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OPTIMISATION OF RESIDENTIAL ROOF INSULATION LAYER THICKNESS BASED ON ECONOMIC ANALYSIS BY GREY RELATION METHOD

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Abstract

This study is focussed on the optimisation of residential building roof insulation layers that includes weathering tile, wood wool and phase change material through grey relation analysis and numerical simulation techniques. The optimum thickness of insulation layers is determined for the quality objective of minimization of insulation material cost and energy consumption cost by air conditioning system over a life time of 10 years. For optimisation of roof insulation layer, the insulation layers are varied to five levels with reference to the height of concrete (HC) layer and various combination of roof layers are obtained from Taguchi's L_{25} orthogonal array. The 25 combination of roof structures are analysed by Numerical simulation technique to determine yearly heating transmission load and in turn used to calculate the cost of energy consumption for cooling a period of 10 years. As a result, the optimum thickness value of roof insulation layer - weathering tile, wood wool and phase change material are predicted as 0.33*HC, 0.33*HC and 0.066*HC respectively and this optimum value will have the cost of insulation and yearly electricity cost of cooling for 10 years as 92 $/m^2$ and 12.45$/m^2$ respectively.

Keywords: Optimum insulation thickness, Grey relational analysis, Numerical simulation, wood wool, phase change material.

1. Introduction

Solar radiation is the major factor that rises the indoor temperature and in turn leads to poor thermal comfort for occupants. Solar radiation entrapped through the roof, exterior walls and window openings and adds heat to the indoor air. Especially, buildings nearer to the equatorial region are highly affected by solar radiations. In this context, reduction of heat entrapment in to the building with the use of thermal insulators is having good merit and hence most research works are getting focus in the building insulation.

D'Orazio et al. [1] studied the effect of roof covering on the thermal performance of highly insulated roof with roof tiles, EPS and fire wood. Al Sanea et al. [2] used cement plaster, moulded polystyrene and heavy weight concrete for insulation. Similarly, Sage-Lauck and Sailor [3] used phase change material, Toguyoni et al. [4] employed local materials like clay and straw and Ming Chou et al. [5] made a new design of metal sheet that includes PCM, polyurethane, corrugated roof for roof cooling. From this, it is well identified that using thermal insulation in building's exterior walls and roof reduces the rate of heat transmission across the building envelop. And also increasing the thickness of roof insulation layer reduces the heating load and their by reduces the cost of energy consumed by air conditioning unit. However, increasing the thickness of insulation layer raises the cost of initial investment and hence an optimum thickness should be identified and practised.

Yu et al. [6] optimised the roof insulation layers that include expanded polystyrene,extruded polystyrene, foamed polyurethane, perlite and foamed polyvinyl chloride using solar air cooling and heating degree days method for external walls of the building. Daouas et al. [7] used analytical method based on complex finite fourier transform to estimate the yearly cooling transmission load and determined the optimum thickness of the insulation layers like extended polystyrene and rock wool. Ozel [8] made a cost analysis for optimum thickness and environmental impacts of different insulation material. Prakash and Ravikumar [9] optimised the roof layer thickness for maximum reduction of heat entry with minimum thickness of insulation layers like PCM, wood wool and weathering tile.

With all these information it is identified that the multi objective optimisation study on roof by considering the material cost and effective insulation of solar radiation is rare and hence this work is focused to optimise the thickness of roof layers namely PCM, wood wool and weathering tile for reduced cost and maximum heat insulation.

