MULTILOOP PI CONTROLLER FOR ACHIEVING SIMULTANEOUS TIME AND FREQUENCY DOMAIN SPECIFICATIONS

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Abstract

Most of the controllers in control system are designed to satisfy either time domain or frequency domain specifications. This work presents the computation of a multiloop PI controller for achieving time and frequency domain specifications simultaneously. The desired time and frequency domain measures are to be specified initially to the design. To obtain the desired value of the performance measures the graphical relationship between the PI controller and the performance criteria is given. Thus by using graphical method a set of PI controller parameters to meet the desired performance measures are obtained in an effective and simpler way. The coupled tank has become a classic design of control engineering for multivariable process. The proposed control strategy has been implemented in the same coupled tank process and validated through simulation studies.

Keywords: Multiloop, Coupled tank, Coefficient diagram method, Gain margin.

1. Introduction

The PI/PID controller is the most popular controller mode used in process control applications because of its remarkable effectiveness and simplicity of implementation. Although significant developments have been made in the advanced control theory, more than 95% of industrial controllers are still PID, mostly PI controllers [1- 4]. PI control is sufficient for a large number of control processes, particularly when dominant process dynamics are either of the first or second order and the design requirements are not too rigorous [2]. Although this controller has only three parameters for PID, it is not easy to find their optimal values without a systematic procedure [5]. As a result, good PI /PID tuning methods are extremely desirable due to their widespread use. Generally, most industrial processes are Multi-Input and Multi-Output (MIMO) systems that are

Nomenclatures	
K	Process gain
k_i	Integral gain
k_p	Proportional gain
T	Equivalent time constant
Greek symbols	
γ_i	Stability indices
γ_i^*	Stability limit indices
τ_p	Process time constant
Θ	Process dead time
Abbreviations	
CDM	Coefficient Diagram Method
FTDP	Frequency time domain performance
MIMO	Multi input multi output
PI	Proportional integral
PID	Proportional integral derivative
SISO	Single input single output

usually controlled by two ways. One is multiple Single-Input Single-Output (SISO) controllers to constitute a multiloop control system, while the other uses multivariable controllers to become a multivariable control system. Multivariable control approaches use a full matrix PID controller. Even though multivariable control method gives a good closed loop response, it has two important deficiencies. The first one is the cross coupling of the process variables that makes it difficult to design each loop independently. In other words, adjusting controller parameters of one loop affects the performance of another, sometimes to the extent of destabilizing the entire system. The second one is for n input and m output MIMO system 3 nm parameters should be tuned for a full matrix PID controller. When the interactions between the variables of the process are modest, a diagonal controller (Multiloop) is often adequate. Over the years different methods have been proposed for designing controllers for a multiloop system [2-6]. One simplest method to design a multiloop control system is Biggest Log modulus Tuning (BLT) method [7]. Though the PI controllers for SISO systems are designed for satisfying predefined specifications [8] the same is not available for MIMO systems.

In this work a design of multiloop controller for coupled tank process is proposed, the controller so designed will achieve the desired frequency and time domain specifications simultaneously.

2. Coefficient Diagram Method

Coefficient Diagram Method (CDM) is a polynomial approach proposed by Manabe [9]. The controller designed using CDM will have the smallest degree, the smallest bandwidth and will have a closed loop time response without any overshoot. The basic block diagram of a CDM control system is shown in Fig. 1.

In this figure, *y* is the output, *r* is the reference input, *u* is the control and *d* is the disturbance signal. The plant transfer function *G(s)* =*N(s)/D(s)* where *N(s)* is the numerator and *D(s)* is the denominator of *G(s)* which do not have any common factors. *A(s)* is the forward denominator polynomial of the controller transfer function while *F(s)* and B(s) are the reference numerator and the feedback numerator polynomials of the controller transfer function respectively. As the controller transfer function has two numerators, it resembles a two Degree Of Freedom structure (2DOF).

Fig. 1. Block diagram of CDM.

