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SIMULTANEOUS OPTIMIZATION OF HOT AND COLD OUTLET AIR TEMPERATURES OF VORTEX TUBE USING GREY RELATIONAL ANALYSIS AND TAGUCHI METHOD

G. SURESH KUMAR[1], G. PADMANABHAN[2], B. DATTATREYA SARMA[3]

[1]G.Pulla Reddy Engineering Collge (Autonomous), Kurnool – 518 007, India
[2]S.V. University Collge of Engineering (Autonomous), Tirupati- 517 502, India
[3]Sri Venkateswara College of Engineering, Nellore- 524 316, India
*Corresponding Author: gudimettaskumar@outlook.com

Abstract

Vortex tube separates inlet pressurized gas into hot and cold streams to let out at the two ends. It is mainly used for spot cooling purposes. In the hitherto studies on vortex tube, either cold temperature or temperatures difference of outlet gas was considered the response variable. In this work, suggesting that vortex tube can also be used for heating applications, the two response variables viz., cold-outlet air temperature and hot-outlet air temperature are optimized simultaneously using Taguchi method and grey relational analysis. Various input controllable parameters / factors may be conceived, the values (levels) of whom may affect the response variables. Five controllable parameters with three levels each and L-27 Orthogonal Array are used for experimentation in the present work. Grey Relational Grades are computed. Except hot-tube Length, all the parameters considered are found significant from ANOVA table. Through confirmatory test, experimental results are validated.

Keywords: Vortex Tube, Hot and Cold Temperatures, Orthogonal Array, Grey Relational Analysis, ANOVA.

1. Introduction

Compressed air or any gas is the input for the vortex tube. Vortex tube separates the input compressed gas into hot and cold streams. The quantity and temperature of these two air streams can be controlled using a conical valve. Compressed air from compressor supplied to the vortex tube passes through the nozzle, located in

Nomenclatures

D_t	Internal diameter of the hot tube, in mm
L	Length of the hot tube , in mm
P	Pressure of compressed Inlet air, in KPa (Kgf / cm^2)
D_o	Internal Diameter of the Orifice, in mm
D_n	Internal Diameter of the Nozzle, in mm
T_h	Temperature of hot outlet air, (^{0}C)
T_c	Temperature of cold outlet air, (^{0}C)

Abbreviations

OA	Orthogonal Array
GRA	Grey Relational Analysis
ANOVA	ANalysis Of VAriance

the vortex chamber, tangential to an internal counter bore. Thus vortex motion is given by nozzle to the inlet air. This spinning stream of air passes down the hot tube as outer vortex. Some of the warmed air is allowed to leave the vortex tube at the hot tube end using a conical valve built into the hot tube. Remaining air is reverted, down the tube as a smaller inside vortex in the low-pressure area within the outer vortex. This inner vortex loses heat and escapes as cold air as shown in Fig. 1.

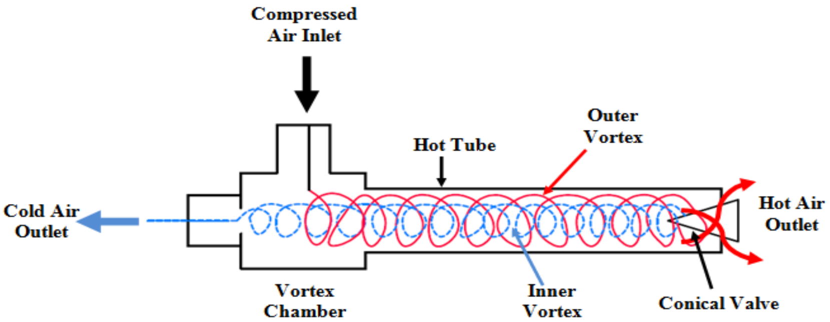

Fig. 1. Working of Counter flow Vortex tube.

The inner vortex is under lower pressure than the outer vortex (because of centrifugal force) and hence the temperature of the inner vortex air is lower than that of the outer vortex air.

The vortex tube can be either parallel flow vortex tube or counter flow vortex tube. The vortex tube used in the present work is of counter flow type whose schematic is the same as shown in Fig. 1.

