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FINITE ELEMENT MODELLING OF A TURBINE BLADE TO STUDY THE EFFECT OF MULTIPLE CRACKS USING MODAL PARAMETERS

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Abstract

The presence of crack in a structure tends to modify its modal parameters (natural frequencies and mode shapes). The fact can be used inversely to predict the crack parameters (crack depth and its location) from measurement of the changes in the modal parameters, once a functional relationship between them has been established. The machine components like turbine blade can be treated as a cantilever beam. Vibration analysis of cantilever beam can be extended successfully to develop online crack detection methodology in turbine blade. In the present work, finite element model of a cantilever beam for flexural vibrations has been considered by including two transverse open U-notches. The modal analysis has been carried out on cantilever beam with two U-notches and observed the influence of one U-notch on the other for natural frequencies and mode shapes. It is found that a certain frequency may correspond to different crack depths and locations. Later, by using a central difference approximation, curvature mode shapes were then calculated from the displacement mode shapes. The location and depth corresponding to any peak on this curve becomes a possible notch location and depth. The identification procedure presented in this study is a useful tool for detection of multiple cracks in a turbine blade.

Keywords: Finite element, Natural frequency, Mode shapes, Cantilever, Multiple cracks

1. Introduction

Any localized crack in a structure reduces the stiffness in the cracked area. These features are related to variation in the dynamic properties, such as, decreases in

Nomenclatures

a	Depth of the crack, mm
$a//h$	Crack depth ratio
b	Thickness of the crack, mm
c	Location of the first crack from the fixed end, mm
c/l	First crack location ratio from fixed end
d	Location of the second crack from the fixed end, mm
d/l	Second crack location ratio from fixed end
e	Element edge length, mm
h	Height of the cantilever beam, mm
i	Node number
k	Curvature of bending
l	Length of the cantilever beam, mm
l/h	Length to height ratio of the beam
w	Width of the beam, mm
ω_1	Natural frequency of the first mode of vibration, Hertz
ω_2	Natural frequency of the second mode of vibration, Hertz
ω_3	Natural frequency of the third mode of vibration, Hertz

natural frequencies and variation of the modes of vibration of the structure. One or more of above characteristics can be used to detect and locate cracks. This property may be used to detect existence of crack or faults together with location and its severity in a structural member. Rizos et al. [1] measured the amplitude at two points and proposed an algorithm to identify the location of crack. Pandey et al. [2] suggested a parameter, namely curvature of the deflected shape of beam instead of change in frequencies to identify the location of crack. Ostachowicz and Krewczuk [3] proposed a procedure for identification of a crack based on the measurement of the deflection shape of the beam.

Ratcliffe [4] also developed a technique for identifying the location of structural damage in a beam using a 1D FEA. A finite difference approximation called Laplace’s differential operator was applied to the mode shapes to identify the location of the damage. Wahab and Roeck [5] investigated the application of the change in modal curvatures to detect damage in a prestressed concrete bridge. Lakshminarayana and Jebaraj [6] carried out analytical work to study the effect of crack at different location and depth on mode shape behaviour.

Kishen and Sain [7] developed a technique for damage detection using static test data. Nahvi and Jabbari [8] established analytical as well as experimental approach to the crack detection in cantilever beams. Babu and Prasad [9] used differences in curvature mode shapes to detect a crack in beams. Wang et. al. [10] incorporated finite element method for dynamic analysis and the mode shape difference curvature information for damage detection/diagnosis.

This paper deals with the technique and its application of natural frequencies and mode shapes to a cantilever beam. The paper of Pandey et al. [2] showed a quite interesting phenomenon: that is, the modal curvatures are highly sensitive to damage and can be used to localize it. They used simulated data for a cantilever and a simply supported beam model to demonstrate the applicability of the method. The damaged beam was modelled by reducing the E-modules of a certain element. By plotting the difference in modal curvature between the intact and the damaged case, a peak appears at the damaged element indicating the presence of a fault. They used a central difference approximation to derive the curvature mode shapes from the displacement mode shapes.

