

DE BRUIJN SEQUENCE BASED PEAK POWER CONTROL AND INTERFERENCE MITIGATION IN MC-CDMA SYSTEM

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Abstract

In Multicarrier Code Division Multiple Access (MC-CDMA) system, the performance can get degraded due to a high Peak to Average Power Ratio (PAPR) which is equivalent to Peak to Mean Envelope Power Ratio (PMEPR) and Multi Access Interference (MAI). There are several methods to reduce the PAPR and MAI. Out of these, one of the methods is to use an efficient spreading code technique to reduce the PAPR and MAI. Here, the PAPR and MAI have been analysed using Walsh Hadamard sequence, Gold sequence, Gordon, Mills and Welch (GMW) sequence and de Bruijn sequence as the spreading code. Simulation results show that the de Bruijn sequence can provide lower PMEPR than that provided by the other spreading sequences. It is also found that the same de Bruijn sequence provides better MAI performance than the other sequences. So, in this paper it is proposed that the de Bruijn sequence can be used as a spreading sequence for MC-CDMA to reduce the PMEPR along with MAI in a MC-CDMA system.

Keywords: De Bruijn sequence, GMW sequences, Gold sequence, MAI; MC-CDMA, PAPR, PMEPR Walsh-Hadamard sequence.

1. Introduction

In a limited bandwidth wireless environment, the number of users is increasing day by day. So, there is a need to design a system, which can provide immunity from interference arising from adjacent signals, and high data rate. Keeping all these requirements in consideration, an improved system of data transmission is designed, called as Multi Carrier - Code Division Multiple Access (MC-CDMA). This MC-CDMA has all these qualities, which is making it quite popular in data transmission

Nomenclatures

$c_m(b)$	b^{th} path gain for the m^{th} user
c_m^s	Spreading sequence for the user s , m be the length of the Sequence
d_n^s	Data symbol for the user s - for PAPR analysis
$\tilde{d}_m(\mathbf{k})$	Estimated k^{th} data of m^{th} user
\tilde{I}_m	Multi-Access Interference (MAI) faced by m^{th} user due to $(M-1)$ users
$R(\cdot)$	Real part of the variable
$rx(t)$	Received signal
s_m	m^{th} user's orthogonal spreading code – for MAI analysis
x_m	Data symbol of m^{th} user for MAI Analysis

Greek Symbols

$\beta(t)$	Additive White Gaussian Noise
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systems in present day scenarios. This system acquires benefits from both, orthogonal frequency division multiplexing (OFDM) and code division multiple access (CDMA) [1].

Some of the features, which are increasing the popularity of MC-CDMA system, include high data rate, excellent bandwidth utilization, ability to prevent multi path interference and fading [2]. The basic idea behind this system is the conversion of frequency selective, broad band channels into a number of parallel, frequency non-selective narrow band channels. Even though MC-CDMA system of data transmission has got many advantages, it faces two serious problems viz. high PAPR and MAI. Due to these shortcomings, there is a reduction in the resolution of signal and deterioration of battery life. It also affects the system performance adversely. The main reasons behind these drawbacks are flawed cross-correlation and auto correlation properties of the spreading sequences or codes. One way to minimize this is to choose an efficient spreading sequence (or) a code possessing good correlation properties [3, 4].

In this paper, we have concentrated on spreading codes with good correlation properties. In MC-CDMA systems, the most commonly used spreading codes are Walsh Hadamard [5] and Gold sequences (or) Gold codes [5, 6]. GMW sequence and de Bruijn sequence have been used for cryptographic purposes earlier, [7, 8, 11] but in this paper, the GMW sequence and de Bruijn sequence are being used as spreading sequence in the MC-CDMA system for the first time. The de Bruijn sequence provides low PMEPR and minimizes the interferences, so de Bruijn sequence outperforms all other spreading codes. Research is still going on in this field to select an effective binary spreading code to reduce the PAPR and MAI in MC-CDMA system.