From Fig.1, the output of the controlled closed loop CDM system is

$$y(s) = \frac{N(s)F(s)}{P(s)} \, r(s) + \frac{A(s)N(s)}{P(s)} \, d(s) \tag{1}$$

where *P(s)* is the characteristic polynomial of the closed loop system and is given by

$$P(s) = D(s)A(s) + N(s)B(s) = \sum_{i=0}^{n} a_i s^i \tag{2}$$

The CDM design parameters namely, equivalent time constant (τ), stability indices (γ_i) and stability limit indices (γ_i^*) [10] are given by

$$\tau = \frac{a_1}{a_0} \tag{3}$$

$$\gamma_i = \frac{a_i^2}{a_{i+1} a_{i-1}} \, , i = 1 \sim (n-1), \gamma_0 = \gamma_n = \infty \tag{4}$$

$$\gamma_i^* = \frac{1}{\gamma_{i-1}} + \frac{1}{\gamma_{i+1}} \tag{5}$$

The above parameters have to be specified prior to design and then used for determining the target characteristic polynomial of the closed loop system. Using Eq. (2), the characteristic polynomial is obtained as

$$P_{target}(s) = a_0 \left[\left\{ \sum_{i=2}^{n} \left(\prod_{j=1}^{i-1} \frac{1}{\gamma_{i-j}^j} \right) (\tau s)^i \right\} + \tau s + 1 \right] \tag{6}$$

The equivalent time constant specifies the time response speed. The stability indices and the stability limit indices affect the stability and the time response. The variations of the stability indices due to plant parameter variations specify robustness.

3. Controller Design

Two steps are involved to design a PI Controller satisfying the required time and frequency domain specifications. First step involves the computation of global

and local stability regions using the stability boundary locus approach. In the second step, the CDM method is used to design PI controller parameters for which the step responses have acceptable overshoot and settling time. Frequency Time Domain Performance (FTDP) map can be obtained by combining the above two steps. FTDP map is a graphical tool which shows the relation between the stabilizing parameters of the PI controllers and the chosen frequency and time domain performance criteria on the same (k_p, k_i) plane. Thus from the FTDP map one can choose a PI controller providing the desired specifications such as gain margin, phase margin, maximum overshoot and settling time values all together. 2DOF control scheme is shown in Fig. 2 which is the rational equivalence of the control system shown in Fig. 1. Here $G_C(s) = B(s)/A(s)$ is the main controller and $G_f(s) = F(s)/B(s)$ is the setpoint filter. It can be shown that the steady state error to a unit step change in the setpoint and the disturbance become zero robustly if,

$$\lim_{s \to 0} G_c(s) = \infty \text{ and } \lim_{s \to 0} \frac{G_f(s)}{G_c(s)} = 0 \tag{7}$$
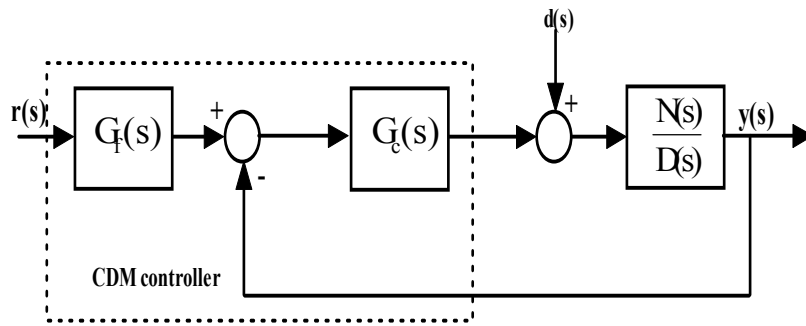

Fig. 2. Equivalent block diagram of CDM.

If $G_c(s)$ includes an integrator then these conditions can be satisfied. Thus $G_c(s)$ can be shown as

$$G_c(s) = k_p + \frac{k_i}{s} = \frac{k_p s + k_i}{s} \tag{8}$$

In the type of the conventional PI element and $G_f(s)$ is an appropriate element satisfying Eq. (7).

