COLLISION-AVOIDANCE FOR MOBILE ROBOTS USING REGION OF CERTAINTY: A PREDICTIVE APPROACH

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Abstract

In on-line environment, obstacles may exhibit different trajectory. Trajectory analysis of the obstacle is essential in determining its future location. If this analysis is accurate the futuristic region where robot and obstacle collision is likely to occur can be estimated. This enables the mobile robot to take corrective action prior to collision. In this approach, the motion pattern of the obstacle is analysed by taking into account the past co-ordinates traversed by the obstacle. Then the futuristic region where the obstacle is likely to occupy is predicted. This region is termed as region of certainty. Simulation results shows that the approach gives more reliable prediction as many number of sample points representing the past positions travelled by the obstacles are taken into consideration. The algorithm yielded better performance under higher obstacle velocity conditions and the results were compared with distance time transform method.

Keywords: Mobile robot, On-line environment, Trajectory analysis, Region of certainty.

1. Introduction

On-line path planning algorithms enables the mobile robot to eliminate collisions as well as to plan its path to reach the target. These algorithms are widely used due to their ability to tackle the environment where the surrounding data is unknown.

Artificial potential field, collision cone approach, relative velocity paradigm are some of the widely used path planning real time algorithms. Khatib [1] proposed an artificial potential field method in which mobile robot moves along the direction of the resultant force generated by attractive and repulsive fields. Problems such as path oscillations, trap situations are encountered in this approach. Fujimura [2] proposed an approach considering the obstacles to be instantaneously static. The trajectory of the obstacles is known to the robot a priori.
Collision-Avoidance for Mobile Robots Using Region of Certainty

Nomenclatures

\[ V \] Velocity of the obstacle, m/s

Greek Symbols

\[ \alpha, \beta \] Motion variables, rad.
\[ \omega \] Angular velocity of the mobile robot, rad./s
\[ \theta \] Angle made by the line connecting the mobile robot and the obstacle with respect to the x-axis, rad

In the collision cone approach proposed by Chakravarthy and Ghose [3], the direction of robot motion is diverted by avoiding the collision cone.

The relative velocity paradigm proposed by Fiorini and Shiller [4] steers the relative velocity between the robot and the obstacle outside the collision cone. In both the relative velocity and collision cone approaches, the motion of the obstacles is considered to be linear. Guzzi et al. [5] proposed a navigation algorithm for mobile robots operating in environment containing humans as circular obstacles.

Some prediction algorithms are used to determine the future location of the obstacle. These algorithms find the future location of the obstacle by analysing the past motion trend. Prediction models such as hidden Markov stochastic models proposed by Zhu [6], the Grey prediction by Luo and Chen [7] can be used for motion prediction. The Kalman Filter by Kalman [8] is used to predict future orientation and location of the obstacle. Hui-zhong et al. [9] suggested the auto regressive model which predicts the future location of the obstacle in real time. Gong and Geng [10] proposed the concept of predicting the band trajectories of the obstacle by analysing the past position traversed by it. However the approach assumes that the obstacle velocity is constant.

Reliable prediction becomes difficult when the random nature associated with the obstacle motion is high. In fact when the degree of randomness of obstacle motion increases, accuracy in predicting its future location reduces. The proposed predictive method considers multiple moving obstacles which follows distorted paths. Based on the past positions travelled by the obstacle, a futuristic region is created. This region is termed as region of certainty where the probability of finding the obstacle is high.

2. Proposed Algorithm

The mobile robot is assumed to be a point object (C-space approach) destined to reach the target from the starting point. Obstacles are assumed to be circles amplified by a factor to accommodate the physical size of the robot [3, 4, 9, 12-17]. Obstacle motion can be described by a set of line segments as shown in Fig. 1. An analysis of these line segments defines parameters such as the velocity, direction of motion, acceleration and deceleration of the obstacles. Let \( P_1, P_2, \ldots, P_n \) be the first, second and \( n \)th locations of the obstacle after entering into the robot’s sensor range.