So, this paper is done in two parts. First part of this paper studies about the changes in natural frequencies because of the presence of multiple cracks in the cantilever beam. The second part of this paper is concerned with investigation of the accuracy when using the central difference approximation to compute the modal curvature and determine the location of the cracks. The application of this technique to turbine blades in which more than one fault positions exist is investigated using a continuous beam with simulated data. The results of this scenario will be analyzed in this paper and multiple cracks will be detected and localized by using the measured change in modal curvatures.

The objective of this paper to find a methodology for predicting crack parameters (crack depth and its location) in a cantilever beam from changes in natural frequencies and curvature mode shapes has been developed. Parametric studies have been carried out using ANSYS Software to evaluate modal parameters (natural frequencies and mode shapes) for different crack parameters. Curvature mode shapes were then calculated from the displacement mode shapes to identify crack location and its severity in the cantilever beam.

2. Theory

Steam turbines have number of stages and employs number of blades ranging from few centimeters in length in high pressure turbine to almost one meter long blades in the low pressure turbine. And the natural frequencies of high pressure turbine blades are high compared to low pressure turbine blades, so even small decrease of natural frequency of low pressure turbine blade makes frequency of the blade closer to operating frequency. Also, for the same crack parameters the eigen value changes are more appreciable for high pressure turbine blade when compared to low pressure turbine blade. It means that the crack in a high pressure turbine blade changes the dynamic behavior more compared to that of low pressure turbine blade. So, the condition of high pressure turbine blade is predicted more perfectly than that of low pressure turbine blade using vibration response.

So, modal analysis of low pressure steam turbine blade was carried out which has been modelled with airfoil cross section and tapered length. The first three natural frequencies are of the turbine blade of low pressure steam turbine last stage blade is given in Table 1 for different angle of twists. Figure 1 confirms the results of turbine blade.

From the Table 1 it was found that first mode frequency for turbine blade is almost same with increased angle of twist. Second mode frequency is increasing with increased angle of twist and Third mode frequency is decreasing with increased angle of twist.

Later same analysis was carried out for cantilever beam with different angle of twists. The low pressure steam turbine is modelled as cantilever beam with l/h=60. The material properties are assigned to the blade and boundary conditions are defined same as that of turbine blade. Natural Frequency values for cantilever beam of rectangular cross section are analyzed and the first, second and third modes are given in Table 2.

Table 1. Natural frequencies of turbine blade with different angles of twist (for uncracked blade).

Angle of Twist	ω_1	ω_2	ω_3
0^o	74.093	268.71	690.59
15^o	72.775	229.47	685.18
30^o	72.395	236.55	719.23
45^o	72.281	269.16	690.62
60^o	71.596	301.53	677.20

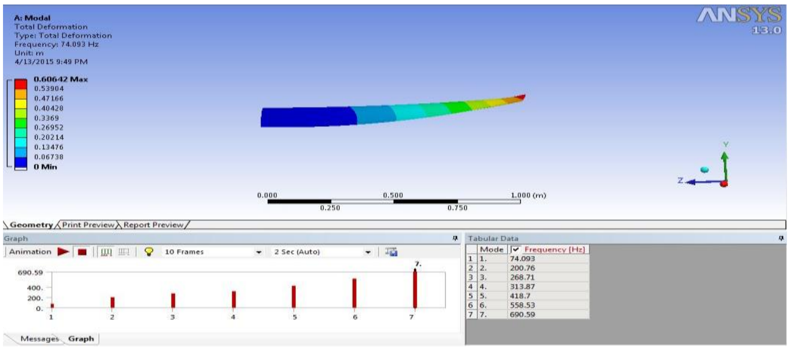

Fig. 1. First mode shape of turbine blade with 0^o angle of twist.

Table 2. Natural frequencies of twisted cantilever beam with different angles of twist (for uncracked beam).