3.1. Global stability regions

Consider a control system shown in Fig. 1 where

$$G_p(s) = G(s)e^{-\theta s} = \frac{N(s)}{D(s)}e^{-\theta s} \tag{9}$$

and a PI controller of the form of Eq. (8). The problem is to compute the global stability region which includes all the parameters of the PI controller which stabilize the given system. The closed loop characteristic polynomial P(s) of the system is

$$P(s) = sD(s) + (k_p s + k_i)N(s)e^{-\theta s}$$

$$= a_n s^n + a_{n-1} s^{n-1} + \cdots a_1 s + a_0 \tag{10}$$

where all or some of the coefficients $a_i, i = 0,1,2\ldots.n$ are the function of k_p, k_i and $e^{-\theta s}$ depending on the order of $N(s)$ and $D(s)$ polynomials, where $N(s)$ and $D(s)$ are the numerator and denominator polynomial of the process by decomposing it into their corresponding even and odd part by replacing s by $j\omega$ gives

$$G(j\omega) = \frac{N_e(-\omega^2) + j\omega \, N_o(-\omega^2)}{D_e(-\omega^2) + j\omega \, D_o(-\omega^2)} \tag{11}$$

Equating the real and imaginary part of the *closed* loop characteristic polynomial to zero and solving for k_p and k_i we get

$$k_p = \frac{(\omega^2 N_0 D_0 + N_e D_e) \cos(\omega\theta) + \omega(N_0 D_e - N_e D_0) \sin(\omega\theta)}{-(N_e^2 + \omega^2 N_0^2)} \tag{12}$$

$$k_i = \frac{\omega^2 (N_0 D_e - N_e D_0) \cos(\omega\theta) - \omega(N_e D_e + \omega^2 N_0 D_0) \sin(\omega\theta)}{-(N_e^2 + \omega^2 N_0^2)} \tag{13}$$

The stability boundary locus $l(k_p, k_i, \omega)$ in the (k_p, k_i)-plane can be obtained using Eqs. (12) and (13). From Eqs. (12) and (13) it is observed that the stability boundary locus depend on the frequency ω which varies from 0 to ∞. Since the controller operates at the frequency range of below the critical frequency ω_c or ultimate frequency, the stability boundary locus can be obtained over the frequency range of ω varies between 0 to ω_c.

3.2. Local stability regions

The performance of the controller can be evaluated by measuring the phase and gain margin. These two are the important frequency domain performance measure. Let the gain-phase margin tester $G_c(s) = Be^{-j\phi}$, which is connected in the feed forward path of the control system shown in Fig. 1. Then the equation for k_p and k_i are

$$k_p = \frac{(\omega^2 N_0 D_0 + N_e D_e) \cos(h) + \omega(N_0 D_e - N_e D_0) \sin(h)}{-B(N_e^2 + \omega^2 N_0^2)} \tag{14}$$

$$k_i = \frac{\omega^2 (N_0 D_e - N_e D_0) \cos(h) - \omega(N_e D_e + \omega^2 N_0 D_0) \sin(h)}{-B(N_e^2 + \omega^2 N_0^2)} \tag{15}$$

where $h = \omega\theta + \phi$

To obtain local stability regions from global stability region one needs to set $\phi = 0$ in Eqs. (14) and (15) for a given value of gain margin B. On the other hand by setting B=1 in the equation for k_p and k_i one can obtain the stability boundary locus for the given phase margin ϕ .

3.3. Time response performance

CDM is a classical control method used to design a controller for achieving desired time domain performances. The experimental identification of the most industrial process expressed in First Order Plus Dead Time (FOPTD) as

$$G(s) = \frac{ke^{-\theta s}}{\tau_p s + 1} \tag{16}$$

The term $e^{-\theta s}$ presents in the model has to be approximated such that the order of the model is not increased because the CDM requires higher order controller for higher order model. If pade approximation or Taylor denominator approximation is used definitely the order of the model get increased results in complex controller. So Taylor numerator approximation $e^{-\theta s} = 1-\theta s$ is used which will not affect the denominator of the model. Hence the model transfer function represented in Eq. (16) becomes

