector of rotor slip
$Q_{(v_i)}$	Vector of reactive power, Var
$P_{g(N)}$	Probability of generating unit with N discrete status
$f_{(ni)}$	Probability of dispatch
f	Damping frequency, hertz

Greek Symbols

ρ	Air density, kg/m^3.
ξ	Damping ratio
λ	Eigenvalues

Abbreviations

MCS	Monte Carlo Simulation
SRS	Simple Random Sampling
LHS	Latin Hypercube Sampling
CDF	Cumulative Distribution Function
SCIG	Squirrel Cage Induction Generator
DFIG	Doubly Fed Induction Generator
SMIB	Single Machine Infinite Bus

1. Introduction

The electric power system is naturally confronted with many uncertainties as a result of randomness associated with its operation. Random changes in load take place all the times, with subsequent adjustment of power generations; there are also variations in the transmission line parameters as a result of changes in the environmental conditions. Any of these stochastic processes can cause a change in the equilibrium state of a power system. In the transition period, synchronism may be lost, or growing oscillation may occur over a transmission line, which could eventually lead to instability of the power system. This kind of instability is referred to as small signal instability. With the recent deregulation of electricity markets, electric power from renewable energy sources has been integrated into the grid. Hence, the power system is now faced with greater uncertainties and is operated closer to security limits than in the past following the intermittent nature of most of the renewable energy sources such as wind, solar, tidal, small hydro, and so forth. Of these various sources, wind energy is the most stochastic due to incessant changes in weather conditions. As more wind power is injected into the grid, the possible implications on the overall power system dynamics is of great concern. A key aspect of such dynamics is the study of the small signal stability.

There are various methods that have been proposed in literature to study the uncertainty regarding the small signal stability of a power system [1-7]. Of these methods, the Monte Carlo Simulation (MCS) is the most popular [8]. This is because of its ability to handle large complex non-linear power systems [2] with high levels of flexibility and accuracy [7]. However, many sample data are required to obtain reasonable results which could make it to be computationally intensive and time consuming [9;10]. In [6], probabilistic small signal stability was conducted using MCS, the uncertainty considered includes nodal load and conventional generation dispatch. The authors later conducted the same study using a grid computing framework in [5] to enhance the computational speed and memory space lacking in the conventional MCS approach. Macdonald [11] sets out to examine three aspect of the sampling method used in MCS applied to a typical building simulation problem. It was concluded that Monte Carlo analysis in building simulation application is most effective with SRS techniques. The probability small signal stability in all the aforementioned researches was conducted using MCS with different sampling techniques. However, none considered the impact of the sampling techniques on the small signal stability of a power system incorporating intermittent wind power generation. This paper therefore explores the usage of LHS techniques in generating random numbers from known probability distribution for the uncertainty assessment of small signal stability of wind connected power system using MCS and then compares the technique with the popular SRS techniques in term of accuracy, robustness and speed.

The MCS algorithm for small signal stability analysis begins with the modelling of the input parameters using random numbers. Subsequently, the load flow analysis is carried out using the Newton Raphson algorithm, followed by the eigenvalue computation using the method of modal analysis, and finally, the results are analysed statistically. The procedure used for the analysis is depicted in Fig. 1.

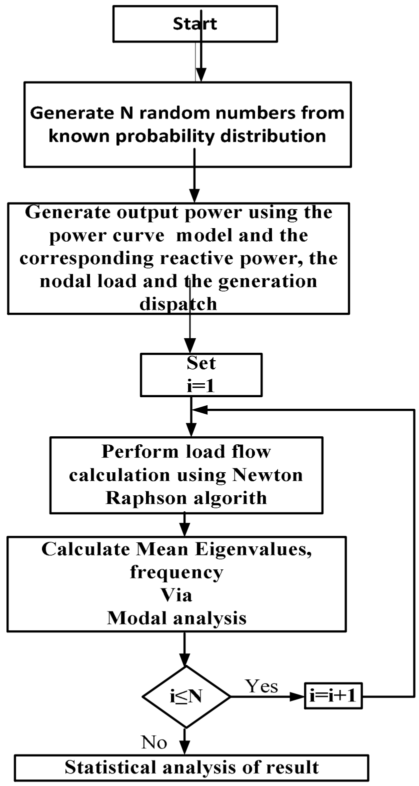

Fig. 1. Flowchart depicting the procedure for MCS Algorithm for Small Signal Stability.

2. Methodology

Monte Carlo simulations are set of computer algorithms for solving various kinds of uncertainty problems using random numbers. It is a numerical method which involves three basic steps as depicted in Fig 2: first, a random number from a given probability distribution is generated. Then, a mathematical model is solved deterministically to obtain the quantities of interest. Finally, a statistical analysis is performed. The first two steps are repeated a finite number of times.

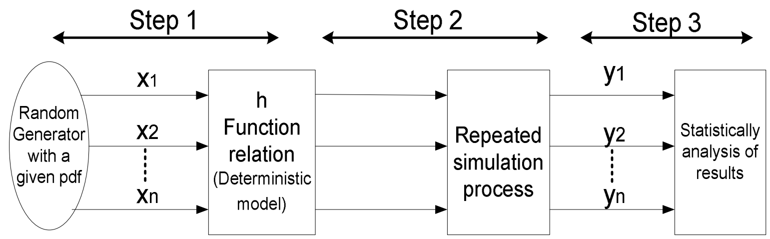

Fig. 2. Monte Carlo Simulation Steps.