2. Residential Roof Structure Model- Optimisation of Roof Layer Thickness

Traditional residential buildings are having the roof constructed with concrete and overlaid with weathering tiles. This roof structure doesn't reduce the entry of solar radiation effectively and the indoor temperature is having fluctuation as a function of solar radiation. Hence a modified form of roof structure with insulation layer like wood wool and inorganic eutectic phase change material is added between the concrete and weathering tile and their thickness are optimised. In this study, the optimisation is done by considering only the transmission part of the air conditioning load since other loads do not influence the optimum thickness of insulation. [10]. The optimum thickness thus obviously gives the minimum total cost, which includes the cost of insulation material and cost of energy consumption over the lifetime of the building [11]. The roof structure under investigation is shown in Fig. 1. The identified control parameters for the optimisation are the thickness of weathering tile, wood wool and phase change material. These three parameters are varied to five levels of variation and Taguchi's L_{25} orthogonal array is employed. Taguchi's methodology is a strategy of planning, conducting, analyzing and interpreting experiments so that, sound and valid conclusions can be drawn efficiently, effectively and economically. It is the powerful tool, which can offer simultaneous improvements in quality and cost [12]. Hence in this study the Taguchi's L_{25} orthogonal array is used and the thickness of the control parameters is generalized with the concrete layer thickness and their corresponding values for five levels are given in Table 1. The multiple objectives of this optimisation study include yearly energy cost for cooling and cost for insulation. These objectives are predicted from the Eqs.(1) and (2) [8].

The early energy cost of cooling per unit area,

$$C_{A,C} = \left(C_E Q_G / COP\right) \times PWF \tag{1}$$

where Q_g is the total heat gain per area (W/m^2), C_E is the cost of electricity ($/kW h), 0.1894 $/kW h [8] and COP is the performance of cooling system, 2 and PWF is the present worth factor.

$$PWF = \frac{(1+r)^N - 1}{r(1+r)^N} \tag{2}$$

$$r = \frac{i-g}{1+g} \tag{3}$$

where i is the interest rate, 8.75% [8]; g is the Inflation rate, 7.50% [8] and N is the life time, 10 years and *r* is the interest rate adopted for the inflation rate.
Cost of insulation [8],

$$C_{ins} = \sum C_i L_i \tag{4}$$

C_i is the cost of insulation material per unit volume ($/m^3) and L_i is the insulation thickness (m)

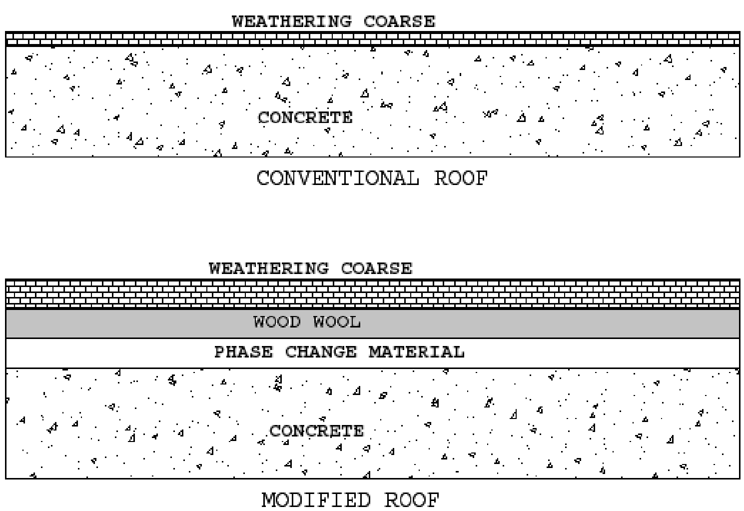

Fig. 1. Roof model- Traditional and modified roof with insulation layers.

Table 1. Modified roof layer- Insulation layers levels of variation.

Control parameter	Level 1	Level 2	Level 3	Level 4	Level 5
Weathering tile thickness (WC)	0.066*Hc	0.13*Hc	0.2*Hc	0.26*Hc	0.33*Hc
Wood wool thickness (WL)	0.066*Hc	0.13*Hc	0.2*Hc	0.26*Hc	0.33*Hc
Phase change material thickness - Inorganic eutectic (PCM)	0.066*Hc	0.13*Hc	0.2*Hc	0.26*Hc	0.33*Hc

The above parametric values are employed in the Taguchi's L_{25} orthogonal array and the thermal performance of roof with various layer thicknesses are analysed by numerical simulation technique and through which the yearly energy cost for cooling by air conditioning unit is calculated.