This paper is structured as follows. Section 2 describes peak power analysis of MC-CDMA system. Section 3 describes the binary spreading sequences. Section 4 describes MAI analysis in MC-CDMA. Section 5 describes simulation results. Section 6 describes the conclusion.

2. Peak Power Analysis of MC-CDMA System

The MC-CDMA transmitter, for peak power analysis, illustrated in Fig. 1 consists of serial to parallel conversion, followed by multiplication of data with spreading sequences. Other components include adder, interleaver and Inverse Fast Fourier Transform (IFFT) block. Let M be the length of the sequence or code and S be the number of active users such that $S \leq M$. Let there be N data symbols such that $d^s = [d_1^s, d_2^s, \dots, d_N^s]$. The spreading sequence for the user s is represented by $c^s = [c_1^s, c_2^s, \dots, c_M^s]$. All the spread data is added up and given to a block interleaver before IFFT operation is done on them. The IFFT output length is $K = N \times M$. The data to be transmitted is $x(t)$, which is the output of IFFT block. The transmitted symbol i.e. one OFDM symbol can be represented as

$$x(t) = \sum_{n=1}^N \sum_{m=1}^M \sum_{s=1}^S d_n^s c_m^s e^{j2\pi[N(m-1)+(n-1)t/T_s]} \\ = \sum_{k=0}^{K-1} \sum_{s=1}^S d_n^s c_m^s e^{j2\pi kt/T_s} \quad 0 \leq t < T_s \quad (1)$$

where T_s is the symbol period.

The PAPR of $x(t)$ is given by

$$PAPR = \frac{\text{Max}|R(x(t))|^2}{1/T_s \int_0^{T_s} |x(t)|^2 dt} \quad (2)$$

where $R(\cdot)$ represents the real part of the variable. It is direct that [9]

$$PAPR \leq PMEPR = \frac{\max_{0 \leq t < T_s} |x(t)|^2}{1/T_s \int_0^{T_s} |x(t)|^2 dt} \quad (3)$$

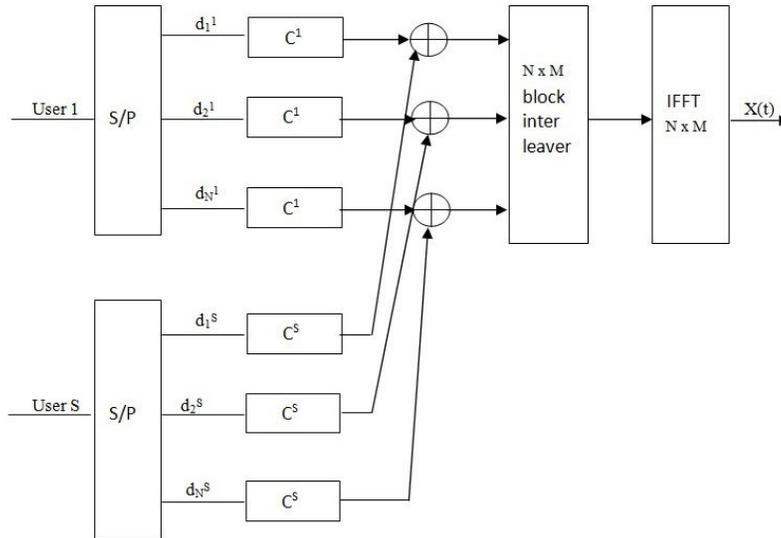


Fig. 1. MC-CDMA transmitter for peak power analysis.