The main parts of the vortex tube are shown in Fig. 2. The main parts of the vortex tube are

1. Cold tube	2. Orifice
3. Air inlet fitting	4. Vortex chamber
5. Nozzle	6. Hot tube
7. Value Housing	8. Conical valve.

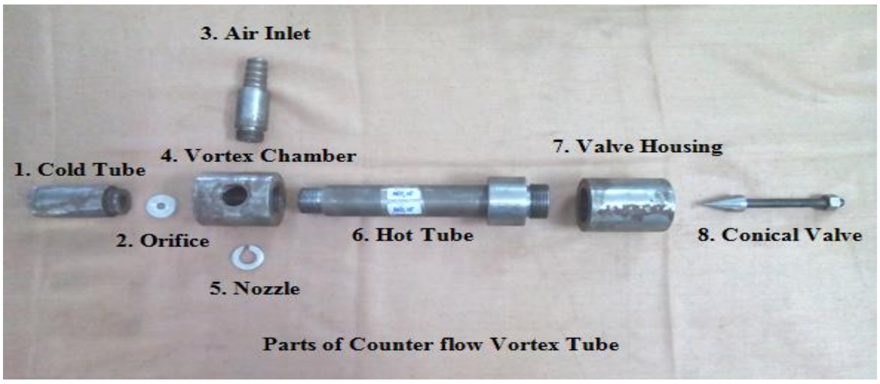

Fig. 2. Main parts of Counter flow Vortex tube.

2. Literature Review

Attention of several studies was focused on optimizing the performance of the vortex tube for various design features, input parameters and so on. Some other studies tried to analyze the heat transfer phenomenon within the vortex tube using Computational Fluid Dynamics (CFD) and other numerical / mathematical procedures. Optimization of cold outlet air/gas temperature of the vortex tube has been the major concentration of these studies as the vortex tube is mainly used for spot cooling. Few studies attempted to optimize the temperature difference between the hot and cold outlet temperatures for different input parameters. Hamdan et al. [1] conducted experiments to study the effect of nozzle parameters on energy separation performance of vortex tube. Results indicate that tangential orientation of nozzle gave maximum energy separation. Further they concluded that under the study conditions, optimum number of nozzles is around 4 and the symmetry/asymmetry of nozzles has minimal effect on the energy separation. Effect of conical shape of the vortex tube from hot towards cold end was studied by Guen et al. [2]. They concluded that 20^0 cone angle gave the best temperature difference. Using Response Surface Methodology, effect of diameter of nozzle, diameter of orifice and inlet pressure on temperature difference was studied by Prabakaran et al. [3]. Diameters of nozzle and inlet pressure were found to have significant effect statistically on temperature difference. Pinar et al. [4] in their work observed that the quality characteristic, temperature difference, increased with the increase of inlet pressure and cold mass fraction and decreased with the increase in nozzle number. L27 Orthogonal Array (OA) was used for experimentation and Taguchi method, for parameter optimization. Mondal et al. [5], Yang et al. [6] and other researchers / authors in their articles / books [10-13] explained how grey relational approach can be used to optimize multiple objectives simultaneously in different areas of applications. A numerical study was carried out by Pourmahmoud et al. [7] considering six different L/D ratios with six straight nozzles. An L/D ratio of 9.3 gave the best temperature separation and under the study conditions, the optimum cold mass fraction was arrived at 0.288. In their other work, Pinar et al. [8] conducted studies with the temperature difference between the hot and cold outlet temperatures as response variable. Taguchi method was used for analysis. The study concluded that all the three parameters considered and their two way interactions have significant effect on maximizing the response, statistically. The effect of orifice diameter and pressure was studied by Prabakaran et al. [9] and found that they have significant effect on the performance of the vortex tube. The effect of cooling the vortex tube on the temperature separation (Ti – Tc) was studied by Eiamsa-ard et al. [14]. It was found that cooling efficiency in the vortex tube was higher when the tube was externally cooled than when it was not cooled, operating under similar conditions. Sahu et al. [15] suggested that waste compressed air in industries be used as input to vortex tube, as, using a separate air compressor doesn't make sense vowing to its low Coefficient of Performance (CoP). In a review paper on performance of vortex tube, Kshirsagar et al. [16] revised the literature relating to the effect of various geometric parameters like number of nozzles, cold orifice diameter, and also inlet air pressure on the performance of the vortex tube. Also they concluded that no theory is perfect in explaining the phenomenon of heat transfer occurring inside the vortex tube and as such one has to conduct experiments to know / understand the effect of various parameters.

