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THE EFFECT OF SOIL VARIABILITY ON THE ULTIMATE BEARING CAPACITY OF SHALLOW FOUNDATION

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Abstract

Conventional shallow foundation design adopts a deterministic approach where representative soil parameters are chosen to calculate the ultimate bearing capacity. However, natural soils are non-homogenous and non-isotropic. Sometimes, large variability is observed in soils and this uncertainty has led to difficulty in foundation design. This paper presents a study on the probabilistic evaluation of foundation safety based on Monte Carlo simulation (MCS) and point estimation method (PEM). Terzaghi and Vesic theory were adopted in this study to compute the ultimate bearing capacity. A hypothetical problem with square footings founded on clayey soils and sandy soils were studied. The study shows the importance of taking account the impact of soil variability in shallow foundation design. Higher coefficient of variation of soil properties results in higher mean and larger standard deviation of ultimate bearing capacity. The probability of failure reduced when the coefficient of variation of soil properties is smaller. In general, PEM estimates higher mean ultimate bearing capacity compared to MCS.

Keywords: Shallow foundation, Ultimate bearing capacity, Monte Carlo simulation, Point estimation method.

1. Introduction

The design of shallow foundation has to fulfil two design criteria, namely the ultimate bearing capacity (i.e., limit state or safety) and the tolerable settlements (i.e., serviceability). This presentation focuses on the earlier. Conventional design adopts deterministic analyses that normally use representative soil parameters to

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Nomenclatures

B	Footing width, m
c'	Effective cohesion, kN/m^2
D_f	Depth of groundwater table, m
D_w	Depth of footing, m
E	Mean
N_c, N_q, N_γ	Bearing capacity factors
P_f	Probability of failure
q	Overburden stress, kN/m^2
q_{ult}	Ultimate bearing capacity, kN/m^2

Greek Symbols

ϕ'	Effective friction angle, ^{o}
γ	Unit weight, kN/m^3
ρ_{ij}	Correlation coefficient of the random variables x_i and x_j
σ	Standard deviation
σ^2	Variances

Abbreviations

COV	Coefficient of variation
FS	Factor of safety
MCS	Monte Carlo simulation
PEM	Point estimation method

determine the ultimate bearing capacity. These analyses are based on the assumption of uniformity: properties of soils are isotropic and homogenous. However, natural soil properties exhibit inherent spatial variability and this can significantly influence the failure mechanisms and the collapse load [1]. The inherent variability of geotechnical parameters as one of three main sources of uncertainty (other two sources are measurement errors and transformations uncertainty) are described in details by Phoon and Kulhawy [2].

The essential statistical parameters related to the variability of soils are: (1) expected value or mean, E, (2) coefficients of variation, *COV,* and (3) frequency distribution. Table 1 shows the range of most frequent values of the coefficient of variation of some geotechnical soil properties. The variability for unit weight, γ, is rather low while for shear strength parameters (effective cohesion, c', and effective friction angle, ϕ'), the variability is high. Clay soil is characterized by a higher coefficient of variation than silt and sand. Rethati [3] observed on large sets of data for geotechnical parameters and found a general symmetry characteristic which can be represented well with a normal distribution. Occasionally, the log-normal distributions are used.

Over the years, different attempts have been made to evaluate the safety of shallow foundation using the probabilistic approaches by assessing certain level of reliability, identified numerically by reliability index or probabilistic failure [4-10]. This study aims to investigate the impact of variations in soil properties on the ultimate bearing capacity of a square shallow foundation. The two deterministic analyses: Terzaghi theory and Vesic theory; and two probabilistic methods were adopted: Monte Carlo simulation (MCS) and point estimate method

(PEM) to compare the results. The assumptions made in this study are that there is no cross-correlation (independent) in soil properties and the spatial variability of data can be presented by a normal distribution.

Table 1. Range of the Most Frequent Values of Coefficients of Variation of Some Soil Properties [11].