The futuristic obstacle motion can be obtained by finding out two variables \( \alpha \) and \( \beta \), where,
\[ \alpha = \sum_{i=2}^{n} (x_i - x_{i-1}) \]  \hspace{1cm} (1)

\[ \beta = \sum_{i=2}^{n} (y_i - y_{i-1}) \]  \hspace{1cm} (2)

\[ p_i(x_i, y_i) \] denotes any arbitrary point traversed by the obstacle \((i = 1, 2, \ldots, n)\)

From Table 1, depending on the sign of \(\alpha\) and \(\beta\), the direction of motion can be found out.

- If \(\alpha = 0\), \(\text{sign} (\alpha) = \text{sign} (x_i)\)
- If \(\beta = 0\), \(\text{sign} (\beta) = \text{sign} (y_i)\)

The direction of obstacle motion is obtained from Table 1. Using this information region of certainty is generated.

![Fig. 1. Piece-wise trajectory of the moving obstacle.](image)

<table>
<thead>
<tr>
<th>sign ((\alpha))</th>
<th>sign ((\beta))</th>
<th>Direction of motion</th>
</tr>
</thead>
<tbody>
<tr>
<td>positive</td>
<td>positive</td>
<td>Towards 1st quadrant</td>
</tr>
<tr>
<td>negative</td>
<td>positive</td>
<td>Towards 2nd quadrant</td>
</tr>
<tr>
<td>negative</td>
<td>negative</td>
<td>Towards 3rd quadrant</td>
</tr>
<tr>
<td>positive</td>
<td>negative</td>
<td>Towards 4th quadrant</td>
</tr>
</tbody>
</table>

2.1. Region of certainty

Region of certainty can be defined as a circle sector where the obstacle is likely to occupy in the future instant. This enables the robot to stay outside the region to
complete the obstacle-avoidance manoeuvre. The radius of the region is dependent upon the maximum value of obstacle velocity attainable. From Fig. 2(a), point C represents the position of the obstacle at time \( t_j \), point B represents the position of the obstacle at time \( t_{j+1} \). The line segment CB denotes the maximum distance \( (V_{\text{max}}) \) traversed by the obstacle within time \( t_{j+1} - t_j \) (\( \Delta t \)).

Within the time (\( \Delta t \)), if the obstacle velocity is less than \( V_{\text{max}} \), the obstacle position lies inside the circle sector. Since all the points in the lines AC, AD and the curve CD represents all the probable points traversed by the obstacle, the region is amplified in order to avoid any collisions happening along AC, AD and CD. This amplification also takes account of the physical size of the robot and the obstacle. Figure 2(b) shows the completed form of region of certainty.

![Fig. 2. (a) Probable region of obstacle existence, (b) Region of certainty.](image)

### 2.2. Identification of the most imminent collision

In the event of presence of multiple obstacles the most imminent collision must be determined. When the Euclidean distance between the obstacle and the robot is equal to the radius of region of certainty associated with that obstacle, then it is considered to be the most imminent one.

### 2.3. Collision-avoidance model

Based upon the location of the robot two cases are identified.

*Case 1: The current location of the robot lies on the curve XY as depicted in Fig. 3.*

From Fig. 3, point R denotes the mobile robot and point O denotes vertex of the region of certainty of the most imminent obstacle. The region XYO indicates the amplified region of certainty. The line RO connects the vertex of the region O and the robot R.

![Fig. 3. Robot’s current location lies on the region of certainty.](image)
Statement 1: If \( \{ R \in R \mid X < R < O \} \) and \( \{ R \in R \mid O < R < y \} \) holds true for the point \( R \) to lie on the curve \( XY \)

Arc length \( XR = V_{max} \cdot \theta \) \hspace{1cm} (3)

Arc length \( RY = V_{max} \cdot \left( \frac{\pi}{2} - \theta \right) \) \hspace{1cm} (4)

If \( XR < RY \) then the direction of motion is towards \( X \)

\[ \theta = \tan^{-1} m_1 \] \hspace{1cm} (5)

\[ \omega \geq \frac{\theta}{\Delta t} \] \hspace{1cm} (6)

where \( \omega \) is the angular velocity of the robot and \( m_1 \) is the slope of the line \( RO \).