3. Binary Spreading Sequences

3.1. Walsh Hadamard sequence

Walsh Hadamard code is in the form of a matrix, where each element of this matrix is either +1 or -1. This matrix is always a square matrix. Any row of the Walsh Hadamard matrix can be extracted and multiplied with the data which is to be transmitted using Walsh Hadamard code as the spreading sequence [6]. The general form of this matrix is given by: $H(2^m)$, where m is any natural number.

$$\text{In general, } H_{2^m} = \begin{pmatrix} H_{2^{(m-1)}} & H_{2^{(m-1)}} \\ H_{2^{(m-1)}} & -H_{2^{(m-1)}} \end{pmatrix}$$

For example,

$$H_2 = \begin{pmatrix} 1 & 1 \\ 1 & -1 \end{pmatrix}$$

3.2. GMW sequence

GMW sequences are pseudorandom binary sequences with a high linear complexity and low auto correlation value, which is very useful in many communication systems [4]. GMW sequence has linear complexity, which provides for a high level of cryptographic security against attack on data and is extensively used in spread spectrum communication [10]. The construction procedure of the GMW sequence is as follows.

- (i) Construct the M -sequence. The length of the sequence is $L = p^q - 1$, where $p = 2$.
Here $q = u.v$, $u \geq 3$, $v \geq 2$; Form the zinger difference set
 $D (\alpha = p^q - 1, \beta = p^{q-1} - 1, \delta = p^{q-2} - 1)$.
- (ii) Construct the decomposition matrix of M -sequence
 $A = [a_{ij}] \quad i=0, 1, \dots, Z-1. \quad j=0, 1, 2, \dots, W-1$.
Matrix has Z rows where $Z = \frac{p^q - 1}{p^s - 1}$ and W columns where $W = p^s - 1$.
- (iii) Matrix elements are: $a_{ij} = \{0 \text{ if } (i + Zj) \in D, 1 \text{ otherwise}\}$

3.3. Gold code

Gold Code or Gold sequence is a binary code. The Gold code is generated with the help of pseudo noise (PN) sequence i.e. Gold code is the concatenation of two PN sequences. This code has the property of smaller cross correlation in a set, so that multiple signals can be broadcasted at an identical frequency. Algorithm for generating the Gold code is:

- i) Let $a = \{a(t)\}$ and $b = \{b(t)\}$, be two sequences such that their period is $m = 2^p - 1$.
- ii) Modulo-2 addition of a with the m repeated shifted versions of b is performed. Through this way, a new sequence can be obtained. The period of this sequence is m .

If the new generated sequence, along with the original sequences (i & ii) are considered, we obtain $n + 2$ numbers of sequences. The resulting set of sequences attained is called Gold code or Gold sequence.

3.4. De Bruijn sequence

A binary de Bruijn sequence is cyclic sequence with order n . The period of the de Bruijn sequence is 2^n and the n -bit pattern appears exactly once [11]. De Bruijn sequence has good randomness properties, so this sequence is used in generation of cryptographic keys [11] and other engineering applications[8,11].The de-Bruijn sequence is constructed by using feedback shift registers (FSRs) [12], Hamiltonian path of n -dimensional de Bruijn graph also known as Good's diagram [8,14] and combinatorial algorithm [13]. One efficient way to generate the de Bruijn sequence is through combinatorial algorithm. Alhakim proposed an algorithm to generate the de Bruijn sequence, named as Prefer Opposite algorithm [11]. This is basically a family of combinatorial algorithms proposed by Fredricksen [13]. In this paper, a novel method of using de Bruijn sequence in spread spectrum communication is proposed by which peak power and interference can be reduced in MC-CDMA system.

The algorithm is as follows

- Step 1: Assign $a_1=a_2,\dots,a_n=0$ (Zero)
- Step 2: Choose $k=n+1$
- Step 3: Let $a_{k-n+1},\dots,a_{k-1}\bar{a}_{k-1}$ be a pattern then, check the pattern. If the pattern has not appeared before then $a_k = \bar{a}_{k-1}$. Increment the value of k by one and return to Step 3.
- Step 4: Otherwise, Let $a_{k-n+1} \dots a_{k-1} a_{k-1}$ be a pattern then, check the pattern. If the pattern has not appeared earlier in the sequence then $a_k = a_{k-1}$. Increment the value of k by one and return to Step 3.
- Step 5: Otherwise, stop.