Angle of Twist	ω_1	ω_2	ω_3
0^o	171.89	1075.8	3006.0
15^o	170.65	918.72	2994.6
30^o	170.52	945.18	3142.6
45^o	170.78	1083.5	3008.7
60^o	169.28	1208.2	2947.7

The same results of turbine blade can be observed for cantilever beam as frequency values given in Table 2. First mode frequency is almost same with increased angle of twist. Second mode frequency is increasing with increased angle of twist and Third mode frequency is decreasing with increased angle of twist. Hence one can conclude that vibration response of a component will purely depend upon mass and stiffness of the component. It is merely affected by size and shape of the component. So, from this analysis one can conclude that modal analysis of cantilever beam can be validated for turbine blades.

The low pressure steam turbine blades on turbine-generator units at an electricity generating station have had a history of failures. Early experience was with failures at the root attachment of the blade, but subsequently failures also occurred at the outer of two lacing wire holes. So, it is required to know the behavior of turbine blades with multiple cracks.

Stress corroded blades will have loss of material. After material removal the damage location can be mapped into U-notch. For this type of damages regular elements with finer mesh can be opted for modelling. For shape edge cracks one had to select crack tip elements to model the cracked geometry. In the present study, U-notch has been modelled. Figure 2 shows a cantilever beam of rectangular cross section, made of mild steel with two U- notches. To find out mode shapes associated with each natural frequency, finite element analysis has been carried out using ANSYS software for uncracked and cracked beams.

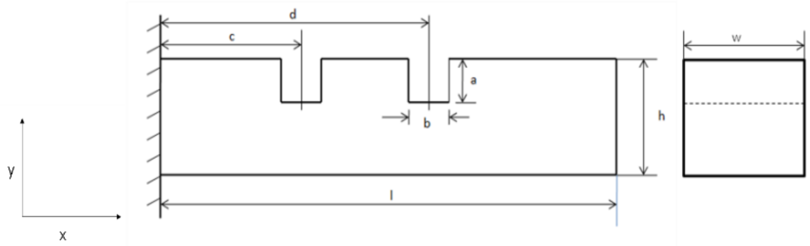

Fig. 2. Cantilever beam with two U-notch cracks.

The mode shapes of the multiple cracked cantilever beams are obtained for U-notches located at normalized distances (c/l & d/l) from the fixed end with a normalized depth (a/h). Figure 3 shows the discretised model (zoomed near the position of U-notches) of cantilever beam. Parametric studies have been carried out for beam having length (l) = 260 mm, width (w) = 25 mm and thickness (h) = 4.4 mm. The breadth (b) of each U-notch has been kept as 0.32 mm. The U-notch locations from the fixed end (c & d) of the cantilever beam have been taken in different combinations near the fixed end, free end and middle of the beam. The intensity of U-notch (a/h) was varied by increasing its depth over the range of 0.25 to 0.75 in the steps of 0.25. This represented the case of a varying degree of crack at particular location. For each model of the two U-notch locations, the first three natural frequencies and corresponding mode shapes were calculated using ANSYS software.

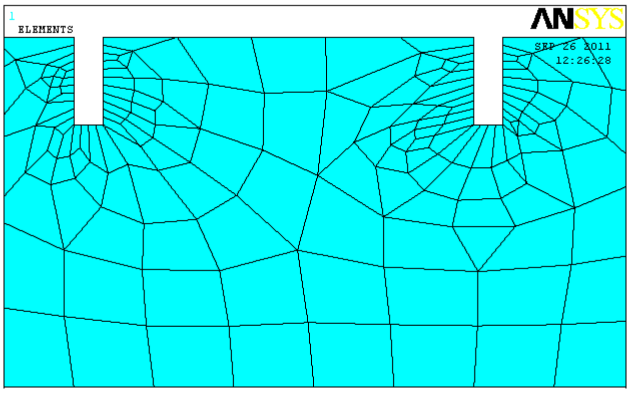

Fig. 3. Discretised model of beam with two U-notches.

