Journal of Engineering Science and Technology
Vol. 11, No. 10 (2016) 1360 - 1372

COMPARISON OF THREE METHODS TO DETERMINE THE INERTIAL PROPERTIES OF FREE-FLYING DYNAMICALLY SIMILAR MODELS

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Abstract

Inertial characteristics of the flying vehicle play an important role in dynamics of the vehicle. Therefore, experimental determination of the moments of inertia (MI) is one of the main tasks to define the accurate mathematical model. Especially, in order to ensure the similitude of mass-inertia parameters in the free-flying dynamically similar models (FDSM), the accuracy of test methods of MI determination has paramount importance. Thus, the present study attempts to examine three common techniques of MI estimation. The investigated methods include, compound pendulum, physical double pendulum and bifilar torsional pendulum. These methods have been used in the current conducted experiments, in order to measure the MOI of a light unmanned aerial vehicle (UAV). Additionally, a novel technique has been proposed to evaluate the experiment error using added loads. The results of the test indicate that the bifilar torsional pendulum method is more accurate than the others, and the error can be reduced to less than 2% and 1% about *x*- and *y*-axis, respectively.

Keywords: Free-Flying Dynamically Similar/Scaled Model, Moments of Inertia, Experimental, pendulum, Error analysis.

1. Introduction

The Moments of inertia (MI) of flying the vehicles determine the torque needed for a desired angular acceleration about axes of rotation. Therefore, they are essential characteristics to study on flight dynamics and development of the control system. Experimental determination of MI is not a new concept, and it dates back to the age of aviation. In 1926, spinning investigation first revealed the necessity of experimental determination of MI of aircraft [1, 2].

Nomenclatures

c	Distance between joint and CG, m
g	Acceleration of gravity , m/s^2
I	Moment of inertia, kg.m^2
I'	Moment of inertia with added loads, kg.m^2
I_{xx}, I_{yy}, I_{zz}	Moment of inertia about *x*-, *y*-, and *z*-axes, kg.m^2
I_L	Moment of Inertia of added loads, kg.m^2
I_{L_0}	Exact moment of inertia of added loads, kg.m^2
L	Distance between pivot point and point of added load, m
l	Distance between axis of oscillation and CG, m
l_1	Length of wire, m
m	Mass, kg
P	Added weight for changing of angle of inclination, kg
r	Half of distance between wires, m
R^2	Coefficient of determination
R_g	Radius of gyration, m
T	Period of oscillation, s
T_1	Period of oscillation in first mode , s
T_2	Period of oscillation in second mode, s
w	Weight, kg
x, y, z	Coordinates along *x*-, *y*-, and *z*-axes, m

Greek Symbols

α	Angle of inclination, deg.
Δ	Absolute error
δ	Relative error

Abbreviations

CG	Center of Gravity
FDSM	Free-flying dynamically similar/scaled model
KhAI	Kharkiv Aviation Institute
MI	Moments of inertia
NACA	National Advisory Committee for Aeronautics
UAV	Unmanned aerial vehicle

The experiments, based on pendulum method, were used by NACA for this purpose and were modified and improved during several decades [2-6]. In the modified methods, the effect of entrapped and additional mass of air, were taken into account [7].

In the east, the Russians in addition to the compound and bifilar pendulum methods successfully attempted the problem by developing a special kind of compound pendulum having two degrees-of-freedom [8-10]. Additionally, Barnes and Woodfield [11] have used the method of spring restrained oscillations for determining the MI of Delta-2 Fairy's aircraft. Peterson [12] presented the mass properties measurement of space craft X-38 using bifilar pendulum, single point suspension, spring table and dynamic inertia method. Additionally, Mendos et al. [13] present an experimentally procedure to determine the MI of a small UAV, using a two-axis motion simulator. Nevertheless, the majority of other previous works on various vehicles have been based on the pendulum method [14-22]. In addition, expensive professional ready-made equipment and instruments are available in the market for space or aviation industries for measurement of mass-inertial properties of objects [23]. Similarly, Shokrollahi et al. [24] demonstrated the estimation of axial MI of a UAV using the inverted torsion pendulum machine.