3. Thermal Performance of Roof Structure- Numerical Simulation Technique.

The roof with various combinations of roof thickness as given in Table 3 is modelled in the 2-dimensional approach in Gambit software and imported to Fluent software for analyzing the thermal performance in transient approach. The material properties of the roof materials are given in Table 2. The model is meshed with quadrilateral element with size of 0.025m as it is identified as a result independent mesh size.

Table 2. Material properties of roof construction materials [16].

Roof material	Density kg/m^3	Thermal conductivity W/mK	Specific heat J/kg.K
Concrete	2300	1.279	1130
Weathering tile	1300	0.25	1300
Wood wool	500	0.1	1000
Phase change material [13]	1640	1.09(0-27° C); 0.54(28-60°C)	1440(0-26.5°C); 125,000 (26.5-28°C); 1440 (28-60°C)

As a boundary condition T_{sol} air temperature was predicted from the solar radiation data for the month of May at Chennai [17] and specified to the surface of the roof. The predicted values of T_{sol} air and solar radiation data are shown in Fig. 2. The bottom surface of the roof is specified as convective boundary with the temperature of 298 K and heat transfer coefficient 10W/m^{2}K. The roof is analyzed under steady state condition initially and later it is simulated in a transient mode for one complete 24 hours. To avoid the effect of initial condition on the result, the simulation has been carried out repeatedly for 24 hours till the temperature distribution at the end of the two consecutive days will be same. All the cases are iterated up to a convergence level of 10^{-6}. This simulation is also validated with the experimental predictions of roof bottom temperature for one complete 24 hrs made by Pasupathi and Velraj [13]. The close agreement of simulated temperature at the bottom of the roof with experimental predictions is shown in Fig. 2. The roof bottom temperature reduces gradually from time 0hr to 8hr and raises upto a time of 18 hr and again gets reduced. The lowest and highest temperature of the roof bottom is predicted as 299K and 306 K respectively.

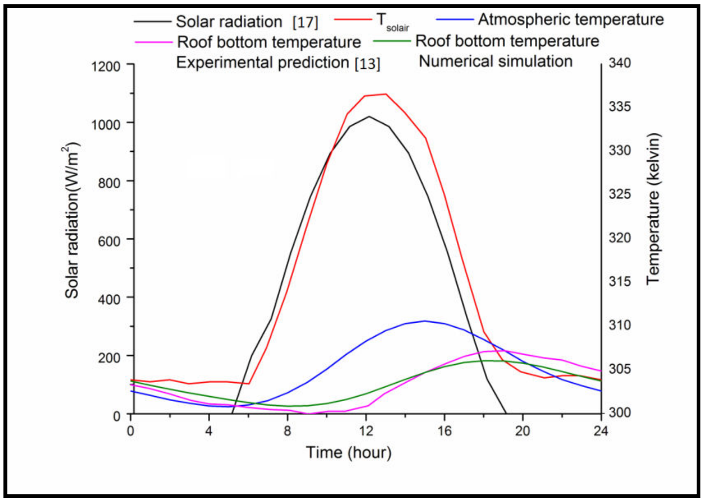

Fig. 2. Metrological data for the month of May at Chennai and validation of Numerical simulation.

Based on the above discussed numerical procedure the thermal performance of roof cases for 25 combinations of roof insulation layer thickness are simulated, their corresponding heat gain through the roof were calculated and shown in Table 3. The temperature across the roof for the roof with concrete and weathering coarse, roof with insulation layers with 0.066 * Hc and 0.33 * Hc are shown in Fig. 3. The maximum and minimum temperature across the roof without insulation layer is predicted as 324.5 K and 303.5 K and for the roof with insulation layer the same is predicted as 331.28 K and 301.78 K (0.066 * Hc) and 337.14 K and 299.5 K (0.33*Hc).