Also, localized changes in stiffness result in a mode shape that has a localized change in slope, therefore, this feature will be studied as a possible parameter for crack detection purposes. For a beam in bending the curvature (*k*) can be approximated by the second derivate of the deflection:

$$k = \frac{d^2y}{dx^2} \quad (1)$$

In addition, numerical mode shape data is discrete in space, thus the change in slope at each node can be estimated using finite difference approximations. In this work, the central difference equation was used to approximate the second derivate of the displacements u along the X direction at node *i*:

$$k_i = \frac{y_{i-1} - 2y_i + y_{i+1}}{e^2} \quad (2)$$

The term $e=x_i-x_{i-1}$ is the element length. In this process meshing and node numbering is very important. Equation 2 required the knowledge of the displacements at node *i*, node *i-1* and node *i+1* in order to evaluate the curvature at node *i*. Thus, the value of the curvature of the mode shapes could be calculated starting from node 2 through node 261 in case of this beam. After obtaining the curvature mode shapes the absolute difference between the uncracked and cracked state is determined to improve crack detection.

$$\Delta(k_i) = |k_i|_{No\ Crack} - |k_i|_{With\ Crack} \quad (3)$$

As a result of this analysis, a set of curvature vectors for different crack localizations are obtained.

3. Results and Discussion

In this analysis, it is assumed that cracks are of U-notched shape. The depth (*a*) and locations (*c* & *d*) of these notches are normalized to the height and length of the cantilever beam respectively. The first three natural frequencies (ω_1, ω_2, & ω_3) for the beam were calculated using ANSYS software and were shown in the Table 3 for different crack depths and crack location ratios.

3.1. Analysis of natural frequencies

The natural frequencies for the first three modes of the uncracked cantilever beam are 171.89, 1075.8 and 3006 respectively. The natural frequencies of the cantilever beam with multiple cracks for different crack location and crack depths are given in Table 3.

It is seen clearly from the Table 3 that the changes in natural frequencies due to crack are appreciable and they decreases as the crack depth ratio increases. Also, when the cracks are located near the fixed end, the first natural frequency (ω_1) is most affected with respect to the severity of the crack as we can see in cases 1, 2 and 3 of the Table 3 for different crack ratios (*c/l*= 0.15 & 0.25). The second natural frequency (ω_2) is most affected as the crack severity is increased at the location close to the middle of the beam. From the Table 3 same can be observed in cases 2, 3 and 4 for *c/l* ratios 0.45 & 0.65. The third natural frequency (ω_3) is more sensitive throughout the beam except when the crack is near the middle portion of the beam which can be seen in all the cases of the Table 3.

Table 3. Natural frequencies for multiple cracks.

Case No.	Crack Location ratio (*c/l* & *d/l*)	Cracks Depth ratio (*a/h*)	Natural Frequencies ω_1	ω_2	ω_3
1	0.15 & 0.8	1.1 & 1.1	165.7	1046.1	2914.0
		1.1 & 2.2	165.6	1037.1	2814.4
		1.1 & 3.3	165.3	983.5	2877.5
		2.2 & 2.2	156.8	1030.5	2818.1
		3.3 & 2.2	123.2	1007.8	2809.0
2	0.25 & 0.45	1.1 & 1.1	165.7	1040.1	2911.7
		2.2 & 1.1	159.5	1038.9	2832.5
		2.2 & 2.2	157.5	1002.7	2825.7
		2.2 & 3.3	146.2	856.0	2797.3
		3.3 & 1.1	132.9	1033.6	2560.2
3	0.25 & 0.65	1.1 & 1.1	166.2	1041.4	2887.4
		1.1 & 2.2	165.7	1006.2	2780.8
		1.1 & 3.3	162.8	849.9	2466.7
		2.2 & 2.2	159.5	1009.3	2710.5
		3.3 & 2.2	132.9	1002.3	2447.1
4	0.65 & 0.85	1.1 & 1.1	167.7	1041.2	2896.3
		1.1 & 2.2	167.8	1037.6	2847.9
		1.1 & 3.3	167.7	1016.0	2563.0
		2.2 & 2.2	167.3	1002.9	2745.3
		3.3 & 2.2	164.3	847.5	2461.8