Uncertainty analysis relating to the MCS can be described by a function given in (1)

$$y = h(x) \tag{1}$$

where h represents the function that describes the behaviour of the model under study, x and y are the vectors of input and output variables respectively, which are given by

$$\begin{aligned} x &= \left[x_1, x_2, ...x_n\right]^T \\ y &= \left[y_1, y_2, ...y_n\right]^T \end{aligned} \tag{2}$$

T denotes the transpose of matrix. The objective is to determine the probability distribution of the output variable y from the known probability distribution of the input variable x in a repeated simulation process.

The uncertainties in the input variable x are defined by the sequence of known probability distribution D, which is given by eq. (3):

$$D = (D_{1,} D_2, ...D_{qx}) \tag{3}$$

where D_i is the distribution associated with the element x_i of x and qx is the number of elements that is contained in x. It should be noted that dependency and additional relationship between the elements of x can be defined.

2.1. Simple random sampling technique

SRS is a standard sampling technique in MCS. It gives each sample of the variable an equal probability of being chosen and it serves as a theoretical basis for the design of other sampling methods. The generation of random samples in SRS mainly depends on the ability to generate uniformly distributed random numbers from the interval [0, 1]. From the uniform random numbers, one can reproduce random numbers for other distribution through a deterministic transformation known as the inversion method.

The relationship between two probability distribution function $p(b)$ and $q(a)$ can be written as (4) based on the fundamental transformation law of probabilities [8].

$$|p(b)db| = |q(a)da| \tag{4}$$

Equation (4) can be re-written as

$$p(b) = q(a)\left|\frac{da}{db}\right| \tag{5}$$

Equation (5) relates a random variable b from the distribution of $p(b)$ to a random variable a from the distribution of $q(a)$. If a is from a uniform distribution [0, 1], then $q(a)$ is constant, hence (5) becomes (6)

$$p(b) = \frac{da}{db} \tag{6}$$

The solution to (6) is given by equation (7), where $P(b)$ is the cumulative distribution function (CDF)

$$a = P(b) = \int_0^b p(c)dc \tag{7}$$

The relationship in (7) yields the target random variable given the source random variable.

The robustness of SRS in the MCS conforms to the law of statistics in determining the variance of the means of the output variables [11]. The mean of the samples can be estimated as (8):

$$\bar{y}_{(srs)} = \frac{1}{N}\sum_{i=1}^{N} y_i \tag{8}$$

where N is the number of samples and y_i is the model output.

Sample variance can be estimated as (9):

$$var\left(y_{(srs)}\right) = \frac{1}{N}\sum_{i=1}^{N}\left(y_i - \bar{y}_{(srs)}\right)^2 \tag{9}$$

The variance in estimating the mean can be determined as (10)

$$var\left(\bar{y}_{srs}\right) = \frac{1}{N} var\left(y_{(srs)}\right) \tag{10}$$

2.2. Latin hypercube sampling

Latin Hypercube Sampling (LHS) is a stratified sampling technique for sampling random variables from the entire distribution. It was first proposed by McKay in

1979 [12] and was developed as a result of concerns in the reactor safety community over the treatment of uncertainty analysis of complex systems [8]. The technique being used in LHS is known as stratified sampling without replacement where the random variable distributions are divided into equal probability intervals. Each sub-interval for each variable is sampled exactly once in a manner such that the entire range of each variable is represented. The sampling procedure can be viewed as follows:

Let $G_1,.......,G_N$ be the N input random variables in a probability problem. The cumulative distribution of G_n which belongs to $G_1,.......,G_N$ can be written as

$$Y_n = F_n(G_n) \tag{11}$$

For a sample size k, the range of Y_n from [0, 1] is divided into k non overlapping intervals of equivalent length; hence the length of each interval is givens as $1/k$

One sample value is selected from each interval randomly without replacement. Subsequently, the sampling values of G_n can be calculated by the inverse function of the equation (11). The nth sample of G_n can be determined by equation (12):

$$G_{nk} = F_n^{-1}\left(\frac{n-0.5}{k}\right) \tag{12}$$

The sample value of G_n can now be assembled in a row of the sampling matrix as (13):

$$[G_{n1}, G_{n2}, ..., G_{nk} ......G_{nN}] \tag{13}$$

Once all the N input random variables are sampled, an Nxk primary sampling G can be obtained, where k is the size of the sample.

LHS can always start with the generation of uniform distributed samples in interval [0, 1]. The CDF can then be inverted to obtain the target distribution [13]. The robustness of the sampling techniques depends on the mean output statistic [11]. Therefore, the mean and the variance of the sample can be evaluated by (14) and (15) respectively.

$$\bar{y}_{(lhs)} = \frac{1}{N}\sum_{i=1}^{N} y_i \tag{14}$$

$$var\left(y_{(lhs)}\right) = \frac{1}{N}\sum_{i=1}^{N}\left(y_i - \bar{y}_{(lhs)}\right)^2 \tag{15}$$

The variance of the mean can be calculated as (16)

$$Var(\bar{y}_{(lhs)}) = \frac{1}{N}var(y_{(lhs)}) - \frac{N-1}{N}cov(A1, A2) \tag{16}$$

where $cov(A1, A2)$ is the covariance between random variables.

3. Uncertainty Model of Input Variables for Small Signal Stability Analysis

The input random variables for the analysis of small signal stability of wind generator-connected power system include the stochastic active and reactive wind power generated by the wind turbine, the change in the conventional generation dispatch and the variation in nodal loads.