Though the vortex tube is popular for cooling applications, side by side, temperature of the hot air coming out from the hot tube may also be useful for heating / preheating purposes. Hence, simultaneous optimization of cold and hot air temperatures is of importance for experimentation and the same is attempted in the present work. Thus this work may usher future applications of vortex tube for simultaneous cooling and heating.

Orthogonal Arrays (OAs) represent a versatile class of combinatorial arrangements useful for conducting experiments to determine the optimum mix of a number of factors in a product to maximize the yield. The OAs should possess orthogonality and balance properties. Taguchi's orthogonal arrays are highly fractional orthogonal designs and they can be used to estimate main effects using only a few experimental runs and the premise is that certain interaction effects are unimportant and can be ignored. Depending on the number of factors and the number of levels of each factor considered, the appropriate OA can be selected. In the present work, L-27 orthogonal array is chosen, to carry out experiments.

2.1 Objective

The objective of this work is to study the influence of the controllable parameters selected, viz., inlet air Pressure, Length of the vortex (hot) tube, internal diameter of the vortex (hot) tube, internal diameter of the orifice / diaphragm, and internal diameter of the nozzle on the responses and to optimize simultaneously the temperatures of air at hot and cold outlets of the vortex tube, thus widening the applicability of the vortex tube.

Hitherto studies on vortex tube, concentrated on the optimization of the temperature difference or cold temperature of vortex tube, and this work is novel in that it considers both hot and cold temperatures of vortex tube for simultaneous optimization suggesting the applicability of vortex tube for heating also.

3. Experimental Work

Five control factors/parameters are identified that are expected to affect the responses. Each factor is taken at three levels. Using designs given by Taguchi, L-27 OA is selected as appropriate for the requirements. Experimentation is carried out and the values of the two responses are noted for all the 27 run conditions. Two repetitions have been carried out. Using grey relational approach, analysis is done and ANOVA table is obtained. Factor effects are studied, optimum levels for the factors are determined. A confirmatory test is run to validate the experimental results with the optimal settings for the factors.

The factors considered with their three levels are as follows:

D_t – Internal diameter of the hot tube (11 mm, 14 mm, 17 mm)
L – Length of the hot tube (120 mm, 150 mm, and 180 mm)
P – Inlet compressed air Pressure, 98.0665 KPa, 196.133 KPa, 294.199 KPa[1Kgf / cm^2, 2Kgf / cm^2, 3Kgf / cm^2]
D_o – Internal diameter of the Orifice (6 mm, 8 mm, and 10 mm)
D_n – Internal diameter of the Nozzle (11 mm, 14 mm, 17 mm)

The response variables are:

T_h –Temperature of hot outlet air, (in degree centigrade), with T_h-1 and T_h-2 being its repetitions
T_c –Temperature of cold outlet air, (in degree centigrade), with T_c-1 and T_c-2 being its repetitions

The standard L-27 OA is modified by substituting level values to get Table1.

Table 1. The L-27 OA (un-coded) for input parameter level combinations with columns for Responses' repetitions.

Test point Number	L	D_t	D_n	D_o	P	T_c-1	T_c-2	T_h-1	T_h-2
1	120	11	11	6	1	25	25	35	35
2	120	11	11	6	2	21	21	38	38
3	120	11	11	6	3	17	16	41	41
4	120	14	14	8	1	21	22	38	41
5	120	14	14	8	2	16	16	41	44
6	120	14	14	8	3	13	13	44	51
7	120	17	17	10	1	26	25	38	41
8	120	17	17	10	2	23	21	41	44
9	120	17	17	10	3	20	19	45	47
10	150	11	14	10	1	25	25	34	35
11	150	11	14	10	2	20	21	37	39
12	150	11	14	10	3	15	15	38	45
13	150	14	17	6	1	20	22	37	37
14	150	14	17	6	2	15	16	43	45
15	150	14	17	6	3	10	12	47	50
16	150	17	11	8	1	26	26	34	35
17	150	17	11	8	2	22	22	36	36
18	150	17	11	8	3	18	18	40	40
19	180	11	17	8	1	24	26	45	42
20	180	11	17	8	2	21	22	48	46
21	180	11	17	8	3	18	18	50	49
22	180	14	11	10	1	25	25	33	40
23	180	14	11	10	2	21	21	36	44
24	180	14	11	10	3	18	16	40	48
25	180	17	14	6	1	19	20	36	39
26	180	17	14	6	2	16	16	41	41
27	180	17	14	6	3	12	11	45	45