The literature study showed the enough maturation of methods to determine MI of various configurations, but emphasized the necessity of error analysis techniques' improvement. In this study, the free-flying dynamically similar models that are the scope of the study need for accurate and low-cost method, especially in academic researches [25]. The MI determination accuracy for FDSM is more important rather than other vehicles, for the reason of dynamic similarity requirements [26].

The purposes of MI determination of flying air vehicles have been presented in Table 1. The most common aim for MI determination is the evaluation of stability and control characteristics of the vehicle. Additionally, after a flight-test program, the mass-inertia characteristics are needed to determine the stability and control derivatives from the flight data. In addition to these purposes, dynamic similarity is the dedicated feature of FDSMs [27].

FDSM is the powerful technique for aeronautical evaluation that has been widely used during the creation of new aerial vehicles, for testing aerodynamic concepts, control systems development and exploring high-risk flight envelopes [28]. FDSMs are not only geometrically scaled replicas; they are specially designed to ensure motion similitude between the sub-scale model and the full-scale aircraft [29]. Thus, the model must have the same CG position, and scaled mass and moments of inertia based on similitude requirements [30, 31].

During the design process, it is attempted to fulfil similarity requirements, but after construction, this must be verified experimentally. Thus, the MI determination and finishing it to the desired values are two of the main ground tests for quality control of FDSM [32]. University of KhAI has more than 50 years of experience in development of FDSM and due to the discussions outlined above, study on investigation and improvement of the accuracy of MI estimation methods was planned. For this reason, a light UAV which developed by KhAI is considered as the test object. In addition, a novel technique of added loads has been used successfully to error analysis and comparison of experimental methods.

Table 1. The purposes of MI determination in flying vehicles.

Purposes	FDSMs	Aircrafts
Stability and Control characteristics' evaluations	+	+
Automatic control system design	+	+
System identification	+	+
Dynamic similarity	+	

2. Methods of MI Determination

The axial moments of inertia definition are presented by Eq. (1).

$$I_{XX} = \int (y^2 + z^2)dm$$
$$I_{YY} = \int (x^2 + z^2)dm \quad (1)$$
$$I_{ZZ} = \int (y^2 + x^2)dm$$

2.1. The compound pendulum

For a compound or physical pendulum, the MI will be given by Eq. (2) [1].

$$I = \frac{T^2 wl}{4\pi^2} \quad (2)$$

The determination of UAV's MI, has been done by swinging it and oscillating as a pendulum. By recording the time period of oscillation, the MI of this assembly about the axis of oscillation is calculated. The MI of the swinging gear alone determined by using the 3D computer model. Then it subtracted from the MI of the assembly which gives the MI of UAV about the axis of oscillation. Furthermore, the swinging gear can be swung as an independent pendulum, and using the same equation, its MI can be calculated. To find the MI about an axis passing through CG, parallel axis theorem has been used. Figure 1 shows the swung UAV to determine the MI about *x*-body and *y*-body axes.

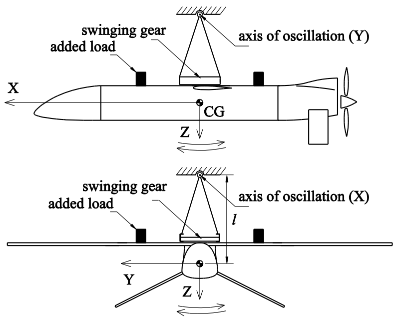

Fig. 1. I_{XX} **and** I_{YY} **Determination by compound pendulum method.**

2.2. The physical double pendulum

For a physical double pendulum, the MI will be given by Eq. (3) [9].

$$I = wcl_1 \left(\frac{g}{4\pi^2 l_1}\right)^2 T_1^2 T_2^2 \quad (3)$$

This method enabled the UAV to perform two simultaneous oscillations in opposite directions about two different axes. In this method, the MI can be obtained directly about the aircraft axis. To determine the MI about *x*-body and *y*-body axes, the UAV is swung as shown in Fig. 2.