3.1. Active and reactive power of wind generator

3.1.1. Wind speed model

The variation in wind speed was modelled using two parameters Weibull distribution as expressed by (17).

$$f_W(v) = \frac{k}{c}\left(\frac{v}{c}\right)^{k-1} exp\left[-\left(\frac{v}{c}\right)^{k}\right] \tag{17}$$

where $f_W(v)$ is the probability of observing wind speed, v. The Weibull shape (k) and scale (c) parameters as used in this paper to generate wind speeds are provided in Table A1 in the appendix. The wind speed is generated from either the SRS $\bar{y}_{(srs)}$ or LHS ($\bar{y}_{(lhs)}$)

3.1.2. Modelling the active power generated by wind generator

Not all the energy in the mass of moving air can be converted into electrical power. The amount of energy that can be converted into useful electrical power (P_e) depends on the coefficient of the performance of the turbine, the transmission efficiency and the generator efficiency which can be written as (18)

$$P_e = \frac{1}{2}\rho A C_p \eta_m \eta_g v^3 \tag{18}$$

where C_p is the coefficient of the performance of the turbine [14], η_m is the mechanical transmission efficiency and η_g is the generator efficiency. For pitch-controlled wind turbine, the generated active power of WECS can be modelled as eq. (19) [15;16].

$$P_{e(v_i)} = \begin{cases} P_r \dfrac{v_i^2 - v_{ci}^2}{v_r^2 - v_{ci}^2} & (v_{ci} \le v_i \le v_r) \\ P_r = \dfrac{1}{2}\rho A C_p \eta_m \eta_g v_r^3 & (v_r \le v_i \le v_{co}) \\ 0 & (v_i \le v_{ci},\ v_i \ge v_{co}) \end{cases} \tag{19}$$

where v_i is the vector of wind speeds generated according to the Weibull distribution of the known parameters, $P_{e(v_i)}$ is the active wind power generated in accordance with the generated wind speeds and the power curve of a wind turbine;

v_{ci} is the cut-in wind speed, v_r is the rated wind speed, v_{co} is the cut-out wind speed, P_r is the rated power. The parameters of the commercially available wind turbine (VESTAS-V82) as provided in Table A2 in the appendix are selected for the simulation.

3.1.3. Reactive power generated by wind generator

Most generators used in wind power applications are asynchronous generators and can be classified into four groups [17] : (i) the SCIG, (ii) the wound rotor induction generator with variable rotor resistance, (iii) the DFIG, and (iv) the induction generator with a full scale converter. The generators in group (i) and (ii) absorb reactive power from the grid. The reactive power absorbed is a function of the active power generated. However, the reactive power for generators in group (iii) and (iv) can be controlled independently using the frequency converter to maintain unity power factor. It can be assumed that there is no exchange of reactive power between the generator and the grid; hence the reactive power is maintained at zero.

In this study, the generators belonging to group (i) and (ii) are considered and they are commonly modelled using RX model when considering them for uncertainty studies [18]. The active power injected into the grid is derived from the power curve of the wind turbine with the prior knowledge of wind speed and its distribution. The reactive power absorbed from the grid can be derived using the steady state equivalent circuit of an asynchronous induction generator as shown in Fig 3.

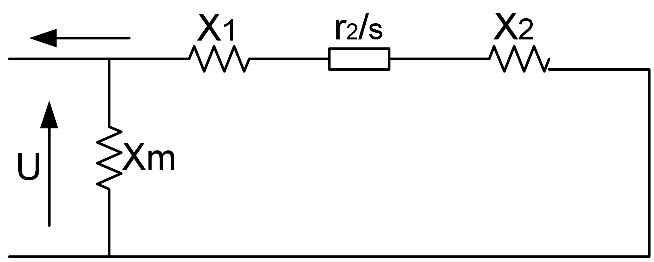

Fig. 3. Simplified steady state equivalent circuit of an asynchronous generator

where U is the terminal voltage, X_1 is the stator reactance, X_2 is the rotor reactance, X_m is the magnetising reactance, r_2 is the rotor resistance, s is the rotor slip of the asynchronous generator. The stator resistance is neglected. From the circuit, the real power injected to the grid is given by (20).

$$Pe_{(v_i)} = \frac{-U^2 \frac{r_2}{s_i}}{\left(\frac{r_2}{s_i}\right)^2 + X^2} \tag{20}$$

where $X = X_1 + X_2$ and $Pe(v_i)$ is the generated active power at different wind speeds according to the power curve in equation (19). The amount of reactive power absorbed from the grid depends on the rotor slip s_i, which also changes as the wind power varies in accordance with the variations in wind speed. Based on equation (20), s_i was derived as (21). The reactive power $Q_{(v_i)}$ absorbed at different wind speeds can be computed as (22).

$$s_i = \frac{-Ur_2 + \sqrt{U^4 r_2^2 - Pe_{(v_i)}^2 X^2 r_2^2}}{2Pe_{(vi)} X^2} \tag{21}$$

$$Q_{(v_i)} = \frac{s_i^2 X(X + X_m) + r_2^2}{s_i X_m r_2} Pe_{(v_i)} \tag{22}$$

Different slip and reactive power can be obtained from the relation between active power and reactive power for different generated active power, depending on the wind speed.