4. MAI Analysis

The MC-CDMA transmitter with multiple users for MAI analysis is shown in Fig. 2. Uncoded Binary Phase Shift Keying (BPSK) MC-CDMA system with M users is considered. Data symbol x_m of the m^{th} user is multiplied with the m^{th} user's orthogonal spreading code $s_m = (s_m(0), s_m(1), s_m(2), \dots, s_m(K-1))$ where s_m is a vector of length K . The spread output is obtained as

$$d_m = x_m s_m = (d_m(0), d_m(1), \dots, d_m(K-1)) \tag{4}$$

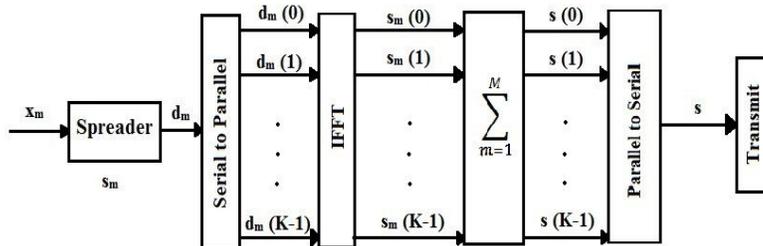


Fig. 2 MC-CDMA transmitter for MAI analysis.

After serial to parallel conversion to obtain T subcarriers, the parallel data is given to a T-point IFFT block. This is done to facilitate OFDM.

The IFFT block output is

$$s_m(p) = \sum_{k=0}^{K-1} d_m[k] \exp(j2\pi k(p+m)/K) \quad (5)$$

Summation of $s_m(p)$ for all users is done to get $s(p)$. Then parallel to serial operation is done to get $s = (s(0), s(1), \dots, s(K-1))^T$. The frequency response of the multipath channel for the m^{th} user is given by $C_{k,m} = \sum_{b=0}^{B-1} c_m[b] \exp(-j2\pi k b/K)$ where B is the number of autonomous paths in the channel. $c_m(b)$ is the b^{th} path gain for the m^{th} user.

The signal received at the receiver is given by

$$rx(t) = \sum_{m=1}^M \sum_{k=0}^{K-1} \sum_{p=0}^{K-1} C_{k,m} d_m(k) \exp(j2\pi k(p+m)/K) + \beta(t) \quad (6)$$

$\tilde{\mathbf{d}}_m = (\tilde{d}_m^0, \tilde{d}_m^1, \dots, \tilde{d}_m^{K-1})$ and is obtained by multiplying the conjugate of the channel response and the received signal vector.

$$\tilde{d}_m^K = C_{k,m}^* rx_k = \sqrt{\frac{2E_s}{N_0}} d_m(k) + \sum_{n=1, n \neq m}^M \sqrt{\frac{2E_s}{N_0}} d_n(k) C_{k,m}^* C_{k,n} + \beta_k \quad (7)$$

where E_s is the energy of the transmitted symbol.

The output of the decision combiner over noisy channel is represented as

$$Z_m = \sqrt{\frac{2E_s}{N_0}} d_m(k) + \check{I}_m + N_m \quad (8)$$

\check{I}_m is the MAI faced by m^{th} user due to $(M-1)$ users while the N_m term is a zero mean Gaussian Random Variable (G.R.V) having $N_0/2$ variance for m^{th} user.

$$\check{I}_m = R \left[\sum_{k=0}^{K-1} \sum_{n=1, n \neq m}^M \sqrt{\frac{2E_s}{N_0}} d_m(k) \right] \quad (9)$$

$$N_m = R \left[\sum_{k=0}^{K-1} \beta_k \right] \quad (10)$$

Using soft detection, we can obtain $\tilde{\mathbf{d}}_m(\mathbf{k})$, which is the estimated k^{th} data of m^{th} user. Then $\tilde{\mathbf{d}}_m$ is despread using \mathbf{s}_m to retrieve the received data symbol $\tilde{\mathbf{x}}_m$.