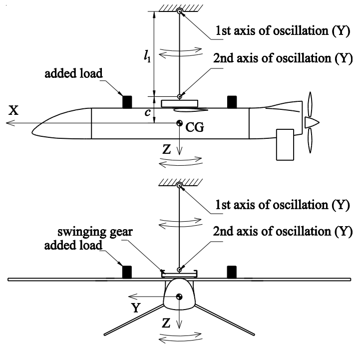

Fig. 2. I_{XX} and I_{YY} Determination by double pendulum method.

2.3. The bifilar torsional pendulum

For a bifilar torsional pendulum, the MI will be given by Eq. (4) [5].

$$I = \frac{wr^2}{4\pi^2 l_1} T^2 \quad (4)$$

To determine the MI about body axes, the UAV has been swung as a bifilar torsional pendulum about passing axes through the CG. It must be ensured that the CG of the aircraft and swinging gear lie on the same vertical line as shown in Fig. 3. By recording the time period of oscillation and using Eq. (4), the MI about the axis of oscillation can be calculated. The swinging gear can be oscillated as an independent pendulum, and using the same equation, its MI can be calculated. To determine the MI about *x*, *y* and *z* body axes, the UAV is swung as shown in Fig. 3.

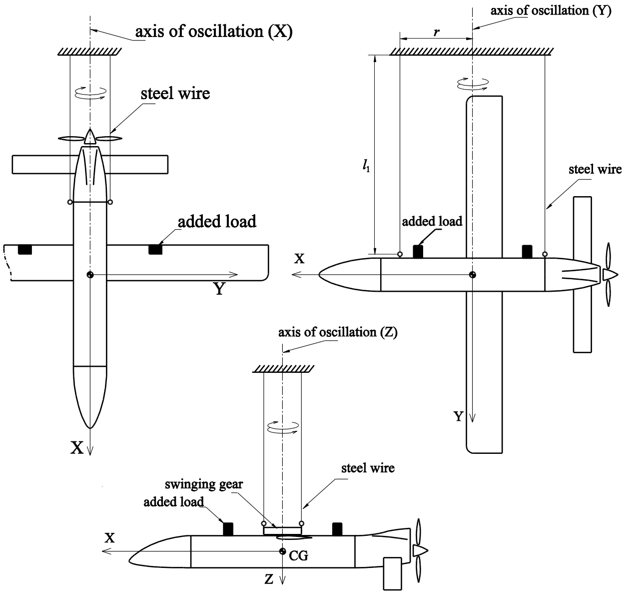

Fig. 3. I_{XX} , I_{YY} and I_{ZZ} Determination by bifilar pendulum method.

2.4. Method of CG determination

For estimation of MI, CG position is a prerequisite, thus, it is determined by using the improved single-point suspension method [33]. Figure 4 presents UAV body axes that defined for this purpose. The *x*-axis has been passed through the pivot point O and point of added weight K. The *z*-axis is perpendicular to *x*-axis. Obviously, supposing the symmetry of UAV, the Y-coordinate of CG is null. For tilting UAV, additional weight is used at point K. By adding various weights; the pitch angle of UAV was changed, thus, by reading the height of two specified point on the fuselage by transit-level; the angle was calculated. It is proved that the change of the added weight versus tangent of pitch angle is linear, as it has shown in Fig. 5. Thus, the equation of the straight line can be written.