3.1.4 Active and reactive power of wind farm

The combination of several Wind Energy Conversion System (WECS) constitutes a Wind Farm (WF) which may be of the same type or different types. The power output of WF consisting of the same types of wind turbines as in the case of this study can be determined as (23) and (24), respectively.

$$P_{e(v_i)wf} = NP_{e(vi)} \tag{23}$$

$$Q_{(v_i)wf} = NQ_{(vi)} \tag{24}$$

3.2 Modelling nodal loads for uncertainty study

It is generally accepted that the electrical loads in power systems can be modelled using normal probability distribution function (25) [4;19]. This distribution has been selected for modelling all the nodal loads.

$$f(y;\mu,\sigma^2) = \frac{1}{\sigma\sqrt{2\pi}} exp^{-\frac{(y-\mu)^2}{2\sigma^2}} \tag{25}$$

where μ is the mean vector, σ^2 is the variance matrix and σ is the standard deviation.

3.3 Uncertainty modelling of conventional generator's dispatch

The variation in the generation output of conventional synchronous generators is mainly due to maintenance plan and unit dispatches as a result of market rules [6]. These uncertainties are commonly modelled using either the binomial distribution or the discrete probability model [6;20]. The discrete probability model is chosen for modelling the output of synchronous generators in this study because it can enhance the fast convergence of the simulation. The model assumes that generation units have distinct statuses in terms of output power. Each of the N generating status assumes a certain probability which adds up to unity as given in equation (26).

$$P_{g(N)} = \sum_{i=1}^{N} f_{(ni)} = 1 \qquad (26)$$

where $P_{g(N)}$ is the probability of generating unit with N discrete status, $f_{(ni)}$ is the probability of dispatch. Discreet probabilities of generation are provided in Table A4 in the appendix.

4. Power System Model and Small Signal Stability Analysis

The dynamic behaviour of a large complex non-linear system such as a power system can be represented by equation (27).

$$\begin{aligned} \dot{x} &= f(x,u,t) \\ y &= g(x,u,t) \end{aligned} \qquad (27)$$

where

$$\begin{aligned} x &= (x_1, x_2, \ldots\ldots, x_n)^T \\ u &= (u_1, u_2, \ldots\ldots, u_n)^T \end{aligned}$$

x is the state vector, u is the input vector and y is the output vector.

Linearising at the system equilibrium yields equation (28)

$$\begin{aligned} \Delta\dot{x} &= A\Delta x + B\Delta u \\ \Delta y &= C\Delta x + D\Delta u \end{aligned} \qquad (28)$$

Subsequently, the system stability subject to a small disturbance is studied based on the state matrix A

$$det(\lambda I - A) = 0 \qquad (29)$$

The values of λ that satisfy equation (29) are the eigenvalues of matrix A. They contain information about the response of the system to small perturbation. The eigenvalue can be real and/or complex. The complex values appear in conjugate pairs if A is real (30).

$$\lambda_i = \sigma_i \pm j\omega_i \qquad (30)$$

where σ_i and ω_i are the real and imaginary part of the eigenvalue λ_i. The frequency of oscillation in, Hz, and the damping ratio are given by equation (31) and (32).

$$f = \frac{\omega_i}{2\pi} \qquad (31)$$

$$\xi = \frac{-\sigma_i}{\sqrt{\sigma_i^2 + \omega_i^2}} \qquad (32)$$

5. System under Study

The performance of LHS in comparison with SRS in small signal stability application of a wind connected power system was first carried out using single machine infinite bus (SMIB) system as depicted in Fig 4. The load on bus 3 was modelled with normal distribution generated using SRS and LHS in turn. 2MW Wind generator made of SCIG is connected to the grid through a 0.69/33kV transformer. The wind speeds that were used to generate the wind power through the turbine power curve were drawn using Weibull distribution via SRS and LHS in turn.

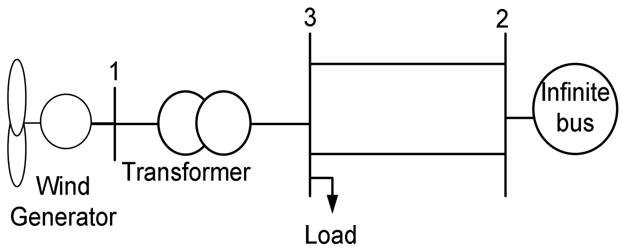

Fig. 4. Single machine infinite bus (SMIB) test system

In order to demonstrate confidence in the result obtained from the SMIB test system in the evaluation of the sampling techniques, a further numerical experiment was conducted using a larger network (IEEE 16–machine 68 bus systems) as depicted in Fig 5.