5. Simulation Results

In multiuser MC-CDMA, the PAPR properties have been discussed with different spreading sequences. The PMEPR can be represented in multiuser scenario as

$$PMEPR = \frac{\max_{0 \leq t < T_s} |x(t)|^2}{(\sum_{k=0}^{K-1} |\sum_{s=1}^S d_n^s c_m^s|^2)} \quad (11)$$

In the simulation, the system parameters involved in PMEPR analysis are explained below. In Fig. 3, 64 subcarriers are used, i.e., $N=2$ and $M=32$. In Fig. 4, 128 subcarriers are used, i.e., $N=1$ and $M=128$. Figures 3 and 4 show the PMEPRs for different spreading codes with different number of users. If number of user increases, it is observed that the PAPR gets reduced. In Figs. 3 and 4, we have analysed with different sequences like Walsh Hadamard, Gold Sequence, GMW sequence and de Bruijn Sequence. It is observed from Fig. 4 that the GMW

sequence performance is relatively equal to that of the Gold sequence. This is because of the large linear span and two level autocorrelation property of GMW sequence. It is perceived that the de Bruijn sequence performance is better than the other sequences (i.e., low PMEPR) on account of superior randomness property. Table 1 shows the comparison of PMEPR for the four different spreading codes (parameters associated with Fig. 3).

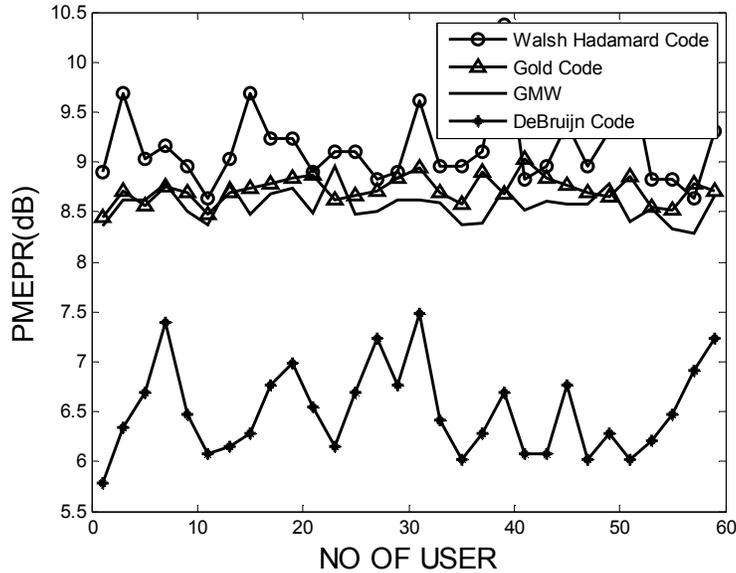


Fig. 3. PMEPR of multiple users with 64 subcarriers.

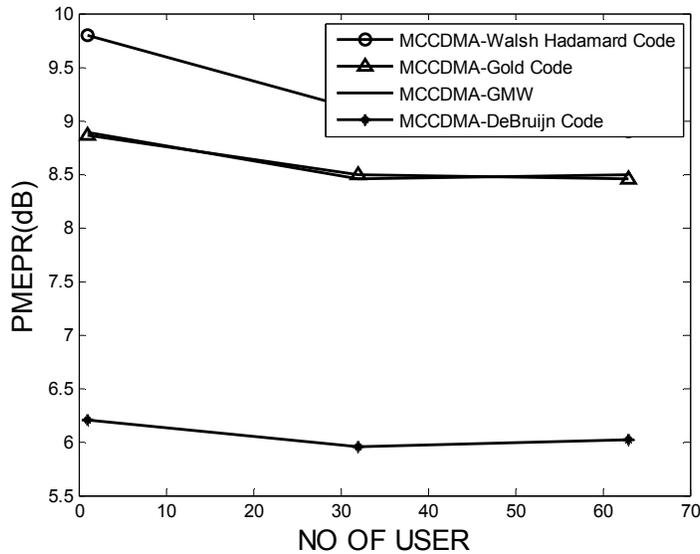


Fig. 4. PMEPR of multiple users with 128 subcarriers.