Soil Property		Range of COV (%)
Unit weight, γ		3-10
Effective cohesion, c'		10-70
Effective friction angle, ϕ':	(i) Clay	10-50
	(ii) Silt	5-25
	(iii) Sand	5-15

2. Bearing Capacity

In general, there are three modes of bearing capacity failure for a shallow foundation: general shear failure, local shear and punching shear. General shear failure case is normally analysed in practice, as the other two modes are accounted implicitly in settlement calculation [12]. The mechanism of general shear failure is indicated in Fig. 1. With this, Terzaghi [13] developed a rational approach to predict ultimate bearing capacity of a square footing:

$$q_{ult} = 1.3c' N_c + qN_q + 0.4\gamma BN_\gamma \quad (1)$$

where q_{ult} is ultimate bearing capacity, q is the overburden stress, B is the footing width, N_c, N_q and N_γ are the bearing capacity factors in which N_c can be derived from the following relationship:

$$N_c = \frac{N_q - 1}{\tan\phi'} \quad (2)$$

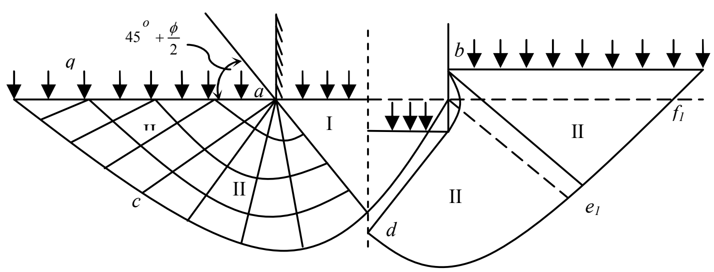


Fig. 1. General Shear Failure Concept [14].

Discrepancies have been noted in the values proposed for N_q, and N_γ:
According to the theory in Terzaghi [13]:

$$N_q = \frac{e^{\frac{270^o - \phi'}{180^o}\pi \tan\phi'}}{2\cos^2\left(45^o + \frac{\phi'}{2}\right)} \tag{3}$$

$$N_\gamma = 2(N_q + 1)\frac{\tan\left(\frac{\pi\phi'}{180^o}\right)}{1 + 0.4\sin\left(\frac{4\pi\phi'}{180^o}\right)} \tag{4}$$

According to the theory in Vesic [14]:

$$N_q = e^{\pi\tan\phi'}\tan^2\left(45^o + \frac{\phi'}{2}\right) \tag{5}$$

$$N_\gamma = 2(N_q + 1)\tan\phi' \tag{6}$$

The location of ground water table affects the bearing capacity of a foundation. This effect is demonstrated in Fig. 2.

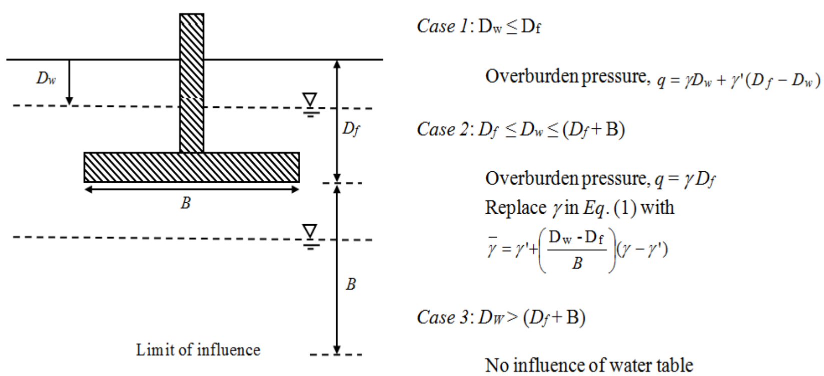


Fig. 2. Influence of Ground Water Table.