3.2. Analysis of mode shapes

The first three natural frequencies and mode shapes of the uncracked cantilever beam and of the cracked cantilever beam at two different element combinations is evaluated in ANSYS. It was found that there is a percentage change in the frequency from the uncracked to the cracked case, but no indication of the exact location of crack is obtained from this without further analysis. Figures 4, 6, 8 and 10 indicate the mode shapes which are almost identical. Hence it is difficult to detect the location of crack. The difference curvature mode shapes are shown in Figs. 5, 7, 9 and 11 which are plotted from the absolute differences between the curvature mode shapes of the uncracked and the cracked cantilever. The maximum difference for each curvature mode shape occurs in the cracked region, which are at point's *c/l* and *d/l*. This characteristic of curvature mode shapes can be very useful in locating the cracked area. So, it can be concluded that by using finite difference approximation crack parameters (depth and location) may be identified for medium size cracks (*a/h*=0.25) in cantilever beam like structures.

3.3. Curvature finite difference approximation

Uncracked Case

By using the same finite element model shown in Fig. 2, linear mode shapes were performed in ANSYS. The numerical results were exported to MATLAB to be processed.

The associated mode shapes were sketched evaluating the displacements in y direction of the 261 equidistant nodes located at the bottom line of the beam. In order to unify the results from the different cases, mode shapes were normalized by setting the largest grid point displacement equal to 1. It can be noticed from figures that all the mode shapes smoothed functions, what indicate the absence of cracks. Cracked mode shapes will be used to compare further results. Since changes in the curvature are local in nature, they can be used to detect and locate cracks in the beam.

Simple Cracked Case

The different combinations of crack location scenarios are selected for studying the effect of localized cracks in the cantilever beam. Although the reduction in natural frequencies is related to the existence of crack and its severity, this feature cannot provide any useful information about the location of the crack. Thus, curvature mode shapes were calculated and compared with the uncracked case. It can be seen that the maximum difference value for each mode shape occurs in the crack locations. In other areas of the beam this characteristic was much smaller. Although the third mode shapes was the most sensitive to the failure it is important that any of the three curvature mode shapes peak at the cracked locations.

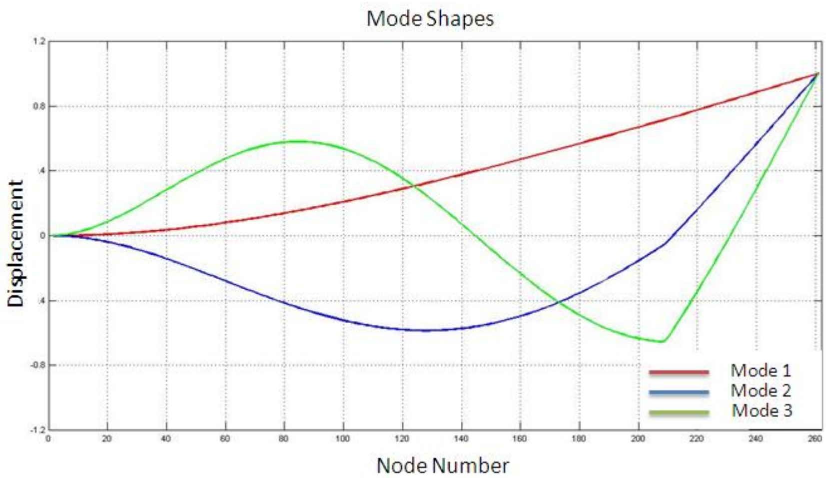

Fig. 4. Mode shapes for crack at *c/l* = 0.15 of depth *a/h* = 0.25 and crack at *d/l* = 0.8 of depth *a/h* = 0.75.