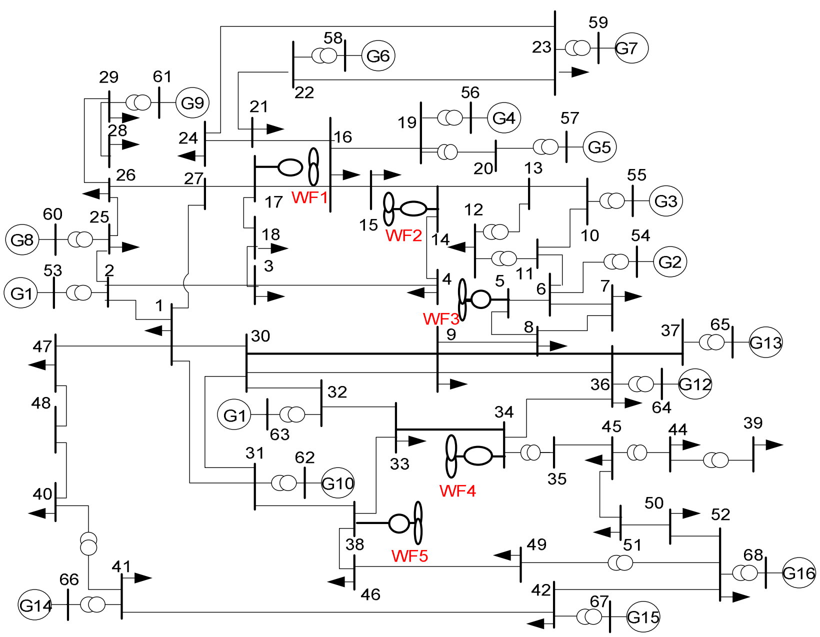

Fig. 5. IEEE 16–machine 68 bus system with 5 wind farms

The system consists of 58 varying quantities, i.e. 5 wind farms generating both active and reactive power in accordance with changes in wind speeds modelled using Weibull distribution, 15 synchronous generators modelled with 5 discrete probability units and 33 electrical nodal loads modelled as a normal distribution. All the synchronous generators were modelled as 4th order and were equipped with an IEEE type 1 exciter model. The network data for both systems can be found in [21]. The Weibull parameters for the generation of wind speed, the wind turbine parameters, the discrete probability for generation dispatch of the conventional generators and the nodal loads mean and standard deviation are provided in Table 5–Table 8 in the appendix. Each WF consists of 100 wind turbines.

6. Simulation Result and Discussion

6.1. Single Machine Infinite Bus (SMIB) System

The modal analysis of wind generator-connected SMIB was performed using different sample sizes of wind speeds and nodal loads generated from required distribution using SRS and LHS in turn. A total of 12 eigenvalues was obtained in each case in which 4 (2 pairs) of the eigenvalues are oscillatory. The mean real part of the eigenvalues, the mean damping ratio and the mean frequency of oscillation of one of the oscillatory modes obtained from the two sampling techniques are compared for different sample sizes. It is generally believed in Monte Carlo simulation uncertainty analysis that the higher the sample size, the better the results [7]. The result obtained using a sample size of 50000 is regarded in this paper as reasonable results (IDEAL).The modal analysis results obtained from each sample size using SRS and LHS in turn are compared to the IDEAL results (i.e acceptable result). The results of the comparison are presented in Figs 6-8. It can be observed from the figures that random variables drawn using LHS techniques produces the same results as that of the IDEAL starting from about 100 sample size. This indicates that, rather than using huge sample size to generate input variables i.e. wind speeds and nodal loads for the small signal stability study, about 100 sample size drawn from LHS technique is enough to produce reasonable results. This will reduce the computer required memory and increase the speed of computation. However, the random samples drawn from SRS techniques fluctuates to about 5000 sample size before it could produce the same result as that of the IDEAL value.

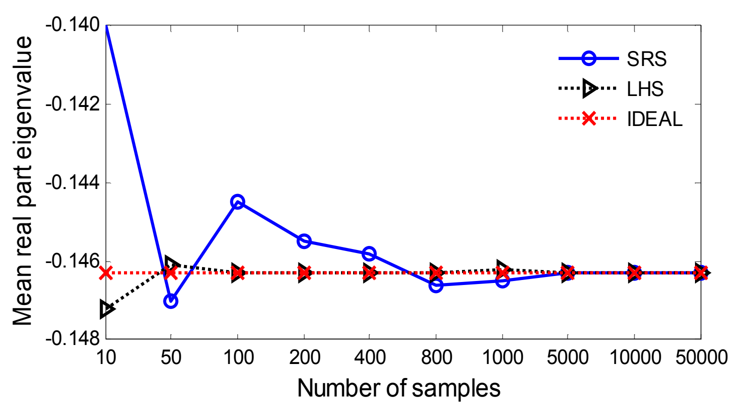

Fig. 6. Comparison of output Mean real part eigenvalue using different sample sizes and sampling method (SRS and LHS) on the SMIB test system

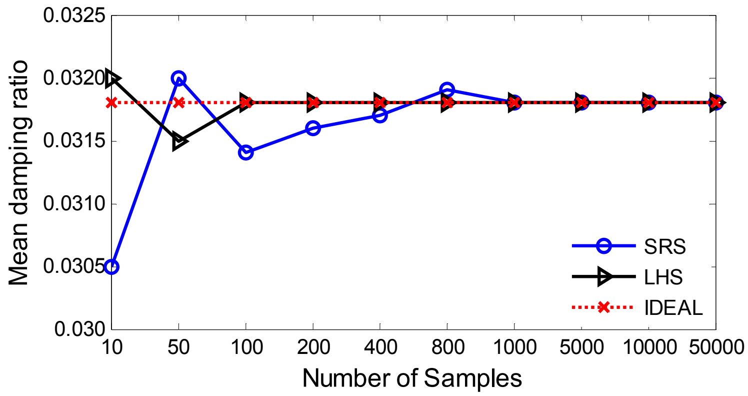

Fig. 7. Comparison of output mean damping ratio using different sample sizes and sampling method (SRS and LHS) on the SMIB test system