3. Probabilistic Analysis

In civil engineering reliability analysis, there are basically three categories of methodologies used in different probabilistic methods to yield measures of the distribution of functions of random variables [15]. The first category is called exact methods which require the probability distribution function of all component variables to be known initially. Examples of these are numerical integrations and Monte Carlo methods. The use of computer is necessary for these methods. The second category is the first order, second-moment methods (FOSM). These methods enable simplification on the implied functional relationship, having the truncated Taylor series expansion of the function. The third category, developed by Rosenblueth [16] and named point estimate method (PEM) allows numerical approximation for the moments of functions of random variables. Two probabilistic methods from two categories above are used in this study to compare the results: Monte Carlo simulation (MCS) and point estimate method (PEM).

3.1. Monte Carlo simulation (MCS)

Classical numerical integration rules such as the trapezoidal rule and Simpson's rule encounter a phenomenon often called the "curse of dimensionality" when dealing with multidimensional problems. In the 1940s, MCS was developed to overcome the above issue. It is a stochastic technique, by no means restricted to numerical integration that based on the use of random numbers with a strong statistical and probabilistic favour. Monte Carlo simulation promises the integration errors for which the order of magnitude, in terms of the number of nodes, is independent of the dimension [17].

In the computational practice era, MCS generates uniform random (pseudorandom) numbers based on deterministic subroutine. In this technique, cumulative probability distribution function (CPF) is determined first, $F(r) = P[x \leq r]$ where x is a random variable, r is a real number and $F(r)$ is the probability that x will take on values equal to or less than r. Harr [15] explained that in CPF, any continuous variate is uniformly distributed over the interval [0, 1]. Therefore, if a random value $R_v(0, 1)$ is generated, the value of $x = r$ satisfying $F(r) = R_v(0, 1)$ would be a random value with probability distribution function $f(x)$ and the CPF is $F(r)$. The procedure is exemplified in Fig. 3. First, the random value $R_v(0, 1)$ is generated. Then, sets $R_v(0, 1) = F(r)$. Finally, the value of $x = r$ is obtained from $F(r)$ corresponding to a particular probability distribution. The convergence is attainable by repeatedly running the model at a large number of trials. The discussion of random number generator, their limitation and use, can be found in [18].

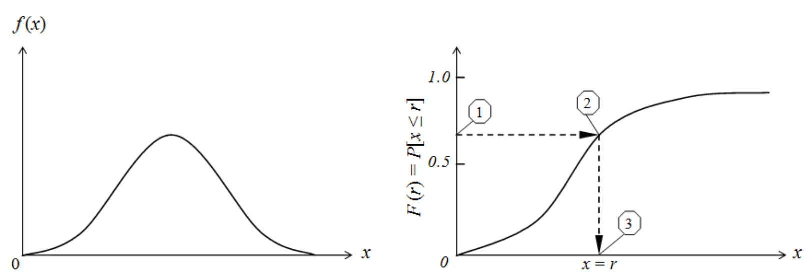


Fig. 3. Procedures for Obtaining Random Number with Required Distribution [15].

3.2. Point estimation method (PEM)

PEM assesses model output uncertainty in terms of its statistical moments (i.e., the mean and the variance) by evaluating the model output value at specified points within the parameter sample space. The points are selected to preserve finite statistical moments of input variables [19]. In general, the particular shape of any probability density function (PDF) used for any random variable is not critical to the analysis, because the PDF is represented by the mean and two hypothetical points located at plus and minus one standard deviation (σ) from the mean (x). Calculation steps are presented below for a function of three random variables, $y = y(x_1, x_2, x_3)$.