MAI analysis is shown in Fig. 5. In this analysis, $M = 4$ and $K = 32$ with uncoded BPSK modulation is assumed in the MC-CDMA system. The multipath channel profile consists of three taps which are $pr_0 = 0$ dB, $pr_1 = -7$ dB and $pr_2 = -10$ dB. The delay considered for the two indirect paths is 1 and 2 samples respectively. The channel is modeled as Rayleigh fading channel. Figure 5 shows the average BER performance comparison of the MC-CDMA system. Gold code, GMW code, Walsh-Hadamard code and the de Bruijn sequence are the spreading codes employed in this system. BER of $1.95E-2$ is obtained in Gold code-spread MC-CDMA at 25 dB SNR while BER of about $1.5E-2$ can be obtained for the same SNR when GMW code is used in the same MC-CDMA model. Better autocorrelation and orthogonality property of the Walsh Hadamard code and good randomness property of de Bruijn sequence further improve the BER performance. A BER of $1.85E-2$ is obtained at 22 dB SNR for WH-spread MC-CDMA compared to $1.3E-2$ for de Bruijn sequence spreaded MC-CDMA. At 25 dB SNR, the de Bruijn sequence-spread MC-CDMA offers $7.8E-3$ BER. This design provides a gain in the performance of about 2.5 dB and 4.5 dB when compared to WH-spread MC-CDMA and GMW-spread MC-CDMA respectively, at a BER of $3E-2$. The simulation results show that the interference levels are lowest when the de Bruijn sequence is used. So, de Bruijn sequence is most favorable for usage as a spreading code in such a system to reduce MAI levels and to improve BER.

Table 1. Comparison of PMEPR (dB) for four different spreading codes with 64 subcarriers.

Number of Users	Walsh-Hadamard Code	Gold Code	GMW Sequence	De-Bruijn Sequence
5	9.03	8.56	8.62	6.89
10	8.64	8.45	8.54	6.08
15	9.68	8.73	8.47	6.27
20	8.86	8.86	8.73	6.98
25	9.09	8.67	8.50	6.68
30	8.89	8.84	8.61	7.48
35	8.96	8.57	8.37	6.02
40	8.93	8.67	8.51	6.08
45	9.38	8.76	8.58	6.40
50	9.50	8.64	8.40	6.02
55	8.32	8.51	8.28	6.47
60	9.31	8.67	8.61	6.87

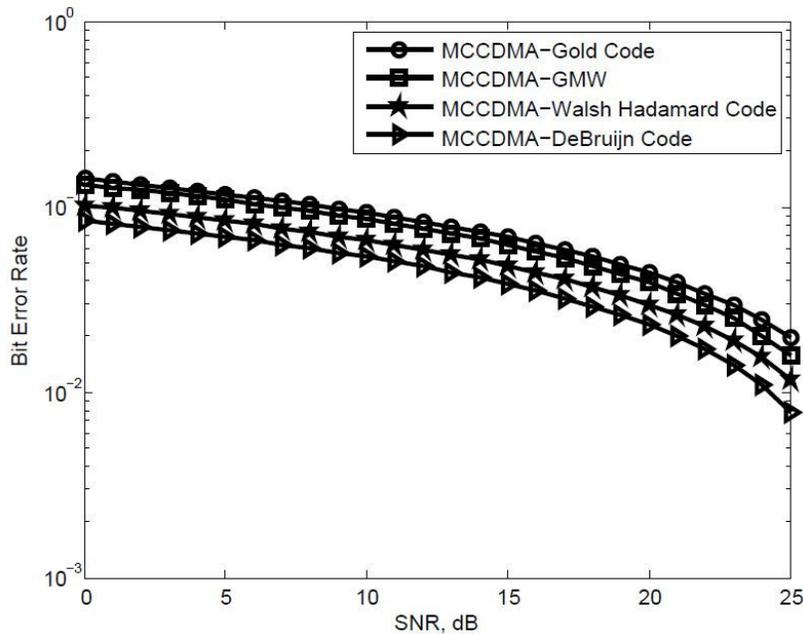


Fig. 5. Illustration of MAI analysis with different spreading codes.