Four columns are added to the right side for noting the repetitions T_c-1, T_c-2 and T_h-1, T_h-2 of the responses. The factors are assigned to columns considering ease of experimentation not violating the properties of the table. Experimental runs order is taken to be the same as given by the table.

4. Results and Discussion

The results obtained after experimentation are analysed using Grey relational analysis (GRA) coupled with Taguchi method.

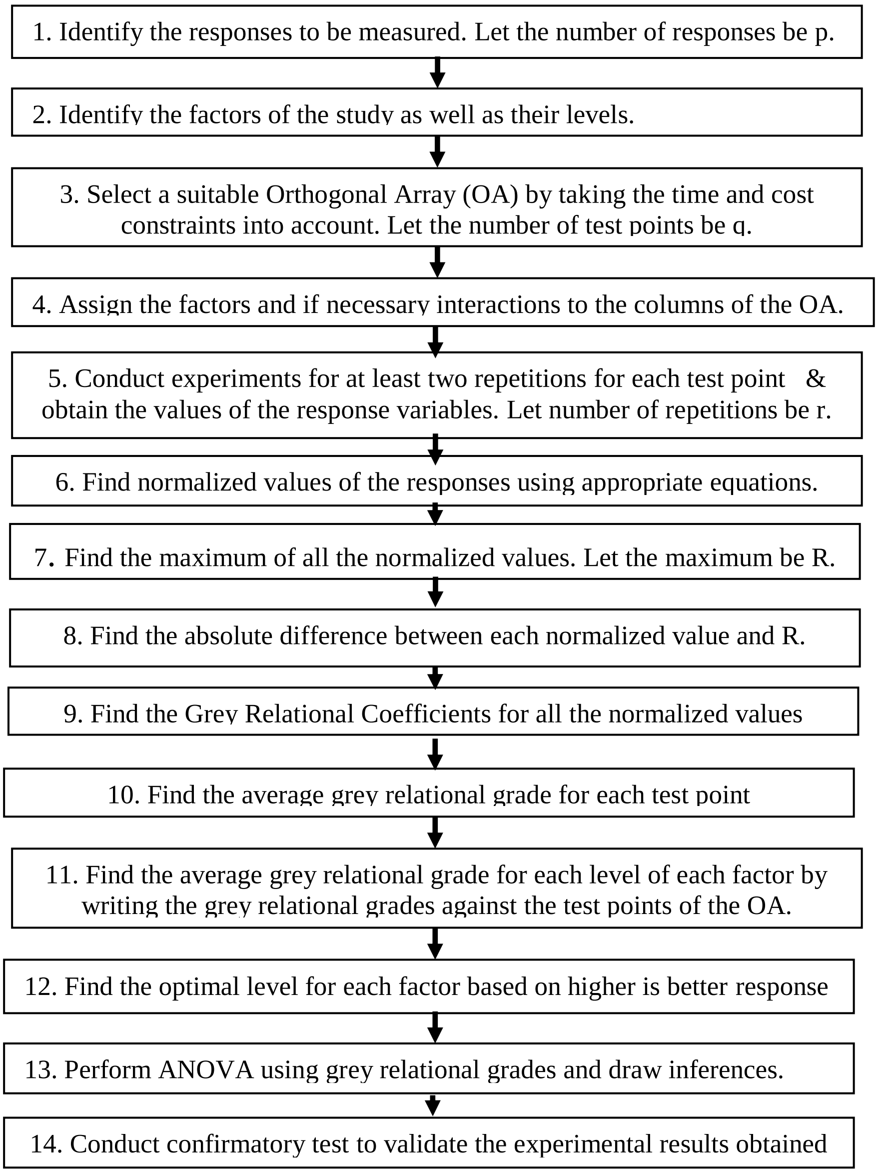

Fig. 3. Flow chart depicting the procedure of Grey Relational Analysis.