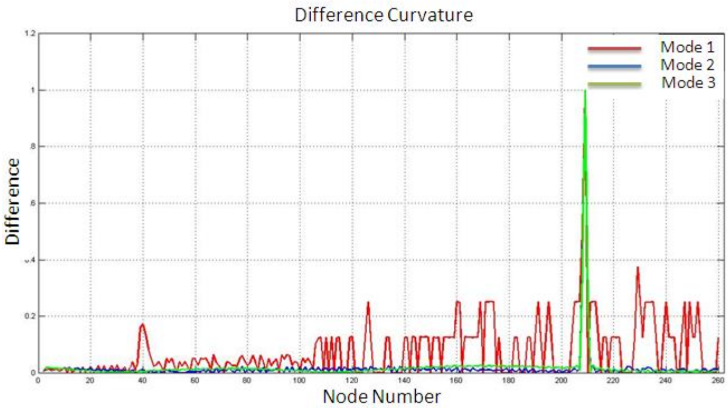

Fig. 5. Difference curvature for crack at *c/l* = 0.15 of depth *a/h* = 0.25 and crack at *d/l* = 0.8 of depth *a/h* = 0.75.

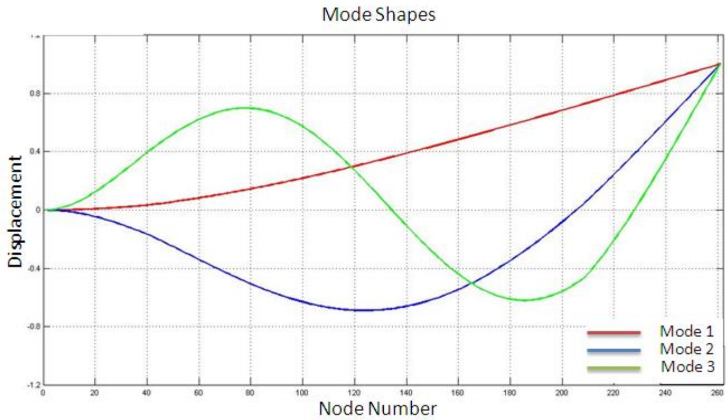

Fig. 6. Mode shapes for crack at *c/l* = 0.15 of depth *a/h* = 0.5 and crack at *d/l* = 0.8 of depth *a/h* = 0.5.

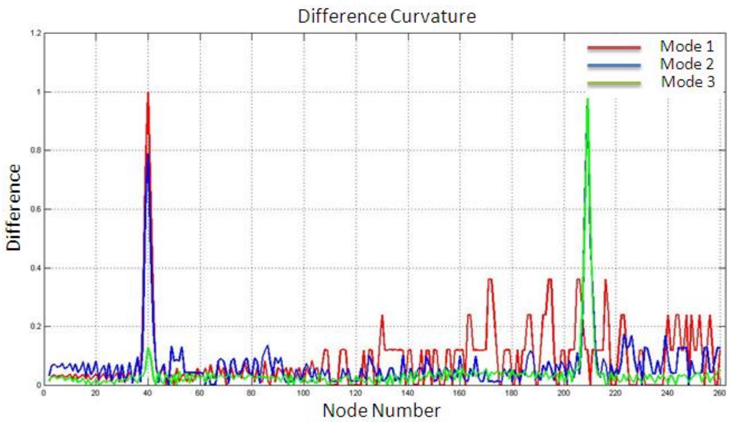

Fig. 7. Difference curvature for crack at *c/l* = 0.15 of depth *a/h* = 0.5 and crack at *d/l* = 0.8 of depth *a/h* = 0.5.