If \( RY < XR \) then the direction of motion is towards \( Y \)

\[ \omega \geq \frac{\pi - \theta}{\Delta t} \] \hspace{1cm} (7)

The maximum attainable angular velocity is restricted by specifications of the actuator given by,

\[ \omega \leq \omega_{max} \] \hspace{1cm} (8)

Case 2: The shortest path of the robot intersects the region of certainty as depicted in Fig. 4.

The line \( RT \) denotes the shortest path of the robot and the line \( RO \) connecting the vertex \( O \) and the robot, where \( (x, y) \) represents any point on the shortest path of the robot. If the line \( OY \) intersects the path,

\[ y = \frac{O_x - R_x}{O_y - R_y}\] \hspace{1cm} (9)

From Eq. (9), we get,

\[ y = \frac{(T_y - R_y)(O_x - R_x)}{T_y - R_y} + R_y \] \hspace{1cm} (10)

The line \( OX \) intersects the path, then

\[ y = \frac{O_x - R_x}{O_y - R_y}\] \hspace{1cm} (11)

Similarly if the line \( OX \) intersects the path, then

Statement 2: If \( \{ y \in R \mid O_y \geq y \geq y_1 \} \) then the shortest path intersects the line \( OY \).
\[ \frac{x - R_x}{T_x - R_x} = \frac{O_x - R_x}{T_x - R_x} \]  
(11)

From Eq. (11), we get,
\[ x = \frac{(T_x - R_x)(O_x - R_x)}{T_y - R_y} + R_x \]  
(12)

**Statement 3:** If \( \{x \in R | O_x \geq x \geq X_x\} \) then the shortest path intersects the line OX.

If Statement 2 and/or Statement 3 are satisfied, then path of the robot is altered. The new path then is ROT. The statements (1, 2 and 3) explained above is applicable if sign \( (\alpha) \) = negative and sign \( (\beta) \) = negative. Section 2.4 discusses the conditional statements for the other 3 quadrants.

### 2.4. Conditional statements for other quadrants

If Sign \( (\alpha) \) is positive and Sign \( (\beta) \) is negative, then

- **Statement:** If \( \{R_x \in R | X_x > R_x > O_x\} \) and \( \{R_y \in R | O_y > R_y > y_y\} \) holds true for the point R to lie on the curve XY.

- **Statement 2:** If \( \{y \in R | O_y \geq y \geq y_y\} \) then the shortest path intersects the line OY.

- **Statement 3:** If \( \{x \in R | O_x \leq x \leq X_x\} \) then the shortest path intersects the line OX.

If Sign \( (\alpha) \) is positive and Sign \( (\beta) \) is positive, then

- **Statement 1:** If \( \{R_x \in R | X_x > R_x > O_x\} \) and \( \{R_y \in R | O_y < R_y < y_y\} \) holds true for the point R to lie on the curve XY.

- **Statement 2:** If \( \{y \in R | O_y \leq y \leq y_y\} \) then the shortest path intersects the line OY.

- **Statement 3:** If \( \{x \in R | O_x \leq x \leq X_x\} \) then the shortest path intersects the line OX.

If Sign \( (\alpha) \) is negative and Sign \( (\beta) \) is positive, then

- **Statement 1:** If \( \{R_x \in R | X_x < R_x < O_x\} \) and \( \{R_y \in R | O_y < R_y < y_y\} \) holds true for the point R to lie on the curve XY.

- **Statement 2:** If \( \{y \in R | O_y \leq y \leq y_y\} \) then the shortest path intersects the line OY.

- **Statement 3:** If \( \{x \in R | X_x \leq X_x \} \) then the shortest path intersects the line OX.