Step 1. Calculate the output value of y using the performance function evaluated to the values of mean plus one standard deviation for each of the three variables.

$$y_{\pm\pm\pm} = y(\bar{x}_1 \pm \sigma[x_1], \bar{x}_2 \pm \sigma[x_2], \bar{x}_3 \pm \sigma[x_3]) \tag{7}$$

Step 2. Calculate the point-mass weights.

$$\begin{aligned} p_{+++} = p_{---} &= \frac{1}{2^3}(1+\rho_{12}+\rho_{23}+\rho_{31}) \\ p_{++-} = p_{--+} &= \frac{1}{2^3}(1+\rho_{12}-\rho_{23}-\rho_{31}) \\ p_{+-+} = p_{-+-} &= \frac{1}{2^3}(1-\rho_{12}-\rho_{23}+\rho_{31}) \\ p_{+--} = p_{-++} &= \frac{1}{2^3}(1-\rho_{12}+\rho_{23}-\rho_{31}) \end{aligned} \tag{8}$$

where ρ_{ij} = the correlation coefficient of the random variables x_i and x_j. In general for n variables, there are 2^n terms of point-mass weights and $n(n – 1)/2$ correlation coefficients.

Step 3. Calculate the expectation (mean) of y.

$$E(y) = p_{+++}y_{+++} + p_{++-}y_{++-} + \ldots + p_{---}y_{---} \tag{9}$$

Equation (8) has $2^3 = 8$ terms, all permutations of the three +'s and –'s.

Step 4. Calculate the variances (σ^2) of y.

$$V(y) = E(y^2) - [E(y)]^2 \tag{10}$$

Step 5.The standard deviation (σ) of y is then calculated by taking the square root of σ^2.

For a function of four random variables $y = y(x_1, x_2, x_3, x_4)$ as in this study, Eq. (8) is extended from 8 terms to 2^4= 16 terms with 4(3)/2 = 6 possible correlation coefficients. The assumptions made in this study are that the soils properties are independent. Hence, the values of correlation coefficient (ρ_{ij}) are zero. Thus, all point-mass weight in this study are equal to $1/2^4 = 1/16$. The expected value and the standard deviation of the 16 values of y would be

$$\begin{aligned} E(y) &= \frac{1}{16}\sum y_{ijkl} \\ \sigma(y) &= \frac{1}{16}\sum [y_{ijkl} - E(y)]^2 \end{aligned} \tag{11}$$

4. Hypothetical Ultimate Bearing Capacity Problems

Two hypothetical problems for foundations on clayey and sandy soils are presented. The foundation geometry for both problems is the same. The foundation is a square footing of width, B = 2.0 m and founded at depth, D_f of 1.0 m below the ground surface. The groundwater table, D_w is located 2.0 m below the ground surface and subject to 0.5 m fluctuations (COV = 25%). The clayey and sandy soil properties are subjected to variations as indicated in Table 1.

The effects of the variations are studied through three cases each for both types of soils with the maximum, median and minimum coefficient of variations. The material properties for the cases are tabulated in Tables 2 and 3. For clayey soil, the expected value of the unit weight, friction angle and cohesion are 19 kN/m^3, 25^o, and 5 kN/m^2, respectively. While for sandy soil, the expected value of the unit weight, friction angle and cohesion are 20 kN/m^3, 30^o, and 3 kN/m^2, respectively. The standard deviations are calculated accordingly based on the *COVs*. The footings are designed to carry 1000 kN column load or equivalent to 250 kPa.

The probabilistic analyses (i.e., MCS and PEM) for the above problems were performed with Excel spreadsheet. Excel offers user friendly feature and many built in functions suitable for a wide range of professional analysis. The statistical function is also readily available in the program. In Monte Carlo simulation, the random number generator can be generated using formula = *rand()* while the normal distributed random variable is simulated by entering *NORMINV(rand(), mu , sigma)* in the formula cell. To produce a reliable result for MCS, many trials were repeated several times with different seeds and the optimum number of trials, N_{opt} = 18000, was obtained which the result did not display any discernible change compared to the larger number of trials. Hence, for the six studied cases, the number of trials fixed at 18000.