The procedure of Grey Taguchi method is given in Fig. 3. Using Taguchi concept, L-27 OA is selected, experimentation is carried out, and results are tabulated. The response values are normalized then, using the appropriate formula based on whether the response is Larger the better, Smaller the better, or Nominal the best.

(1) For the larger-the-better type response

$x_{ijk}^* = [\ x_{ijk} - \min(x_{ijk})\ ] / [\ \max(x_{ijk}) - \min(x_{ijk})\ ]$

(2) For the-smaller-the-better type response

$x_{ijk}^* = [\ \max(x_i^{(o)}(k)) - (x_i^{(o)}(k))] / [\ \max(x_i^{(o)}(k)) - \min(x_i^{(o)}(k))\ ]$

(3) For the nominal-the-best type response (Target / Nominal Value = TV)

$x_{ijk}^* = 1 - [\ \{\ x_i^{(o)}(k) - TV\ \} / \max\{\ \max(x_i^{(o)}(k)) - TV, TV - \min(x_i^{(o)}(k))\ \}]$

Where x_{ijk}^* is normalized value for kth repetition of ith response in jth test point.

i- is an increment for number of responses,

j- is an increment for number of test points, and

k-is an increment for number of repetitions.

x_{ijk} is the response value noted against kth repetition of ith response in jth test point.

In the present work, T_c is of smaller the better type and T_h is of larger the better type responses.

Maximum of the normalized values regardless of response variables, test points, and repetitions is found out. Let the maximum value be R, which is known as reference value. Its formula is given as:

$R = \max(x_{ijk*})$; in the present work, R = 1.

Absolute difference between each normalized value and the reference value R, regardless of the response variables, test points, and repetitions, denoted by D_{ijk} is calculated.

$\text{Max}(D_{ijk})$ and $\min(D_{ijk})$in the present problem are identified to be 1 and 0 respectively.

For each of the test point, response and repetition combination, Grey Relational Coefficient (GRC) is calculated using the appropriate formula.

$GRC_{ijk} = [\min(D_{ijk}) + E^* \max(D_{ijk})] / [\ D_{ijk} + \max(D_{ijk})]$

E value is taken to be 0.5.

Now for each of the 27 test points, Grey Relational Grade (GRG) is calculated, averaging the GRC values of that test point. Also GRGs, and squares of the GRGs are aggregated, and presented below along with total sum of squares.

Total GRG = 13.59317

Total GRG Sq. = 7.357026

SS Total = 0.51353

Corresponding to each level of each factor, average GRGs are computed and the result is presented in Table 2. Corresponding to each level of each factor there will be nine GRG values, adding and dividing the result by nine of which yield average GRG corresponding to that level of the factor.

Now maximum GRG value and difference of maximum and minimum GRG values are determined for each of the factors in order to ascertain the extent of influence of factors (Order of Influence) on the responses as well as to determine the optimal level of each factor.

Table 2. Average of GRGs of factors at each Level (Response Table for mean GRGs).

	L	D_t	D_n	D_o	P
Level 1	0.490939	0.479587	0.427073	**0.537788**	0.386176
Level 2	0.486918	**0.553664**	0.530096	0.517235	0.480903
Level 3	**0.532496**	0.477103	**0.553184**	0.45533	**0.643273**
Max	**0.532496**	**0.553664**	**0.553184**	**0.537788**	**0.643273**
max – min	0.045579	0.076561	0.126112	0.082458	0.257097
Rank	5	4	2	3	1

Now the optimal level of the factor is the one for which average GRG is the maximum.

ANOVA is performed for the GRGs (Table 3) and inference is drawn on the statistical significance of factors on the responses.

Table 3. ANOVA Comparison.