From the heat gain, the cooling cost incurred for the electricity ($C_{A,C}$) for a period of 10 years is evaluated from the Eq. (1) and given in Table 3. Also the material cost of weathering tile, wood wool and PCM for unit thickness is 0.4\$/m^2, 10\$/m^2 and 40\$/m^2 respectively and these material cost are collected from the local sellers. These material cost are used to calculate the total cost of insulation (C_{ins}) by using the Eq. (4) and both costs for various combination of roof insulation layer thickness are given in Table 3.

4. Grey Relational Analysis

Grey relational technique was developed by Geng for multiple objective optimisations. A grey system has a level of information between black and white, where the black represents having no information and white represents having all information [14]. Grey analysis along with Taguchi's technique is a successful method to analyze the relationship between sequences with less data and can analyze many factors that can overcome the disadvantage of statistical method [15]. In this GRA technique, the various responses are normalized and grey relational coefficients are determined. Based on the grey relation coefficients, grey relational grades are predicted and are ranked. These grey relational grades are used to identify the optimal values of control parameters

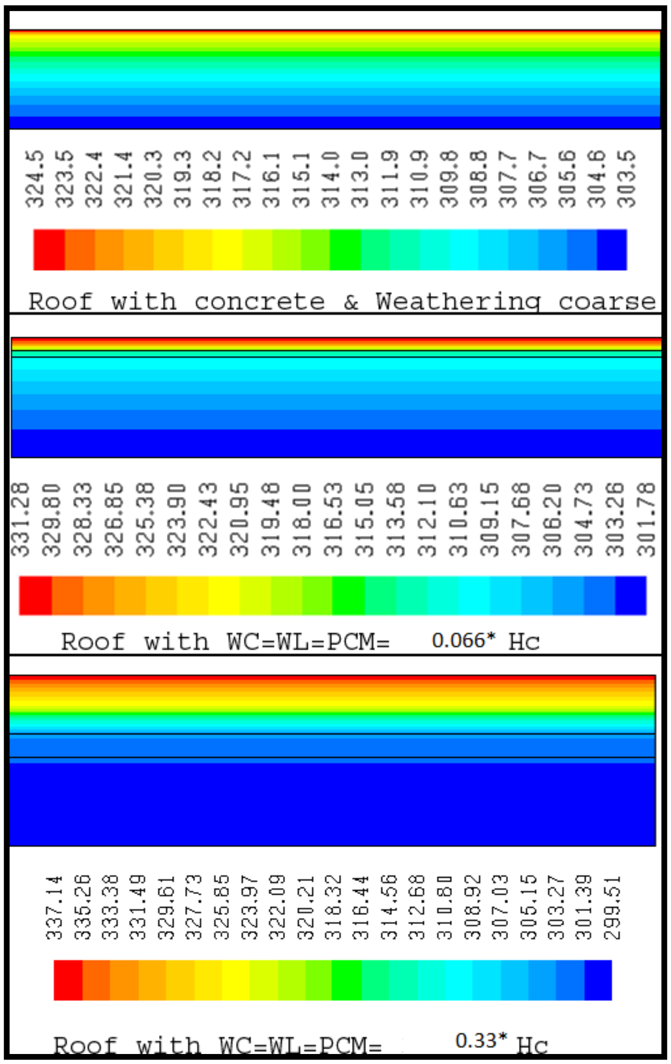

Fig. 3. Temperature profile across the various roof patterns at time =14hrs.

Table 3. Thermal performance and Cost predictions for 25 combinations of insulation layer thickness.