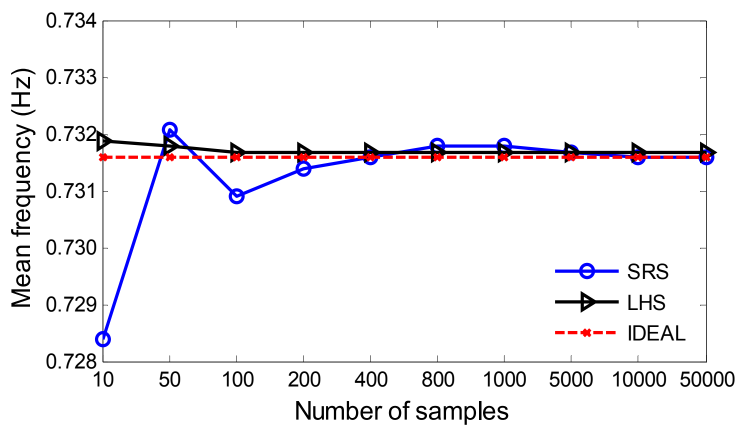

Fig. 8. Comparison of output oscillatory frequency (Hz) using different sample sizes and sampling methods (SRS and LHS) on the SMIB test system

The results of the two oscillatory modes obtained from the modal analysis of wind generator-connected SMIB system are compared in term of computational speed in Table 1. It is evident from the table that the LHS technique achieved the same result in 100 iterations as the result obtained from 50,000 iterations (ideal sample size) with lesser simulation speed and memory. The simulation speed for IDEAL is estimated to be about 2.57 hours while that of LHS and SRS techniques with 100 sample size are estimated as 58.6s and 46.7s respectively. This shows the computational advantage of LHS over the SRS method.

Table 1. Comparison of the 2 oscillatory modes obtained from SMIB using 100 sample size of SRS and LHS

Result		(IDEAL)	100 sample size of SRS	100 sample size of LHS
$E(\lambda)$	i	-1.2037± 0.2205i	-1.023 ± 0.456i	-1.2037± 0.2205i
	ii	-0.1462±4.5970i	-0.1320 ± 4.907i	-0.1463± 4.5972i
$E(\xi)$	i	0.9836	0.8130	0.9836
	ii	0.0318	0.0290	0.0318
$E(f)$	i	0.0345	0.0296	0.0341
	ii	0.7316	0.6819	0.7317
Time		2.57hrs	56.8s	48.7s

6.2. IEEE 16–Machine 68 Bus Systems

To increase the confidence in the initial results presented, large network (IEE 16-machine 68 bus systems) was used with more random input variables. 68 eigenvalues (modes) were obtained in which 24 (12 pairs) of it are oscillatory modes. For the purpose of this study, one oscillatory mode (one with least damping) was picked for analysis. The results of the modal analysis using different sample sizes are compared in Figs 9-11. It is again revealed that reasonable results could be obtained with approximately 100 sample size using LHS. This shows that, irrespective of the size of the electrical network, about 100 sample size is good enough to produce the same result as that of the IDEAL.

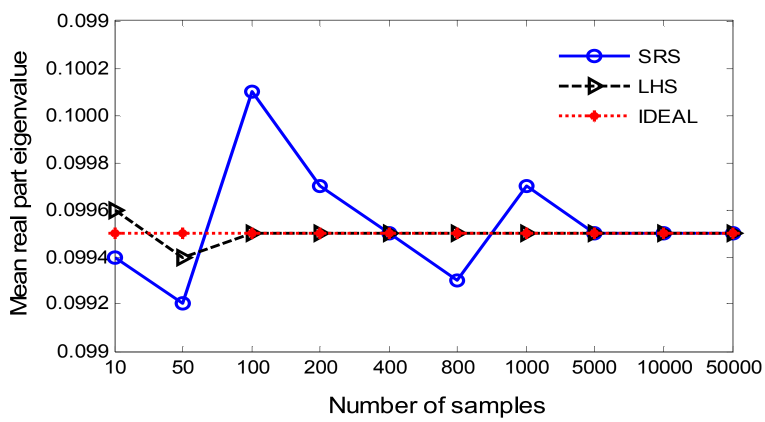

Fig. 9. Comparison of output mean real part eigenvalue using different sample sizes and sampling method (SRS and LHS) on the IEEE 16–machine 68 bus test system.

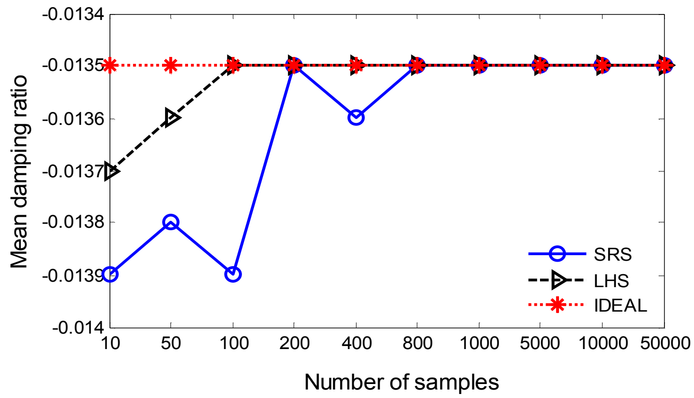

Fig. 10. Comparison of output mean damping ratio using different sample sizes and sampling methods (SRS and LHS) on the IEEE 16–machine 68 bus test system