In this study, the ultimate bearing capacity was computed using Eq. (1) with both the Terzaghi and Vesic theory. In the next section, the effects of the normal random variables are discussed in terms of the mean and standard deviation of ultimate bearing capacity, factor of safety, and probability of failure. The factor of safety, FS is defined as the ratio of ultimate bearing capacity to the bearing stress. In this study, the probability of failure P_f is defined as the ratio of the frequency of FS less than unity to the total number of FS.

Table 2. Soil Properties of Clayey Soils for Three Cases.

			Case 1		**Case 2**		**Case 3**	
		E	*COV* (%)	σ	*COV* (%)	σ	*COV* (%)	σ
γ	(kN/m^3)	19	10	1.9	6.5	1.235	3	0.57
ϕ'	(^{o})	25	50	12.5	30	7.5	10	2.5
c'	(kN/m^2)	5	70	3.5	40	2	10	0.5

Table 3. Soil Properties of Sandy soils for Three Cases.

			Case 4		**Case 5**		**Case 6**	
		E	*COV* (%)	σ	*COV* (%)	σ	*COV* (%)	σ
γ	(kN/m^3)	20	10	2	6.5	1.3	3	0.6
ϕ'	(^{o})	30	15	4.5	10	3	5	1.5
c	(kN/m^2)	3	70	2.1	40	1.2	10	0.3

5. Results and Discussion

Six cases were analysed with two probabilistic methods (MCS and PEM) using two bearing capacity formulas (Terzaghi theory and Vesic theory). The first three cases are of clayey soils while the last three cases are of sandy soils. The results of clayey soils are discussed first, followed by the sandy soils.

5.1. Clayey soils

Tables 4 and 5 show the Monte Carlo simulation results for ultimate bearing capacity, q_{ult} and the factor of safety, FS. The mean q_{ult} for the case with max *COV* (i.e., Case 1) is higher than the cases with median *COV* and min *COV* (Cases 2 and 3). This indicates larger *COV* in the soil properties seems to offer greater resistance to the foundation load. In other words, the mean factor of safety for soils with higher variability is greater than soils with low variability. Based on this result, one might think that by adopting mean value as a deterministic value of the material properties for soils with high variability is a correct approach in foundation design. However, this is incorrect. The reason is that with larger *COV,* the standard deviation or the range of the values of q_{ult} or FS are also larger as shown in Fig. 4. Table 5 shows the probability of failure for Cases 1, 2, and 3 (results of Terzaghi theory) are 26.8%, 14.6% and 0%, respectively. This denotes that the probability of failure of the soils with high variability is actually higher than the soils with lower variability albeit their mean values are larger in terms of ultimate bearing capacity or factor of safety. Hence, in designing a foundation system, the act of ignoring the variability of the soil properties can be dangerous and unsafe.

Table 4. MCS Results of Ultimate Bearing Capacity for Clayey Soils.

q_{ult}	*Case 1*		*Case 2*		*Case 3*	
	Terzaghi	**Vesic**	**Terzaghi**	**Vesic**	**Terzaghi**	**Vesic**
Mean	1088.818	1306.547	650.013	797.570	523.781	642.528
Standard Error	11.164	13.023	3.456	4.295	0.850	1.075
Median	515.743	629.405	499.579	611.952	509.684	624.637
Standard Deviation	1497.821	1747.255	463.610	576.284	114.084	144.243
Sample Variance	2243467	3052900	214934	332103	13015	20805
Kurtosis	8.173	7.082	1.818	1.634	-0.529	-0.535
Skewness	2.725	2.559	1.476	1.429	0.467	0.459
Range	9220.990	10401.546	2233.641	2747.632	485.143	610.821
Minimum	63.564	67.527	148.796	168.021	328.960	395.848
Maximum	9284.554	10469.073	2382.437	2915.653	814.102	1006.668

In Fig. 4, the histogram for the first case is having the largest skewness and kurtosis value (leptokurtic distribution) followed by the second and the third case. Skewness is a measure of symmetry while kurtosis is a measure of whether the data are peaked or flat relative to a normal distribution. All three cases exhibit data that are skewed right and the Case 1 shows the data that are heavily tails. Negative kurtosis value in Case 3 indicates rather flat distribution that declines slowly as verified in the histogram. Histogram of Case 3 also demonstrates results that are closer to normal distribution with least skewness.