Ex. No	Weathering tile thickness, WC	Wood wool thickness, WL	Phase change material thickness, PCM	Heat gain (W/m^2)	$C_{A,C}$ ($)/$m^2$	C_{ins} ($)/$m^2$
1	1	1	1	40.88	29.11	50.40
2	1	2	2	32.77	23.34	100.40
3	1	3	3	27.35	19.48	150.40
4	1	4	4	23.83	16.97	200.40
5	1	5	5	21.25	15.14	250.40
6	2	1	2	36.46	25.96	90.80
7	2	2	3	29.86	21.27	140.80
8	2	3	4	25.33	18.05	190.80
9	2	4	5	24.13	17.19	240.80
10	2	5	1	21.71	15.47	90.80
11	3	1	3	32.90	23.43	131.20
12	3	2	4	27.43	19.54	181.20
13	3	3	5	23.68	16.87	231.20
14	3	4	1	22.99	16.38	81.20
15	3	5	2	18.88	13.45	131.20
16	4	1	4	27.64	19.69	171.60
17	4	2	5	23.42	16.68	221.60
18	4	3	1	22.72	16.18	71.60
19	4	4	2	21.56	15.35	121.60
20	4	5	3	19.36	13.79	171.60
21	5	1	5	27.50	19.59	212.00
22	5	2	1	26.52	18.89	62.00
23	5	3	2	22.93	16.33	112.00
24	5	4	3	20.36	14.50	162.00
25	5	5	4	18.31	13.04	212.00

4.1. Data pre-processing

The various objective responses are normalized between 0 to 1and this is the process of converting the original sequence to a complete sequence. Normalization of responses is based on the quality objective like maximization, minimization and definite target. In this study, the quality objective is to reduce the yearly cost of electrical energy for cooling and cost of the insulation and hence the quality objective is smaller the better. The equation used to normalize the output response under the quality objective of smaller the better is given in Eq. (5) [9].

$$x_i^* = \frac{\max x_i^o(k) - x_i^o(k)}{\max x_i^o(k) - \min x_i^o(k)} \tag{5}$$

Where x_i^0 is the value after the grey relation generation, $\max x_i^o(k)$ is the largest value of $x_i^o(k)$, $\min x_i^o(k)$ is the smallest value of $x_i^o(k)$ and x^0 is the desired value. The subscript i =1,2,….,m and k= 1,2,….,n, where m is the number of experiments and n is the total number of observations of data.

4.2. Computation of grey relational coefficients

The grey relational grade corresponding to each performance characteristics is predicted and the overall evaluation of multi response characteristic based on grey relational grade is calculated from equation 6 to 10 [9].

$$\xi_i(k) = \frac{\Delta_{\min} + \xi\Delta_{\max}}{\Delta_{oi}(k) + \xi\Delta_{\max}} \quad (6)$$

where, $\Delta_{oi}(k)$ is the deviation sequence of the reference sequence $x_0^*(k)$, x_i^* is the comparability sequence and ξ is the distinguishing or identification coefficient. The deviation sequences are calculated through the Eqs. (7) to (9) [9].

$$\Delta_{oi}(k) = \left\| x_0^*(k) - x_i^*(k) \right\| \quad (7)$$

$$\Delta_{\max} = \max_{\forall j \in i} \max_{\forall k} \left\| x_0^*(k) - x_j^*(k) \right\| \quad (8)$$

$$\Delta_{\min} = \min_{\forall j \in i} \min_{\forall k} \left\| x_0^*(k) - x_j^*(k) \right\| \quad (9)$$

From the grey relation coefficients, grey relation grade is calculated from the Eq. (10) [9].

$$\gamma_i = \frac{1}{n} \Sigma_{k=1}^{n} \xi_i(k) \quad (10)$$

The grey relation grades are used to show the relationship among the sequences and are given in Table 4.

4.3. Optimum level selection

The optimum level for thickness of various insulation layers are predicted from the respond table. In the respond table the relation grades are grouped by factor level and then averaged. For example the grey relation grades for the factor weathering tile thickness (WC) can be calculated for the level 1 as follows.

$$\gamma_{wc1} = \frac{1}{5}(0.67 + 0.55 + 0.53 + 0.54 + 0.56) = 0.57$$

Similar method was adopted for each factor under all levels of variations and the response table is generated as shown in Table 5. Since the grey relational grades represented the level of correlation between the reference and the comparability sequences, the larger grey relational grade means the comparability sequence exhibits a stronger correlation with the reference sequence. The respond values for all the insulation layer thickness under five levels of values are given in Table 5. From this table, it is identified that optimum thickness of weathering course, wood wool and PCM layer should be set at 0.33 * HC, 0.33* HC and

0.066 * HC respectively. For this insulation layer thickness, the cost of insulation and yearly electricity cost for 10 years is 92$/m^2 and 12.45 $/m^2 respectively.