$$P = \frac{w}{L}(-X_{CG} - Z_{CG} tg\alpha) \tag{5}$$

Coordinates of CG can be calculated by intersecting the line with main axes, as it has shown in Fig. 5. Thus,

$$\begin{aligned} &\text{tg}\alpha = 0 \ \rightarrow X_{CG} = -\frac{P(tg\alpha = 0)\cdot L}{w}; \\ &P = 0 \quad \rightarrow Z_{CG} = -\frac{X}{tg\alpha(p = 0)} \end{aligned} \tag{6}$$

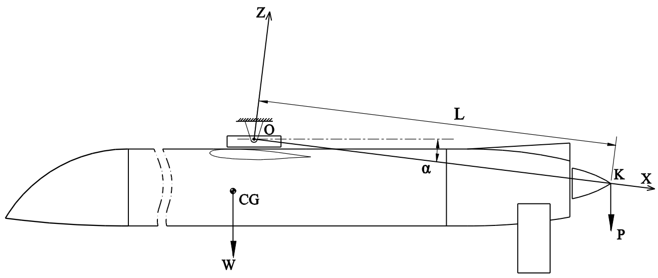

Fig. 4. Determination of CG.

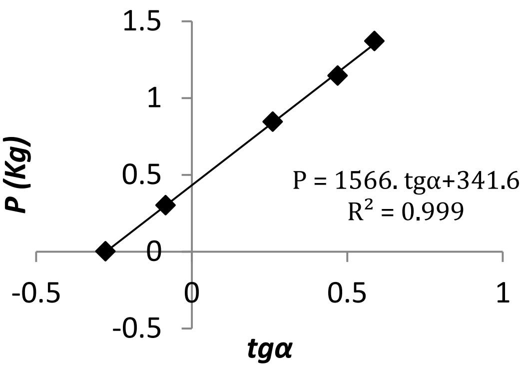

Fig. 5. Graph of CG determination experiment.

3. Error Analysis

The main aim of the present study is the comparison of methods and choosing the most accurate ones to determine axial MI of FSDM. There are several literatures that discussed the accuracy of experimental methods [5 and 9]. However, the error depends on many factors such as test setup, precision of sensors and the skills of operators. In the other word, the same method in the different tests may give a distinctive accuracy. Usually, for calculation of accuracy, the theoretical data of MI is compared with the experimental data, but accuracy of theoretical data is not known exactly. Here, the method of added load is proposed to determine the accuracy of the experiment in the arbitrary method and configuration. Two same metallic loads were fabricated and after each test, were placed in the specific positions. The MI of UAV with and without loads was determined. By subtracting these MI, the MI of loads was determined by experiment and the accuracy of the test will be calculated. The MI of assembly which is determined experimentally in the ambient air consists of four parts [2]. MI of the structure;I_S, MI of the entrapped air; I_E, MI of air moving with the UAV or of the added mass;I_A, and the MI of swinging gear; I_G. By running the test in the vacuum room, the terms of moments relating to air will be ignored.

$$I_{UAV} = I_S + I_E + I_A + I_G \tag{7}$$

The MI of UAV with added loads is the summation of $\mathrm{I_{UAV}}$ and the MI of loads about the axis of oscillation, thus,

$$I'_{UAV} = I_{UAV} + I_L \tag{8}$$

The values of $\mathrm{I_{UAV}}$ and $\mathrm{I_L}$ have the errors which are interested to calculate. Equation (9) shows the summation of exact value and error of $\mathrm{I_{UAV}}$. Equation (10) shows the summation of exact value and error of load. By substituting Eqs. (9) and (10) in Eq. (8), the Eq. (11) is obtained. Subtracting the $\mathrm{I_{UAV}}$ from $\mathrm{I'_{UAV}}$ gives the MI of the loads by experiment. On the other hand, the exact MI of loads has been given. Thus, using Eq. (13), the accuracy of loads' MI determination can obtain in the arbitrary experimental setup.