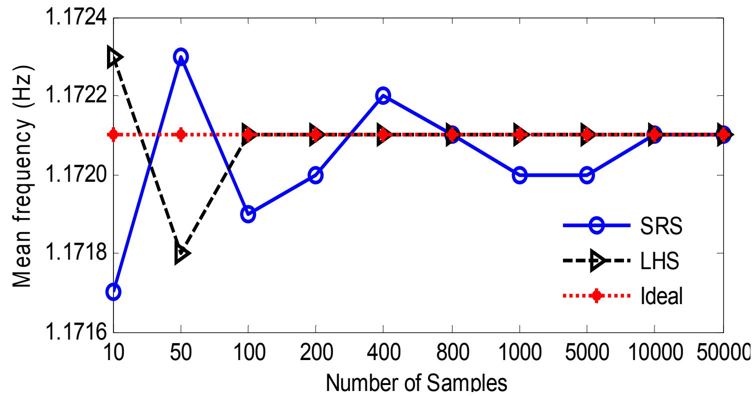

Fig. 11. Comparison of output mean oscillatory frequency using different sample sizes and sampling methods (SRS and LHS) on the IEEE 16–machine, 68 bus test system

Table 2 shows the result of the modal analysis of the least damping ratio of wind connected IEEE 16–machine 68 bus test system and the time taking to complete the computation. The table also indicates that about 100 sample size of LHS present basically the same result as that of IDEAL with lesser simulation time.

Table 2. Comparison of local mode with the least damping ratio obtained from the IEEE 16–machine 68 bus system

Result	50000 sample size (IDEAL)	100 sample size of SRS	100 sample size of LHS
$E(\lambda)$	0.0993± 7.3644i	0.1002 ± 8.105	0.0993±7.3643i
$E(\xi)$	-0.0135	-0.0218	-0.0135

E(*f*)	1.1721	1.310	1.1722
time	5.23hrs	1.08min	58.6s

6.3 Robustness of the sample techniques

The robustness of a sampling technique can be measured in terms of the significant reduction in variance between repeated analyses [11] . To measure the robustness of the output mean results obtained by the two sampling techniques, different sample sizes of 100, 1000, 5000 and 50000 were run repeatedly 100 times and the variance in the output statistic were evaluated using equations (10) and (16) in a repeated MCS. The flowchart of the algorithm is depicted in Fig 12 and was programmed based on the power system toolbox (PST) [21] in MATLAB^{TM} 2011b on an Intel(R) xeon (R), core 2 duo CPU T8300 @ 2.4Ghz, 16 GB of RAM.

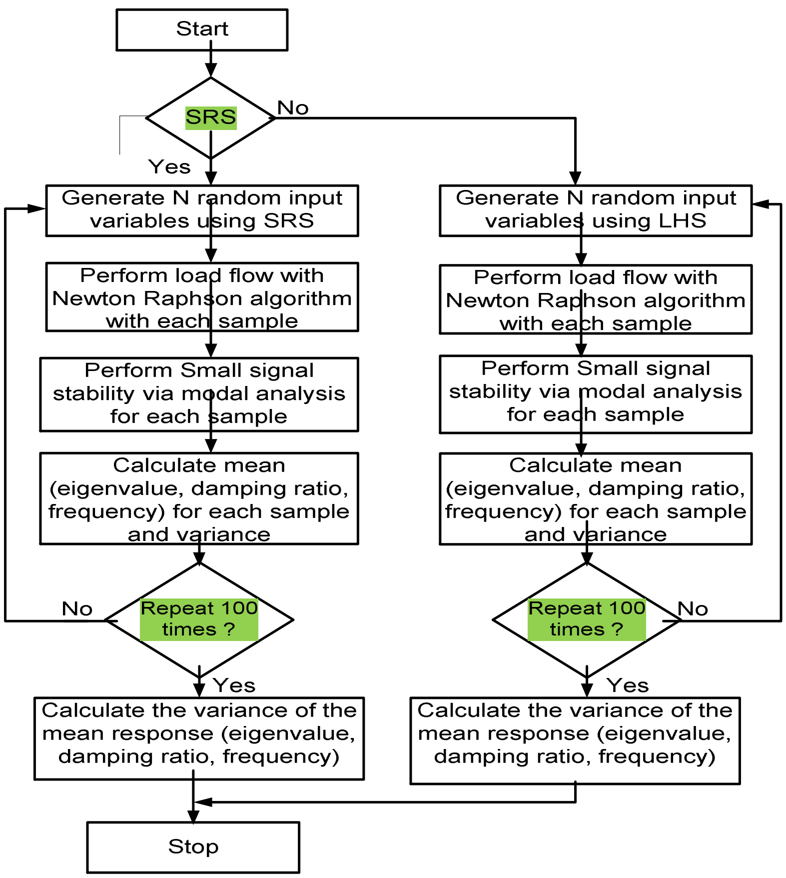

Fig 12. The flowchart to test the robustness of the sampling techniques

The results comparing the robustness of both sampling techniques are given in Table 3 and Table 4 for the SMIB and IEEE 16–machine 68 bus systems, respectively. The LHS method yielded a lower variance compared with the SRS of the same sample size. The tables also indicate that the variance reduced with the increase in sample size. This is expected, but interestingly, the value of variance in the result of LHS with a sample size of 100 yielded a comparable value to that of SRS with a sample size of 50000. This further corroborates the earlier simulation results.