6. Conclusion

In this paper, PAPR and MAI have been analysed for a MC-CDMA system using Walsh Hadamard, Gold code, GMW sequence and de Bruijn sequence for a multiuser scenario. When considering the Walsh Hadamard sequence, the PMEPR values are high, however, for Gold codes and GMW sequences, the PMEPR values are moderately equal but lower than that for Walsh Hadamard sequence. For de Bruijn sequence, the PMEPR values are much lower than that for all the three sequences. Similarly, MAI has been analysed using these four spreading codes. In the interference analysis too, the de Bruijn sequence outperforms the three sequences. So, it can be concluded that the de Bruijn sequence can also act as an efficient spreading sequence for the MC-CDMA system. If the existing channel coding techniques are adopted, the system performance would improve further.

References

1. Hara, S.; and Prasad, R. (1997). Overview of multicarrier CDMA. *IEEE Communication Magazine*, 35(12), 126-133.
2. Fazel, K.; and Kaiser, S. (2008). *Multicarrier and Spread Spectrum Systems* (2nd Ed.), Wiley Publishers, UK.
3. Gao, X.; Yu N.Y.; and Zhiwei M. (2010). Peak power control of MC-CDMA with special class of binary sequences. *Proceeding of 23rd Canadian*

- conference on *Electrical and Computer Engineering*, Calgary, Alberta, Canada 1-4.
4. Gong, G.; and Golomb, S.W. (2005). *Signal design for good correlation for wireless communication, cryptography and radar*, Cambridge University Press, New York.
 5. Tasi, S.H.; Lin, Y.P.; and Kuo, C.C.J. (2006). MAI free MC-CDMA system based on Hadamard Walsh Codes. *IEEE Transactions on Signal Processing*, 54(8), 3166-3179.
 6. Nobilet, S.; Helard, J.F.; and Mottier, D. (2006). Spreading sequences for uplink and downlink MC-CDMA systems: PAPR and MAI minimisation. *European Transactions on Telecommunications*, 13(5), 465-474.
 7. Scholtz, R.A.; and Welch, L.R. (1984). GMW sequences. *IEEE Transactions on Information Theory*, 30(3), 548-553.
 8. Meltem, S.T. (2011). Evolutionary construction of de Bruijn sequences. *Proceeding of the 4th ACM workshop of security and artificial intelligence*, 81-86.
 9. Litsyn, S. (2007). *Peak power control in multicarrier communication*, Cambridge University Press, New York.
 10. No, J.S.; Golomb, S.W.; Gong, G.; Lee, H.K.; and Gaal, P. (1998). Binary pseudorandom sequences of period 2^n-1 with ideal autocorrelation. *IEEE Transactions on Information Theory*, 44(2), 814-817.
 11. Abbas, M.A. (2010). A simple combinatorial algorithm for de-Bruijn sequences. *The American Mathematical Monthly*, 117(8), 728-732.
 12. Golomb, S.W. (2006). Shift register sequences – A retrospective account. *Sequence and their application-SETA-2006, Lecture notes in computer Science*, 4086, 1-4.
 13. Harold, F. (1982). A survey of full length nonlinear shift register cycle algorithms. *Society for Industrial and Applied Mathematics (SIAM- Review)*, 24(2), 195-221.
 14. Golomb, S.W.; Welch, L.R.; Goldstein, R.M.; and Hales, A.W. (1982). *Shift register sequences*. Aegean Park Press.