### 2.5. Working of the algorithm

From Fig. 5, the mobile robot proceeds from the start to the target. As it moves along, robot checks for the presence of any moving obstacles. Once detected, variables \( \alpha \) and \( \beta \) are found out and the direction of motion is identified. When the Euclidean distance between the robot and the obstacle is equal to the radius of its region of certainty, depending on the location of the robot, path is altered. The robot steers around the region and once the region is avoided, it proceeds to the target.
3. Simulation Results

The proposed approach is simulated in MATLAB R2010a version in Intel (R) Core (TM) i3 processor at 2.4 GHz. Sensor range of the robot is assumed to be 25 m and the sampling period is 50 ms. The initial acceleration of the robot is assumed 0.1m/s². After 3s the robot maintains a velocity of 2m/s. The maximum velocity bound of the obstacles are assumed as 2 m/s. For each instant the previous x-coordinate and y-coordinate of the obstacle is added with a factor (a random number times the velocity of the obstacle in x-direction and y-direction) to give the next position of the obstacle. This is done to ensure randomness in the motion.

Figure 6 shows an environment containing 3 moving obstacles; robot (R) and the obstacles (O₁, O₂ and O₃) are represented by black and blue circles.

![Flow chart of the algorithm.](image-url)
respectively. The trail of the robot and obstacles are represented by green and red colours respectively. Figure 6 demonstrates the robot steering around the region of certainty of $O_1$. During encounter with $O_1$, the robot’s location falls on the curve hence the robot moves to the shortest safe point. Figure 7 shows the complete trajectory of the robot avoiding collision with $O_2$ and $O_3$ by steering to the vertex of the region of certainty.

**Fig. 6.** Robot steers around region of certainty of obstacle 1.

**Fig. 7.** Robot reaches the target (T).
4. Comparative Performance of the Algorithm

The proposed method of collision-avoidance is compared with the results obtained for two methods of distance time transform algorithm [18]. The initial position and velocity of robot and obstacles used is shown in Table 2.

<table>
<thead>
<tr>
<th>Parameters</th>
<th>Robot</th>
<th>Obstacle A</th>
<th>Obstacle B</th>
<th>Obstacle C</th>
<th>Obstacle D</th>
</tr>
</thead>
<tbody>
<tr>
<td>Initial position(m)</td>
<td>[0,0]</td>
<td>[0.05,5]</td>
<td>[0.03,6]</td>
<td>[0.07,7]</td>
<td>[0.04,12]</td>
</tr>
<tr>
<td>Velocity(m/s)</td>
<td>0.4</td>
<td>0.3</td>
<td>0.45</td>
<td>0.6</td>
<td>0.9</td>
</tr>
</tbody>
</table>

Figure 8 shows the performance of the algorithm with distance time transform method. The obstacles and its trail is denoted by a blue and white circles respectively. The path travelled by the robot (R) is represented using a series of green circles. The direction of obstacles (A, B, C and D) is represented using a red arrow and the direction of the robot is represented using a black arrow. The result discussed in distance time transform method [18] shows at higher obstacle velocities the collision-avoidance becomes tedious. The proposed algorithm was simulated under the same conditions. Table 3 shows the comparative study of region of certainty approach with distance time transform scheme.

<table>
<thead>
<tr>
<th></th>
<th></th>
<th></th>
</tr>
</thead>
<tbody>
<tr>
<td>Collision with obstacles take place at obstacle velocity 0.9 m/s</td>
<td>Collision with obstacles take place at obstacle velocity 0.9 m/s</td>
<td>Collision is avoided and target is reached at obstacle velocity 0.9 m/s</td>
</tr>
</tbody>
</table>

Fig. 8. Performance of the proposed algorithm.
5. Conclusions
An effective algorithm has been proposed to avoid collisions using the approach of region of certainty. The proposed approach considers the past motion of the obstacle to predict a futuristic region where the obstacle is likely to occupy. Simulation results show that the proposed algorithm is more effective in environments containing obstacles of high degree of freedom. A comparative study shows that the algorithm is capable of operating in environments containing high speed obstacles. This work can be extended for collision-avoidance of multiple robots with many moving obstacles.

References


