DESIGN OF ROBUST COMMAND TO LINE-OF-SIGHT GUIDANCE LAW: A FUZZY ADAPTIVE APPROACH

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Abstract

In this paper, the design of command to line-of-sight (CLOS) missile guidance law is addressed. Taking a three dimensional guidance model, the tracking control problem is formulated. To solve the target tracking problem, the feedback linearization controller is first designed. Although such control scheme possesses the simplicity property, but it presents the acceptable performance only in the absence of perturbations. In order to ensure the robustness properties against model uncertainties, a fuzzy adaptive algorithm is proposed with two parts including a fuzzy (Mamdani) system, whose rules are constructed based on missile guidance, and a so-called rule modifier to compensate the fuzzy rules, using the negative gradient method. Compared with some previous works, such control strategy provides a faster time response without large control efforts. The performance of feedback linearization controller is also compared with that of fuzzy adaptive strategy via various simulations.

Keywords: Fuzzy adaptive control, Negative gradient, Missile guidance, CLOS.

1. Introduction

The principle of command to line-of-sight (CLOS) guidance is to force a missile to fly as closely as possible along the instantaneous line-of-sight (LOS) between the ground tracker and the target. If the missile can continuously stay on the LOS, it eventually hit the target. To provide demanded accelerations for the missile, a guidance controller is utilized at the ground station to take account of tracker information about missile position, target position, as well as the angular velocity and acceleration of the LOS. These acceleration commands can then be transmitted to the missile via a radio link. The CLOS guidance has been regarded
as a low-cost guidance concept because it emphasizes placement of avionics on the launch platform, as opposed to mounting on the expendable weapons [1].

Missile guidance can be defined as the process of gathering information, used to form the suitable commands for missile control system. Such control system compares the actual flight trajectory with a predefined path of target such lead the missile to the target. various control schemes e.g., input–output feedback [2], hybrid controller [3, 4] and adaptive and fuzzy sliding mode controllers [5] have been proposed to solve the guidance problem. Due to the maneuverability of the objectives, the existence of external disturbances, and parameter variations, the aforementioned methods may not ensure the robust performance.

The sliding mode control (SMC) law is a substantial case from the wider class commonly, referred to as variable structure control (VSC), which ensures the robustness against system uncertainties and bounded external disturbances. The robustness is attributed to the discontinuous term in the control input. However, this discontinuous term also causes an undesirable effect called chattering. Removing such drawbacks, fuzzy rules may be used to drive a fuzzy sliding control [6-8].
Intelligent techniques such as fuzzy control, neural network and the combination of fuzzy and adaptive controllers have been extensively applied to nonlinear systems. The self-organizing controllers, based on fuzzy logic or neural networks, are more flexible than conventional ones, due to the ability to choose the rule base, number of fuzzy rules, network structure and the number of neurons [9, 10]. Feedback linearization has been adopted before to control the LOS of a nonlinear missile [11], in which the angular momentum is assumed to be fixed relative to the axis of angular positions roll, pitch and yaw.

A nonlinear supervisory controller has been designed and coupled with the main sliding mode controller in the form of an additional control signal [12]. The supervisory control signal is activated when the beam angle constraint goes to be violated.

In this paper, the mathematical model of missile motion is first given to solve the missile guidance problem. A fuzzy adaptive control is proposed in which the weights are updated via the negative gradient algorithm. To make a comparison study, the feedback linearization technique is also developed. Simulation studies illustrate the benefits of the designed fuzzy adaptive scheme to solve the tracking problem.

2. Formulation of Missile-Target Engagement

Clos missile as a tracking problem is presented for a time varying nonlinear system. Figure 1 depicts the diagram of the missile and target tracking in 3-D. Center's internal frame is specified on the ground tracker. The vertical axis is upwards and the screen horizontally.

Missile center body frame with the axis of the center of mass is characterized in the direct path of the central line. Missile movement in internal frame is expressed as [13]
\[
\begin{bmatrix}
\dot{x}_m \\
\dot{y}_m \\
\dot{z}_m
\end{bmatrix} = \begin{bmatrix}
c\phi_m c\psi_m & -s\phi_m c\theta_m & -c\phi_m \sin \psi_m \\
-c\phi_m \sin \psi_m & s\phi_m c\theta_m & c\phi_m \sin \psi_m \\
-s\phi_m c\theta_m & c\phi_m \sin \psi_m & c\phi_m \sin \psi_m
\end{bmatrix} \begin{bmatrix}
\alpha_x \\
\alpha_y \\
\alpha_z
\end{bmatrix}
\]

\[
\begin{bmatrix}
\psi_m \\
\theta_m
\end{bmatrix} = \begin{bmatrix}
c\phi_m c\psi_m \\
-s\phi_m c\theta_m
\end{bmatrix} \begin{bmatrix}
\frac{s\phi_m c\theta_m}{v_m} \\
\frac{c\phi_m \sin \psi_m}{v_m}
\end{bmatrix} \begin{bmatrix}
\alpha_y \\
\alpha_z
\end{bmatrix} - \begin{bmatrix}
0 \\
g \cos \theta_m
\end{bmatrix}
\]

Missile speed is calculated by

\[
v_m = (x_m^2 + y_m^2 + z_m^2)^{1/2}
\]

and the missile axial acceleration \(a_x\) (forward acceleration) is given by

\[
a_x = (T - D) / m
\]

where \(T\), \(D\) and \(m\) represent the thrust force, drag force and mass of the missile, respectively.