Table 4. Normalized data and overall grey relation coefficients.

Ex. No	Normalized Data		Deviation evaluation		Grey relation coefficient		Overall Grey relation coefficient
	$C_{A,C}$ ($)	C_{ins} ($)	$C_{A,C}$ ($)	C_{ins} ($)	$C_{A,C}$ ($)	C_{ins} ($)	
1	0.00	1.00	1.00	0.00	0.33	1.00	0.67
2	0.36	0.75	0.64	0.25	0.44	0.67	0.55
3	0.60	0.50	0.40	0.50	0.56	0.50	0.53
4	0.76	0.25	0.24	0.75	0.67	0.40	0.54
5	0.87	0.00	0.13	1.00	0.79	0.33	0.56
6	0.20	0.80	0.80	0.20	0.38	0.71	0.55
7	0.49	0.55	0.51	0.45	0.49	0.53	0.51
8	0.69	0.30	0.31	0.70	0.62	0.42	0.52
9	0.74	0.05	0.26	0.95	0.66	0.34	0.50
10	0.85	0.80	0.15	0.20	0.77	0.71	0.74
11	0.35	0.60	0.65	0.40	0.44	0.55	0.49
12	0.60	0.35	0.40	0.65	0.55	0.43	0.49
13	0.76	0.10	0.24	0.90	0.68	0.36	0.52
14	0.79	0.85	0.21	0.15	0.71	0.76	0.74
15	0.97	0.60	0.03	0.40	0.95	0.55	0.75
16	0.59	0.39	0.41	0.61	0.55	0.45	0.50
17	0.77	0.14	0.23	0.86	0.69	0.37	0.53
18	0.80	0.89	0.20	0.11	0.72	0.83	0.77
19	0.86	0.64	0.14	0.36	0.78	0.58	0.68
20	0.95	0.39	0.05	0.61	0.91	0.45	0.68
21	0.59	0.19	0.41	0.81	0.55	0.38	0.47
22	0.64	0.94	0.36	0.06	0.58	0.90	0.74
23	0.80	0.69	0.20	0.31	0.71	0.62	0.66
24	0.91	0.44	0.09	0.56	0.85	0.47	0.66
25	1.00	0.19	0.00	0.81	1.00	0.38	0.69

Table 5. Response table for grey relation grades.

Level	WC	WL	PCM
1	0.569155	0.535064	**0.730321**
2	0.563145	0.564172	0.639286
3	0.598473	0.599222	0.574814
4	0.632743	0.622489	0.54706
5	**0.643735**	**0.685868**	0.515334
Best Level	5	5	1

5. Conclusion

Residential building occupants are now paying more interest to live in a comfort environment and the purchase of air conditioner system is increasing regularly. Nowadays, the severity of solar radiations rising rapidly and the high use of air-conditioning system will increase the electrical energy consumption. By considering the above facts present work is focused to reduce the entrapment of solar radiation with optimum roof insulation layer thickness that lay over the concrete. Roof layer thickness is optimised for reduced cooling cost though Taguchi's grey relational method. From this study the optimum thickness of weathering tile, wood wool and phase change material are predicted as 0.33 * HC, 0.33* HC and 0.066 * HC respectively and this optimum value will have the cost of insulation and yearly electricity cost for 10 years as 92 $\$/m^2$ and 12.45$\$/m^2$ respectively. This study is a useful guide for the construction engineers to construct the roof at optimum thickness with economical and environmental advantages with reduced entrapment of solar radiations.

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