In short, the larger the variability in soil properties, the higher the values for skewness and kurtosis. Therefore, engineers should be careful when dealing with clayey soils of high variability during the design of shallow foundation. Histograms in Fig. 4 are the results of Terzaghi theory, however, a similar conclusion can be made to Vesic theory.

Table 5. MCS Results of Factor of Safety for Clayey Soils.

FS	*Case 1*		*Case 2*		*Case 3*	
	Terzaghi	**Vesic**	**Terzaghi**	**Vesic**	**Terzaghi**	**Vesic**
Mean	4.355	5.226	2.600	3.190	2.095	2.570
Standard Error	0.045	0.052	0.014	0.017	0.003	0.004
Median	2.063	2.518	1.998	2.448	2.039	2.499
Standard Deviation	5.991	6.989	1.854	2.305	0.456	0.577
Sample Variance	35.895	48.846	3.439	5.314	0.208	0.333
Kurtosis	8.173	7.082	1.818	1.634	-0.529	-0.535
Skewness	2.725	2.559	1.476	1.429	0.467	0.459
Range	36.884	41.606	8.935	10.991	1.941	2.443
Minimum	0.254	0.270	0.595	0.672	1.316	1.583
Maximum	37.138	41.876	9.530	11.663	3.256	4.027
Probability of failure	0.268	0.224	0.146	0.090	0	0

In general, the results obtained from Vesic theory are higher than Terzaghi theory for both the ultimate bearing capacity and the factor of safety. For example, the differences in mean value of ultimate bearing capacity computed from Terzaghi theory and Vesic theory for Cases 1, 2 and 3 are 20%, 2% and 23%, respectively. Similar results are obtained with PEM as shown in Table 6. However, PEM gives ultimate bearing capacity values that are 18.0%, 12.4%, and 1.3% higher than MCS results for Cases 1-3 (results of Terzaghi theory). Moreover, the probability of failure with PEM is significantly higher than MCS especially in Cases 1 and 2. For Case 3, both probabilistic approaches produce zero percentage of failure.

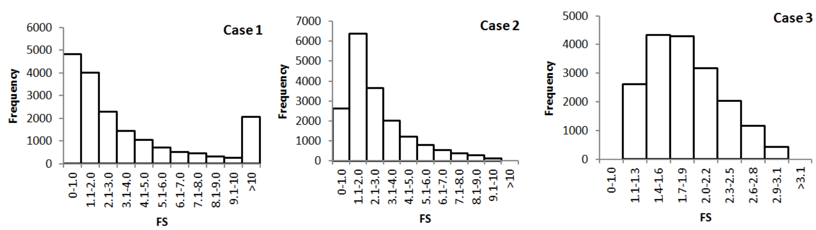


Fig. 4. Histogram of FS Distribution for Cases 1-3 with Terzaghi Theory.

Table 6. PEM Results for Cases 1-3.

	Case 1		*Case 2*		*Case 3*	
	Terzaghi	**Vesic**	**Terzaghi**	**Vesic**	**Terzaghi**	**Vesic**
Ultimate bearing capacity (Mean)	1285.346	1562.77	730.443	898.621	530.677	651.205
Standard deviation	1209.45	1484.146	517.508	649.513	146.390	184.957
Factor of safety (Mean)	5.141	6.251	2.9218	3.594	2.123	2.605
Standard deviation	4.838	5.937	2.070	2.598	0.586	0.740
Probability of failure	0.5	0.5	0.25	0.188	0	0