$$G(s) = \frac{N(s)}{D(s)} = \frac{-k\theta s + k}{\tau_p s + 1} \tag{17}$$

The CDM controller polynomial is chosen as

$$A(s) = l_1 s; \; B(s) = k_1 s + k_0 \tag{18}$$

Thus the configuration in Fig. 2 is transformed to PI based control system. From Eqs. (8) and (18), it can be seen that $k_1 = k_p$, $k_0 = k_i$ and $l_1 = 1$. A characteristics polynomial depending on the parameters k_p and k_i is obtained after substituting the controller polynomials determined by Eq. (18) in Eq. (2). Since the model transfer function is in FOPTD form, the characteristic polynomial is

$$P(s) = \left(\tau_p - K\theta k_p\right)s^2 + \left(1 - \theta K k_i + K k_p\right)s + K k_i \tag{19}$$

Using Eq. (4), γ_1 and τ can be obtained as

$$\gamma_1 = \left(1 + K k_p - K\theta k_i\right)^2 / K k_i \left(\tau - K\theta k_p\right) \tag{20}$$

$$\tau = \left(1 + K k_p - K\theta k_i\right) / K k_i \tag{21}$$

From Eqs. (20) and (21), k_i can be found for γ_1 and τ separately as

$$k_i = \left\{K\left(2\theta + 2K\theta k_p + \gamma_1 \tau - \gamma_1 K\theta k_p\right) - \sqrt{\Delta}\right\} / (2K^2 \theta^2) \text{ for } \gamma_1 \tag{22}$$

where $\Delta = K^2 \left\{\left(2\theta + 2K\theta k_p + \gamma_1 \tau - \gamma_1 K\theta k_p\right)^2 - 4\theta^2 \left(1 + K k_p\right)^2\right\}$ and

$$k_i = \left(1 + K k_p\right) / (K\tau + K\theta) \text{ for } \tau \tag{23}$$

The values of k_p obtained from the global stability region can be substituted in Eqs. (22) and (23), k_p-k_i curve can be plotted. The values of k_p and k_i parameters at the intersection of two curves provided desired τ vs γ_1.

4. Frequency Time Domain Performance Map

The FTDP map is obtained by plotting frequency domain stability region and time domain performance curve in the same k_p-k_i plane. For frequency domain specification, the global stability region and local stability region are obtained using Eqs. (12)-(15). The time domain performance curves which corresponding to overshoot and settling time can be plotted using Eqs. (22) and (23). From the FTDP map, the k_p-k_i values can be chosen which will satisfy the desired frequency and time domain properties.

5. Coupled Tank Process

Control of a coupled tank process a benchmark problem that demands strong robustness and is difficult to control in chemical industry because coupled tank

process is a MIMO system with two inputs and two outputs. The schematic diagram of coupled tank process is shown in Fig. 3. The controlled variables are tanks level of h_1 and h_2. The levels of the tank are maintained by manipulating the inflow to the tanks. β_x is the valve ratio of the pipe between tank1 and tank2, β_1 and β_2 are the valve at the outlet of tank1 and tank2 respectively. The mass balance equation of the coupled tank process is

$$A_1 \frac{dh_1}{dt} = k_{PP1}u_1 - \beta_1 a_1 \sqrt{2gh_1} - \beta_x a_{12}\sqrt{2g(h_1 - h_2)} \tag{24}$$

$$A_2 \frac{dh_2}{dt} = k_{PP2}u_2 + \beta_x a_{12}\sqrt{2g(h_1 - h_2)} - \beta_2 a_2 \sqrt{2gh_2} \tag{25}$$

where A is the cross sectional area of the tank, a_1 and a_2 is the cross sectional area of outlet pipe in tank1 and tank2 respectively, k_{PP1} and k_{PP2} are the gains of pump1 and pump2 and g is the specific gravity. The parameters of the process and its operating points are listed in Table 1 and Table 2.

Fig. 3. Schematic diagram of coupled tank process.

Table 1. Parameters of coupled tank process.