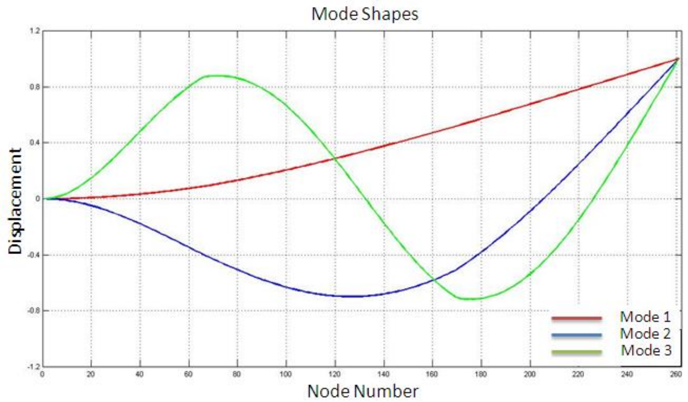

Fig. 8. Mode shapes for crack at *c/l* = 0.25 of depth *a/h* = 0.5 and crack at *d/l* = 0.65 of depth *a/h* = 0.5.

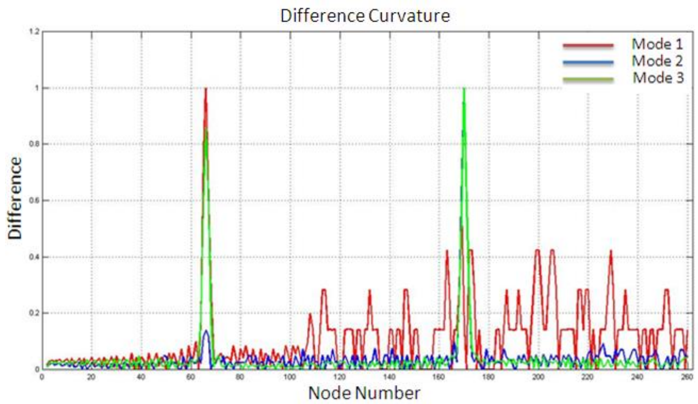

Fig. 9. Difference curvature for crack at *c/l* = 0.25 of depth *a/h* = 0.5 and crack at *d/l* = 0.65 of depth *a/h* = 0.5.

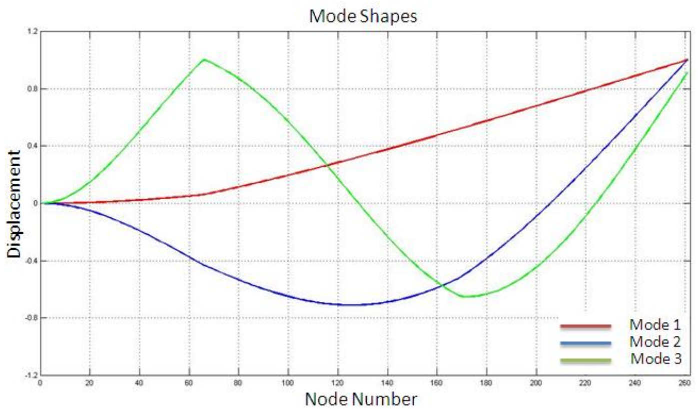

Fig. 10. Mode shapes for crack at *c/l* = 0.25 of depth *a/h* = 0.75 and crack at *d/l* = 0.65 of depth *a/h* = 0.5.

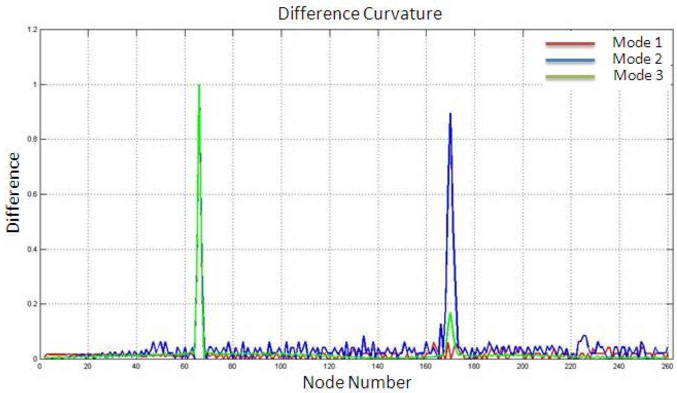

Fig. 11. Difference curvature for crack at *c/l* = 0.25 of depth *a/h* = 0.75 and crack at *d/l* = 0.65 of depth *a/h* = 0.5.