5.2. Sandy soils

Sandy soils exhibit lower variability compare to clayey soils, especially on the value of friction angle where the *COV* for friction angle is 5-15%. The small differences in friction angle, therefore result in small differences in terms of mean values for the ultimate bearing capacity and the factor of safety for Cases 4-6 as indicated in Tables 7 and 8 using Monte Carlo simulation. The kurtosis and skewness values are of smaller magnitudes compared to Cases 1-3. A noteworthy finding is that the expected value for friction angle in sandy soil is 5^o higher than clayey soils, but the mean value for the ultimate bearing capacity is somehow lower in sandy soils compared to clayed soils. For example, in Case 4 with Terzaghi theory, the mean value for ultimate bearing capacity is 552 kN/m^2 while it is 1088 kN/m^2 in Case 1. This implicitly suggested that the expected value is of secondary importance compared to the coefficient of variation. Besides, an engineer can design with a more confident for foundation on sandy soils than on clayey soils.

Table 7. MCS Results of Ultimate Bearing Capacity for Cases 4-6.

q_{ult}	*Case 4*		*Case 5*		*Case 6*	
	Terzaghi	**Vesic**	**Terzaghi**	**Vesic**	**Terzaghi**	**Vesic**
Mean	552.278	677.867	527.024	646.586	512.450	628.342
Standard Error	1.706	2.144	1.057	1.333	0.506	0.636
Median	503.895	617.598	506.847	621.667	508.143	622.941
Standard Deviation	228.860	287.643	141.864	178.813	67.840	85.349
Sample Variance	52377.005	82738.670	20125.365	31974.163	4602.251	7284.468
Kurtosis	0.033	0.009	-0.487	-0.505	-0.740	-0.742
Skewness	0.827	0.819	0.521	0.513	0.247	0.241
Range	1010.811	1265.752	605.880	760.703	284.876	358.506
Minimum	223.416	265.457	294.458	353.756	387.587	470.169
Maximum	1234.227	1531.209	900.338	1114.459	672.463	828.676

Table 8. MCS Results of Factor of Safety for Cases 4-6.

FS	*Case 4*		*Case 5*		*Case 6*	
	Terzaghi	**Vesic**	**Terzaghi**	**Vesic**	**Terzaghi**	**Vesic**
Mean	2.209	2.711	2.108	2.586	2.0498	2.513
Standard Error	0.007	0.009	0.004	0.005	0.0020	0.003
Median	2.016	2.470	2.027	2.487	2.0326	2.492
Standard Deviation	0.915	1.151	0.567	0.715	0.2714	0.341
Sample Variance	0.838	1.324	0.322	0.512	0.0736	0.117
Kurtosis	0.033	0.009	-0.487	-0.505	-0.7397	-0.742
Skewness	0.827	0.819	0.521	0.513	0.2470	0.241
Range	4.043	5.063	2.424	3.043	1.1395	1.434
Minimum	0.894	1.062	1.178	1.415	1.5503	1.881
Maximum	4.937	6.125	3.601	4.458	2.6899	3.315
Probability of failure	0.0315	0	0	0	0	0

The histogram of factor of safety for Cases 4-6 are shown in Fig. 5. They are slightly right skewed and the medium values are close to the mean values which indicate a near symmetrical distribution. The values of factor of safety are less spread around the mean and with less extreme values compared to Cases 1-3.