$$I_{UAV} = I_{UAV_0} + \Delta I_{UAV} \tag{9}$$

$$I_L = I_{L_0} + \Delta I_L \tag{10}$$

$$I'_{UAV} = I_{UAV_0} + I_{L_0} + (\Delta I_{UAV} + \Delta I_L) \tag{11}$$

$$\Delta I = I'_{UAV} - I_{UAV} = I_{L_0} + \Delta I_L \tag{12}$$

$$\delta I_L = \frac{\Delta I_L}{I_{L_0}} \tag{13}$$

4. Experimental Setup

In this research, the MI of UAV was determined by three methods. For this reason, a swinging gear was designed that can be used in all methods. This swinging gear was made up of two 'U' beams and three 'L' beams and installed by four bolts that were used for wing. For swinging the UAV, the steel strings with $1mm$ of diameter were used. For leveling the UAV in the experiments a transit level was used to make the body axis parallel to the axis of oscillation (Fig. 6). A knife edge joint was designed and fabricated, using commercial of-the-shelf turning insert to minimize friction in axis of oscillation (Fig. 7). For noting the time period of oscillation, a video camera with the frame rate of 50 frames per second was used.

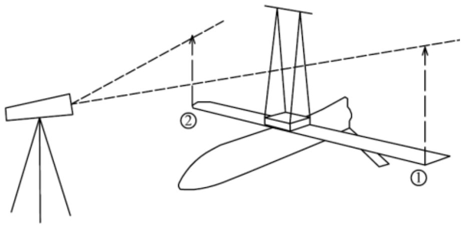

Fig. 6. Using Transit level for leveling of UAV.

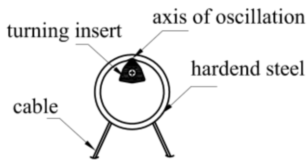

Fig. 7. Knife edge joint.

5. Performing Experiment

To compare the methods, the Astrogon-Sky light UAV was selected to perform the experiment. This UAV was developed in KhAI university with a Launch weight of 10 kgf, length of 1.57 m and wing span of 2 m. The theoretical MI of UAV Astrogon-sky is unknown, but the CG was determined experimentally. The MI in compound pendulum and physical double pendulum are determined only about *x*- and *z*-axes, but in bifilar torsioanl pendulum method, all of axial MI were determined. Totally, seven experiments were planned for original configuration of UAV, and they repeated with added loads. After analyzing the test data, some of the tests repeated with modifications. Figures 8-11 show the swung UAV in the three methods of experiment.

Fig. 8. Double pendulum method.

Fig. 9. Physical pendulum method.

Fig. 10. Bifilar torsional pendulum method.

Fig. 11. Swinging Gear.

6. Results and Discussions

The obtained data from three methods are presented in Table 2. As it has shown, the bifilar torsional pendulum method has the lowest error, and the physical double pendulum has the highest error. Although, Gernet [9] has mentioned the better accuracy for the double pendulum than these results, but in the present study, after several experiments, it could not be obtained the better results. It may be because of light weight of UAV that causes interfering modes of oscillations. Thus, based on the experiences of authors, it does not suggest the application of double pendulum method for light UAVs and FDSMs. Furthermore, the results of physical pendulum method in the first set of experiments were not as expected.

There are several possible sources of error that have been identified. First, the friction about the axis of oscillation results in the error of the oscillatory period. Second, measuring the period of oscillation using the chronometer is not accurate enough. Third, longer length of supporting cable creates greater error when using the parallel axis theorem to calculate the MI about CG. Therefore, by investigating the experiment setup and procedure, some modifications were conducted to improve accuracy. At first, the knife edge joint replaced instead of ball bearing joint. In addition, the period of oscillation recorded by video camera with a rate of 50 frames/sec. instead of a chronometer. In this way, during 50 oscillations, the accuracy can reach 0.0005 second. Furthermore, Gernet [9] has proven that the optimum distance between CG and axis of oscillation, should be less than the radius of gyration. For this reason, in the modified experiment, this distance was chosen near the radius of gyration that is defined by Eq. (14).