A_1, A_2 (cm^2)	a_1, a_2, a_{12} (cm^2)	β_1	β_2	β_x
154	0.5	0.7498	0.8040	0.2245

Table 2. Operating conditions of coupled tank process.

u_1(v)	u_2(v)	h_1(cm)	h_2(cm)	K_{PP1}(cm^3/v-sec)	k_{PP2}(cm^3/v-sec)
2.5	2.0	18.32	12.23	33.336	25.002

6. Simulation Studies

The experimentally identified model for the coupled tank process using reaction curve method is [10]

$$G_p(s) = \begin{bmatrix} \dfrac{16.99\,e^{-12.89s}}{(214.03s + 1)} & \dfrac{6.69\,e^{-72.57s}}{(204.93s + 1)} \\ \dfrac{9.23\,e^{-35.01s}}{(256.44s + 1)} & \dfrac{11.38\,e^{-25.04s}}{(169.15s+1)} \end{bmatrix} \tag{26}$$

   PI controller parameters satisfying time and frequency domain specification such as settling time,overshoot,gain margin and phase margin are obtained using the above identified model. FTDP map has to be built for loop1 and loop2 seperately satisfying the specifications. The global stability region for loop1 and loop 2 are shown in Figs. 4 and 5 are computed using Eqs. (12) and (13). All PI controllers which stabilizes the system are shown in global stability region. PI controller values   has to be choosen from the global stability region which satisfies time and frequency domian performances. The local stability regions for the gain margin and phase margins can be identified within the global stability region using Eqs. (17) and (18) and are shown in Figs. 6-9. The intersections values of k_p and k_i of the local stability regions for gain margin and phase margin will satisfy both GM and PM specifications. The FTDP map with the properties of 2.0<GM<5.0 ,30°<PM<60°, γ (1.4,1.9,3.0) curve and τ (50,75,100) curve is shown in Figs. 10 and 11 for loop1 and loop2 respectively. From the FTDP map values of the k_p, k_i satisfying the above properties are obtained as k_{p1}=0.4606, k_{i1}=0.008261 and k_{p2}=0.2473, k_{i2}=0.00268 for loop1 and loop2 respectively. The closed loop and its interaction outputs for setpoint change in tank1 of 25 cm from its operating value are shown in Figs. 13 and 14. Similarly the setpoint change in tank2 of 17 cm is shown in Fig. 15 and its corresponding interaction output of tank1 is shown in Fig. 16.
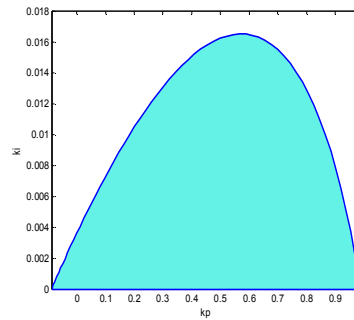

Fig. 4. Global stability region for Loop1.

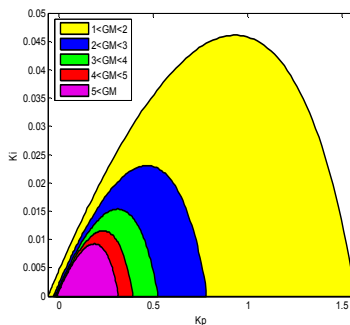**Fig. 5. Global stability for Loop2.**

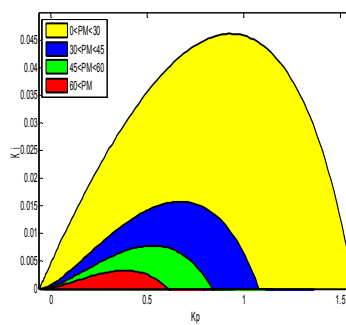**Fig. 6. Local stability regions for some GM values of Loop1.**

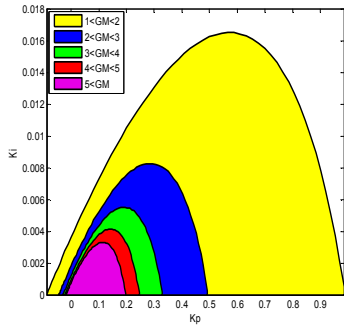**Fig. 7. Local stability region some PM values of Loop1.**