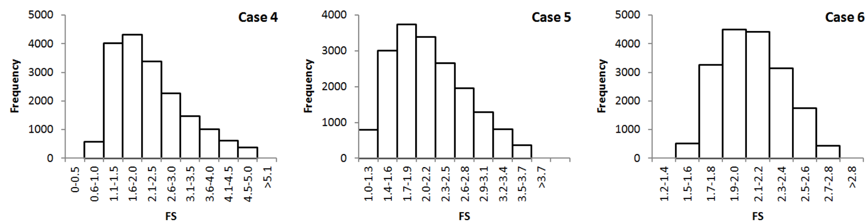


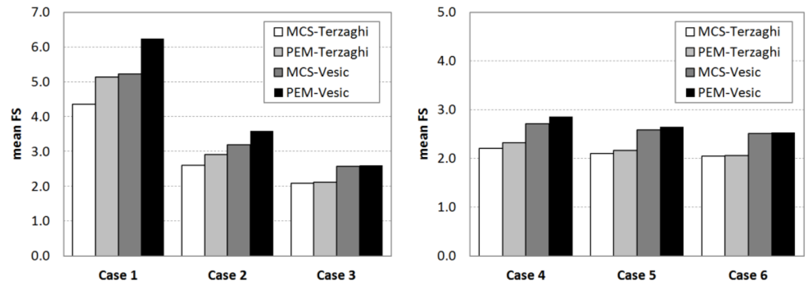
Fig. 5. Histogram of FS Distribution for Cases 4-6.

Table 9 presents the results for the PEM where the values for the probability of failure (zero or near zero probability) agree with the MCS results. In addition, the differences in PEM and MCS results in the ultimate bearing capacity (Terzaghi approach) are 5.4%, 2.4% and 0.7%, respectively for Cases 4-6 which is significantly lower than cases in clayey soils. Similar to clayey soils, the results of Vesic theory for the ultimate bearing capacity and the factor of safety are higher than Terzaghi theory (differ about 23% of all cases), true for both MCS and PEM approaches.

Table 9. PEM Results for Cases 4-6.

	Case 4		*Case 5*		*Case 6*	
	Terzaghi	**Vesic**	**Terzaghi**	**Vesic**	**Terzaghi**	**Vesic**
Ultimate bearing capacity (Mean)	582.329	715.701	540.577	663.597	516.485	633.416
Standard deviation	287.896	361.992	182.592	230.001	88.580	111.139
Factor of safety (Mean)	2.329	2.863	2.162	2.654	2.066	2.534
Standard deviation	1.152	1.448	0.730	0.920	0.354	0.445
Probability of failure	0.0625	0	0	0	0	0

Comparison of results in terms of the factor of safety is shown in Fig. 6. The highest mean FS is found in Case 1 when PEM and Vesic theory were adopted for the analysis while lowest mean FS obtained when MCS and Terzaghi theory were adopted. In all cases, Vesic theory yields a higher mean value than Terzaghi theory while PEM yields a higher mean value than MCS. Two generalisations can be made after six cases have been analysed. First, the influence of theory adopted (Terzaghi or Vesic) on the value of ultimate bearing capacity is more significant in sandy soils than clayey soils due to small variability in sandy soils. In other words, the choice of different theory adopted in design of shallow foundation on sandy soils has greater impact on the ultimate bearing capacity compared to the effect of variability of soil properties that may found on the sandy soils. Second, the impact of probabilistic approach (MCS or PEM) adopted to the value of ultimate bearing capacity is larger in clayey soils than in sandy soils.



(a) **(b)**

Fig. 6. Comparison of Mean of FS (a) Clayey Soils, and (b) Sandy Soils.

6. Conclusion

This paper aims to evaluate the impact of variations in soil properties on the ultimate bearing capacity of a square footing. Monte Carlo simulation and point estimation method are used to analyse the problems of clayey and sandy soils. Few conclusions can be made:

- The mean ultimate bearing capacity or factor of safety is higher when the coefficients of variations of soils properties are larger.
- Probability of failure is higher when the coefficient of variation is larger.
- The higher the coefficient of variation of soil properties, the larger the skewness and kurtosis value for the ultimate bearing capacity.
- Point estimate method estimates higher ultimate bearing capacity than Monte Carlo simulation.
- Vesic theory calculates (20-23)% higher ultimate bearing capacity than Terzaghi method.
- The influence of theory adopted on the value of ultimate bearing capacity is more significant in sandy soils than clayey soils.
- The impact of probabilistic approach adopted to the value of ultimate bearing capacity is larger in clayey soils than in sandy soils.

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