Table 3: Variance of mean output variables for 100 repetitions (M_{run} =100) on SMIB

Sample size	Output variables	SRS	LHS
100	$var\left(\bar{\xi}\right)$	6.90x10-8	1.53x10-10
1000	$var\left(\bar{\xi}\right)$	8.95x10-9	1.83x10-12
5000	$var\left(\bar{\xi}\right)$	1.39x10-9	5.05x10-14
50000	$var\left(\bar{\xi}\right)$	1.26x10-10	1.09x10-15

Table 4: Variance of mean output variables for 100 repetition (M_{run} =100) on the IEEE 16–machine 68 bus system

Sample size	Output variables	SRS	LHS
100	$var\left(\bar{\xi}\right)$	7.57x10-9	3.89x10-11
1000	$var\left(\bar{\xi}\right)$	2.52x10-10	5.99x10-12
5000	$var\left(\bar{\xi}\right)$	8.74x10-11	2.24x10-12
50000	$var\left(\bar{\xi}\right)$	1.82x10-11	4.55x10-13

It can be inferred from the results presented in both tables that LHS is preferable compared to SRS in terms of robustness. It has also been demonstrated that 100 samples of LHS are sufficient to produce a reasonable result for the practical Monte Carlo-based small signal stability application. In this manner, the computation cost is greatly reduced.

7. Conclusion

The comparison in terms of accuracy, speed and robustness of two sampling techniques namely simple Random Sampling (SRS) Latin Hypercube (LHS) sampling techniques commonly used in the MCS has been examined for application in the probabilistic small signal stability analysis. Based on the results presented, it is concluded that LHS is preferable compared to SRS in terms of accuracy, speed and robustness. It has also been demonstrated that a sample size of 100 drawn from the LHS is adequate to produce a reasonable result for practical Monte Carlo based small signal stability applications. This in turn reduced the computational cost of the simulation.

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Appendix A
Parameters used for the study

Table A-1 The Weibull parameters and the location of wind farms on IEEE 16-machine 68 bus system

Bus / Parameters	5	14	17	34	38
Mean (m/s)	6.74	8.73	6.90	7.28	7.59
St. deviation	3.20	4.15	3.09	3.67	3.44
Shape (k)	2.24	2.23	2.40	2.08	2.35
Scale(c), m/s	7.61	9.86	7.80	8.22	8.58

Table A-2 Parameters of the induction generator

Asynchronous generator	Parameters
Rotor resistance r2 (pu)	0.009
Stator reactance x1 (pu)	0.01

Rotor reactance x2 (pu)	0.01
Magnetising reactance Xm (pu)	3.0
Inertia (H) (s)	2

Table A-3 Wind turbine parameters

Wind Turbine (VESTAS-V82)	Parameters
Rated turbine power, P_r (kW)	1650
Hub height (m)	70
Overall efficiency (%)	95
Cut-in wind speed , v_{ci} (m/s)	3
Rated wind speed , v_r (m/s)	13
Cut-out wind speed, v_{co} (m/s)	20

Table A-4 Discreet probabilities of generation dispatch on IEEE 16-machine 68 bus system

States	SG1		SG2		SG3		SG4	
	Mean	**Prob**	**Mean**	**Prob**	**Mean**	**Prob**	**Mean**	**Prob**
1	2.30	0.10	5.50	0.05	6.80	0.10	6.0	0.20
2	2.80	0.15	5.30	0.10	6.20	0.15	6.80	0.20
3	3.20	0.20	5.00	0.15	6.00	0.20	6.50	0.20
4	2.20	0.25	5.45	0.20	7.20	0.25	5.60	0.20
5	2.50	0.30	5.10	0.50	6.50	0.30	6.32	0.20
States	SG5		SG6		SG7		SG8	
	mean	**Prob**	**mean**	**Prob**	**mean**	**Prob**	**mean**	**Prob**
1	5.50	0.10	6.80	0.10	6.00	0.05	5.50	0.10
2	4.80	0.15	6.20	0.15	5.00	0.10	4.50	0.15
3	5.00	0.22	6.00	0.20	5.80	0.10	6.00	0.15
4	6.30	0.23	7.20	0.25	6.20	0.15	6.20	0.20
5	5.02	0.30	7.00	0.30	5.60	0.60	5.40	0.40
States	SG9		SG10		SG11		SG12	
	mean	**Prob**	**Mean**	**Prob**	**Mean**	**Prob**	**Mean**	**Prob**
1	7.60	0.10	6.00	0.20	11.00	0.05	14.30	0.10
2	8.30	0.15	4.90	0.20	9.80	0.10	13.20	0.15
3	6.00	0.20	4.80	0.20	10.50	0.15	12.80	0.25
4	7.20	0.25	5.50	0.20	9.30	0.20	14.00	0.25
5	8.00	0.30	5.0	0.20	10.00	0.50	13.50	0.25
States	SG13		SG14		SG15		SG16	
	mean	**Prob**	**Mean**	**Prob**	**Mean**	**Prob**	**Mean**	**Prob**
1	29.80	0.05	19.30	0.05	8.80	0.1	41.00	0.05

2	36.50	0.05	17.00	0.10	10.80	0.15	39.00	0.15
3	33.80	0.10	18.20	0.15	9.50	0.20	43.00	0.20
4	38.20	0.10	16.00	0.20	11.30	0.25	40.50	0.20
5	35.91	0.70	17.85	0.50	10.00	0.30	40.00	0.40

All values are in pu on the MVA base of 100MVA