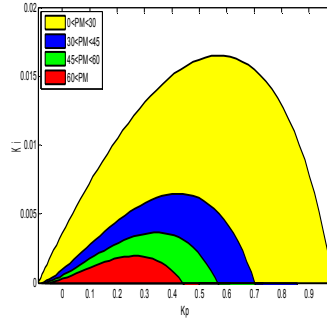**Fig. 8. Local stability regions for some GM values of Loop2.**

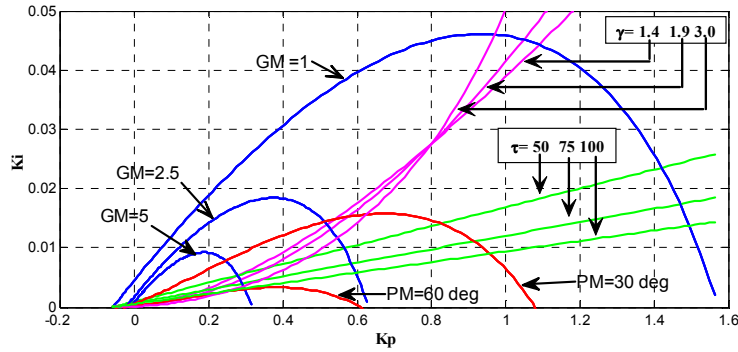**Fig. 9. Local stability regions for some PM values of Loop2.**
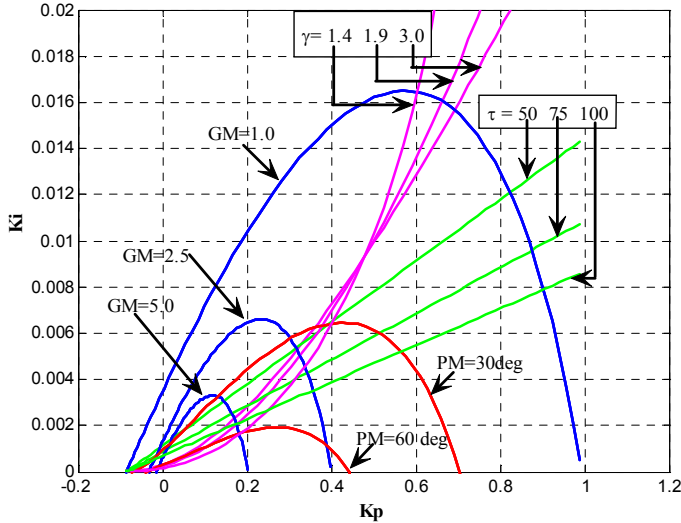

Fig. 10. FTDPM for Loop1.
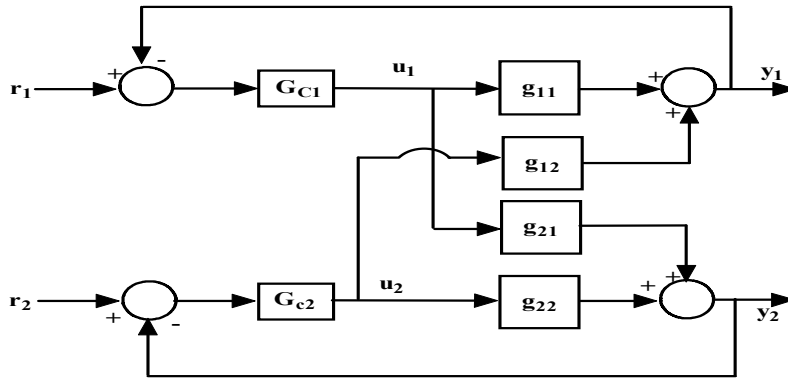

Fig. 11. FTDPM for Loop2.