$$R_g = \sqrt{I/m} \tag{14}$$

The result of experiment with these modifications is presented in Table 3, that shows error of I_{YY} from 18.5 % reduced to 1.4%. Gracy [5] presented the accuracy of±2.5% for I_{XX} and ±1.3% for I_{YY} for physical compound pendulum method. As well, Wolfe et al. [20] observed difference about 2% between the experimental and analytical data. Moreover, Sumit et al. [14] obtained the error of 15% of I_{YY} by comparing the experimental and theoretical data. Thus, the result is acceptable in comparison to the similar works.

In the first series of experiments by bifilar pendulum method, the highest error observed for I_{YY} . Possible sources of error include measurement accuracy of oscillation period, misalignment of CG with axis of oscillation, friction in joints and length of cables. Therefore, some corrections were applied to the experimental setup. The optimum values for length of string and distance between them were reviewed [9, 18]. It is suggested the length of string should be 3 to 9 times more than the distance between strings and distance between strings more than 1.6 of radius of gyration. Thus, the experiment with modified test setup repeated and as it has shown in Table 3, the error of 9.8% for I_{YY} reduced to 0.87%. This was better in comparison with the similar works that mentioned above [5, 14, 20]. It should be noted that in the most of similar works, the errors are calculated by comparing the experimental and theoretical data. This method suffers from uncertainty in the theoretical data as the base of comparison. Instead, the proposed method of added load, do not need the theoretical data to evaluate the error of experiment.

Table 2. Data of the first series of experiments.

	MI	**Without load($\mathbf{kg.m^2}$)**	**With added load($\mathbf{kg.m^2}$)**	**Error of MI**
Bifilar torsional pendulum	I_{XX}	0.647	1.1	2%
	I_{YY}	1.8	1.9	9.8%
	I_{ZZ}	2.44	2.63	6.2%
Physical pendulum	I_{XX}	0.642	0.983	7.6%
	I_{YY}	1.75	1.925	18.5%
Physical double pendulum	I_{XX}	0.527	0.656	66%
	I_{YY}	1.92	2.19	20.8%

Table 3. The modified experiment data.

	MI	Without load	With load	Error of MI
Bifilar torsional pendulum	I_{YY}	1.815	2.088	0.87%
Physical pendulum	I_{YY}	1.838	2.077	1.4%

7. Conclusions

In this paper the comparison of experimental methods to determine MI has been presented. The procedure consisted of the implementation of three methods to estimate the MI of a light UAV and evaluation of the results. It was discussed the possible sources of error and the solution for avoiding them. Some concluding observations from the investigation are given below.

- Time period of oscillation, affects significantly on the accuracy of experiments. By replacing commercial of-the-shelf camera instead of a chronometer, the error of operators can be eliminated.
- Knife edge joint was used to minimize the friction that noticeably improved the results.
- The results of experiments, which were verified by comparison with the published data show that the bifilar torsional pendulum method is more accurate than the other methods. It is simple enough but difficult for large and heavy vehicles. By this method, MI about all axes can be determined.
- The physical pendulum method can provide reasonable results by minimizing friction and selecting the optimum length of cables. By using this method, it is possible to determine the MI about *x*- and *y*-axes. Furthermore, determination of MI about Z is possible but needs enough space and more complex tools.
- The double pendulum method is difficult in performing because of interfering the oscillation modes that causes the noticeable error in results. In the present work as it has shown, the errors in this method are higher than the other methods. Therefore, the bifilar torsional and physical pendulum methods are recommended for FDSM problems and UAVs.

- The major novelty of this study is proposing the method of error analysis using the added load technique. Unlike previous methods, it validates MI experiments in arbitrary configuration and presents the reliability of results, without need for theoretical data of MI.

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