Source of Variation	SS	DoF	MSS	F-cal	F-table	Inference
					(α = 0.025)	
L	0.011462	2	0.005731	1.857077	4.69	Insignificant
D_t	0.034066	2	0.017033	5.519366	4.69	Significant
D_n	0.081153	2	0.040576	13.14854	4.69	Significant
D_o	0.033162	2	0.016581	5.372999	4.69	Significant
P	0.304308	2	0.152154	49.30465	4.69	Significant
Error	0.049379	16	0.003086			
Total	0.51353	26				

F- cal > F-table → Factor is Significant

From the above ANOVA table, all the factors except the length of the vortex tube are found to be statistically significant on the responses viz., temperatures of air at hot and cold outlets of the vortex tube. Here, the results obtained are for 97.5 % confidence limits ($\alpha = 0.025$). If for a factor, calculated F value is greater than F- table value corresponding to $\alpha = 0.025$, that factor is statistically significant, i.e., the changes in levels / values of that factor will have significant effect on the response under study. The objective stated in the beginning is achieved through performance of ANOVA.

The order of significance for factors is P > D_n > D_o > D_t > L and L - 3, D_t- 2, D_n– 3, D_o – 1, & P - 3 are the optimal factor – level combinations as is given by Table 2. Optimal factor – level combinations are also given by Fig. 4.

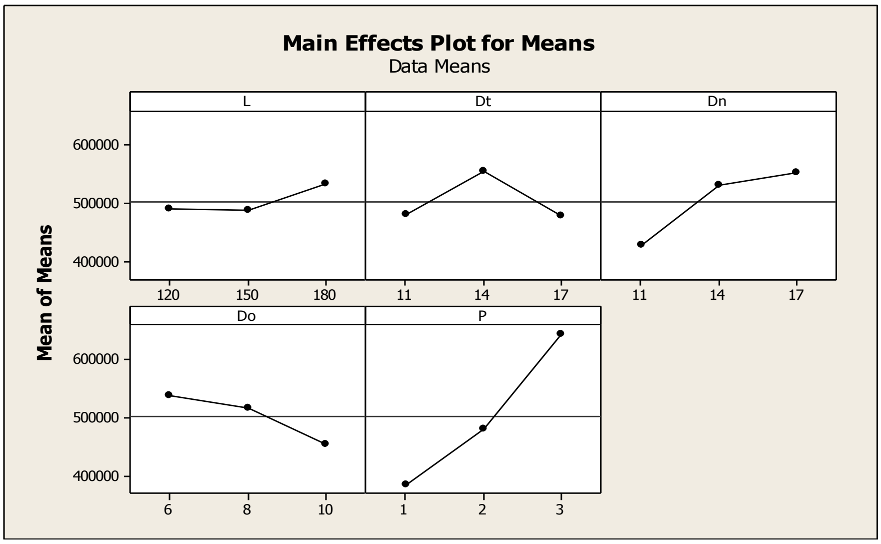

Fig. 4. Main effects plot for mean GRGs

The Main effects plot for the Means (Fig. 4) indicates the following:

- Higher the inlet air Pressure and internal diameter of Nozzle, better are the response values.
- Peak points in the graph show that the optimum response values are achievable with the optimal factor settings at P = 294.199 KPa (3Kgf / cm^2), D_t = 14 mm, D_n =17 mm, D_o = 6 mm, L = 180 mm.

4.1 Confirmatory Test

The confirmatory test is run twice with the above optimal levels for factors set (two repetitions) and the results are tabulated in Table 4.

Table 4. Confirmatory Test (run at the optimal settings) Results.

Repetition / Test point Number	1	2
Hot outlet temperature - T_h (ºc):	48	50
Cold outlet temperature – T_c (ºc):	12	11

GRG for these results is computed and it is observed that this GRG (0.881818) is higher when compared with the GRG values of all the 27 test points. Thus the results are validated.

5. Conclusions

It is concluded that all the parameters considered except length, L, are found to have significant effect on the response statistically. For simultaneous optimum hot and cold outlet temperatures, the settings of the parameters should be: **L -** 180 mm; $\mathbf{D_t}$**–** 14 mm**;** $\mathbf{D_n}$– 17 mm; $\mathbf{D_o}$– 6 mm; and **P –** 294.199 KPa (3Kgf / cm^2). The results of this work are useful to enhance the applications domain of the vortex tube and to motivate the industries to use the vortex tube for suitable simultaneous heating and cooling applications.

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