Fig. 12. Block diagram of multiloop control System.
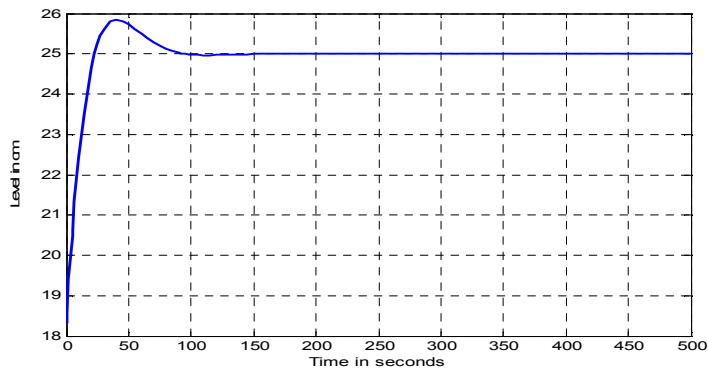

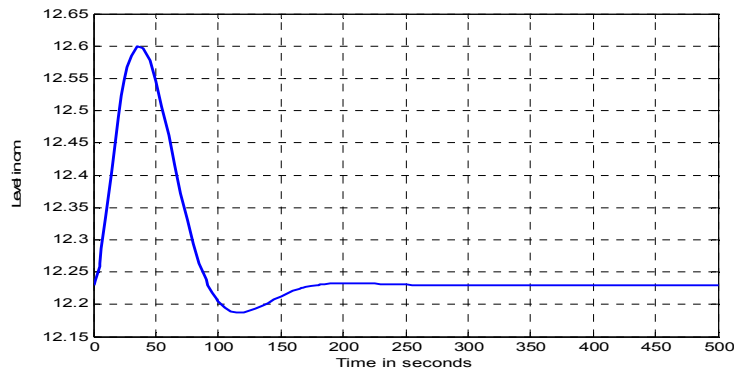**Fig. 13. Closed loop response of coupled for setpoint change in tank1.**

**Fig. 14. Closed loop interaction response of coupled tank
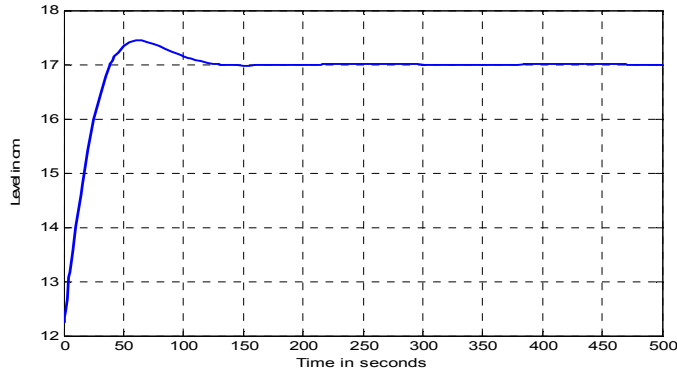for setpoint change in tank1.**

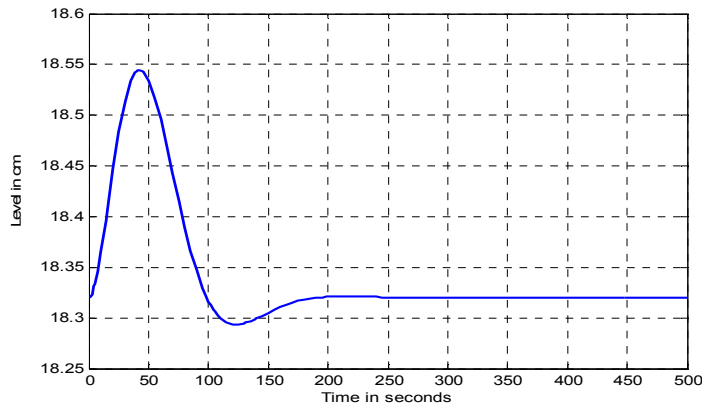**Fig. 15. Closed loop response of coupled tank for setpoint change in tank2.**

**Fig. 16. Closed loop interaction response of coupled tank
for setpoint change in tank2.**

7. Conclusions

A new graphical method has been used to design a multiloop controller for multivariable coupled tank process. In this method has advantage in its ease of finding the controller parameters without solving the equations and satisfying both time domain and frequency domain performances. It has been shown that the FTDPM is obtained first by computing global stability regions for loop1 and loop2 by stability boundary locus method then local stability region is also computed for frequency domain performances then finally CDM method is applied to achieve time domain performances. FTDPM gives a visual chance for the designer to choose the controller parameters. Simulation results clearly show the supremacy of the controller performances to the multivariable system.

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