To examine the curvature mode shape technique for cantilever beam having several crack locations, the span of the beam is discretized by 260 elements as shown in Fig. 12.

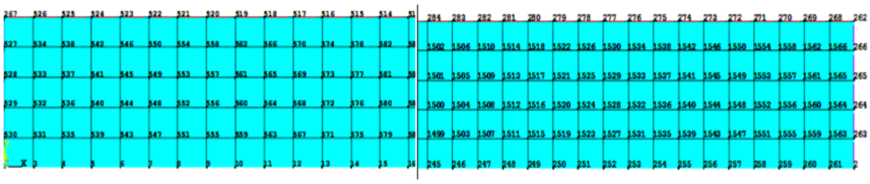

Fig. 12. Discretised model of beam showing node numbers at the bottom line considered for evaluating curvature mode shapes.

Two crack locations are assumed at a time. From the natural frequency analysis, it was found that when the cracks are located near the free end the first natural frequency is mostly affected. The second natural frequency is most affected as the crack severity is increased at the location close to the middle of the beam. And the third natural frequency is more sensitive throughout the beam except when the crack is near to the middle portion of the beam. This comparison indicates the presence of faults, but to localize them the difference in modal curvature should be plotted. Firstly, two cracks at *c/l* = 0.15 of depth *a/h* = 0.25 and at *c/l* = 0.8 of depth *a/h* = 0.75 were considered. The first three displacement mode shapes are shown in Fig. 4. The difference in modal curvature between the uncracked and the cracked continuous beam is plotted in Fig. 5 for the first three modes. For mode 1 in Fig. 5, it can be observed that the peak at *c/l* = 0.15 is very small comparing to that at *d/l* = 0.8. And also, for mode 1 the modal curvature at *c/l* = 0.15 is less and has very small values at the nodes next to it. In contrast, at *d/l* = 0.8 high modal curvature takes place. This indicates the severity of the crack depth ratio *a/h* = 0.75 at *d/l* = 0.8 compared to crack depth ratio *a/h* = 0.25 at *c/l* = 0.15. Also, first mode shape modal curvature is more sensitive near the fixed end compared to second and third mode shape modal curvatures. Again second and third mode shape modal curvatures are more sensitive near the free end compared to first mode shape modal curvature. The same can be observed for some other cracked scenarios shown in Figs. 6 and 7, Figs. 8 and 9 & Figs. 10 and 11. So, depending on the absolute ratio between the modal curvature values for a particular mode at two different locations, one peak can dominate the other. Therefore, one can conclude that in case of several crack locations in a structure, all modes should be carefully examined.

The crack is assumed to affect stiffness of the cantilever beam. The stiffness matrix of the cracked element in the Finite element model of the beam will replace the stiffness matrix of the same element prior to damaging to result in the global stiffness matrix. Thus frequencies and mode shapes are obtained by solving the Eigen Value Problem (*EVP*) $[K] - \omega^2 [M] = 0$. So it can be seen in Figs. 4 and 10 very clearly the changes in slopes and deviations in mode shape at crack location for crack depth ratio 0.75.

4. Conclusions

A method for identifying multiple crack parameters (crack depth and its location) in a cantilever beam using modal parameters has been attempted in the present paper. Parametric studies have been carried out using ANSYS Software to evaluate modal parameters (natural frequencies and mode shapes) for different multiple crack parameters.

- A change in certain mode frequency may correspond to different crack depth and location. But for multiple crack effects, mode shape analysis needs to be carried out.
- Due to the irregularities in the measured mode shapes, a curve fitting can be applied by calculating the curvature mode shapes using the central difference approximation. The curvature mode shapes technique for crack localization in turbine blades can be investigated.
- The results confirm that the application of the curvature mode shape method to detect cracks in turbine blades seems to be promising. The identification procedure presented in this paper is believed to provide a useful tool for detection of multiple cracks in turbine blades.

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