### 2.1. Error tracking system

A tracking error is defined to convert the CLOS guidance problem into a tracking problem. The CLOS guidance involves guiding the missile along the LOS to the target. The LOS frame is shown in Fig. 2.

![Fig. 2. Definition of tracking error [13].](image)

The \(X_L\) axis is forward along the LOS to the target, and the \(Y_L\) axis is horizontal to the left of the \(X_L - Z_L\) plane. As indicated in Fig. 2,
coordinates \((e_1,e_2)\) which represent the missile position in the LOS frame, can be related to \((x_m,y_m,z_m)\) as

\[
\begin{bmatrix}
  e_1 \\
  e_2
\end{bmatrix} =
\begin{bmatrix}
  -s \sigma_i & c \sigma_i & 0 \\
  -s \gamma_i c \sigma_i & -s \gamma_i s \sigma_i & c \gamma_i
\end{bmatrix}
\begin{bmatrix}
  x_m \\
  y_m \\
  z_m
\end{bmatrix}
\]  \(4\)

where \(\sigma_i\) is the azimuth angle of LOS to target, \(\sigma_m\) denotes the azimuth angle of LOS to missile, and \(\gamma_i\) is the elevation angle of LOS to target. Since \(e_1\) and \(e_2\) cannot be measured directly, these quantities must be computed indirectly using the polar position data of the missile available from the ground tracker as

\[
e =
\begin{bmatrix}
  e_1 \\
  e_2
\end{bmatrix} =
\begin{bmatrix}
  R_m c(\Delta \gamma + \gamma_i) s \Delta \sigma \\
  R_m s(\Delta \gamma + \gamma_i) c \gamma_i - R_m c(\Delta \gamma + \gamma_i) s \gamma_i c \Delta \sigma
\end{bmatrix}
\]  \(5\)

in which \(R_m\) is the missile range from ground tracker, \(\Delta \gamma\) is the difference of missile and target elevation angle, and \(\Delta \sigma\) denotes the difference of missile and target azimuth angle. Note that \(\|e\|_2\) represents the distance from the missile to the LOS. Therefore, the missile will eventually hit the target if the tracking error is driven to zero before the target crosses the missile.

To define the missile guidance as a tracking problem, missile system variables are defined as

\[
x =
\begin{bmatrix}
  x_1 & x_2 & x_3 & x_4 & x_5 & x_6 & x_7 & x_8
\end{bmatrix}^T
\]  \(6\)

\[
= 
\begin{bmatrix}
  x_m & y_m & z_m & \dot{x}_m & \dot{y}_m & \dot{z}_m & \psi_m & \theta_m
\end{bmatrix}^T
\]

The desired output of the controller consists of two acceleration commands \(a_{yc}\) and \(a_{zc}\). The control law is shown as

\[
u_T = [u_{T1} \quad u_{T2}]^T = [a_{yc} \quad a_{zc}]^T
\]  \(7\)

### 2.2. Feedback linearization controller

Consider the nonlinear equation

\[
\dot{x} = f_0(x,t) + \sum_{j=1}^{m} f_j(x,t)u_i; \quad e = E(x,y_d)
\]  \(8\)

in which equations \(u = (u_1,u_2) = (u_{awc},u_{exc})\) and \(y_d = (y_{d1},y_{d2}) = (\sigma_i,\gamma_i)\), \(i = 1,\ldots,m\). In \(8\), \(y_d\) is the desired path. With regard to the descriptions, provided in the section 2, equation \(8\) can be modified to the form

\[
\dot{x} = f(x,t) + \sum_{j=1}^{2} g_j(x)u_j
\]  \(9\)

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\[ e = h(x,t) \]

where \( f(x,t), g_j(x) \) and \( h(x,t) \) are

\[ f(x,t) = \begin{bmatrix}
    x_4 \\
    x_5 \\
    x_6 \\
    a_x(t) s x g x_8 \\
    a_x(t) s x_8 - g \\
    - g c x_8 \\
    \left( x_4^2 + x_5^2 + x_6^2 \right)^{1/2}
\end{bmatrix} \]

\[ h(x,t) = \begin{bmatrix}
    h_1(x,t) \\
    h_2(x,t)
\end{bmatrix} = \begin{bmatrix}
    -x \sigma_j + x \xi \sigma_j \\
    -x s \gamma_j c \sigma_j - x s \gamma_j s \sigma_j + x s \gamma_j
\end{bmatrix} \]  

(10)

Indeed, for (9) the vector fields are defined by

\[ X_0 = \frac{\partial}{\partial t} + \sum_{i=1}^{8} f_i(x,t) \frac{\partial}{\partial x_i} \]

\[ X_j = \sum_{i=1}^{8} g_{j,i}(x) \frac{\partial}{\partial x_i} \]

where \( f_i(x,t) \) and \( g_{j,i}(x) \) are the \( i \)th components of \( f(x,t), g_j(x) \). By some manipulation and using the vector fields [11], the tracking error in (9) can be put compactly into the following form:

\[ g_1(x) \]

\[ g_2(x) \]

\[ \left( x_4^2 + x_5^2 + x_6^2 \right)^{1/2} \]

\[ \left( x_4^2 + x_5^2 + x_6^2 \right)^{1/2} \]
where \( F_1(x,t) = X_1^2 h_1 \), \( F_2(x,t) = X_2^2 h_2 \), \( G_{11}(x,t) = X_1 X_0 h_2 \), \( G_{12}(x,t) = X_2 X_0 h_2 \), \( G_{21}(x,t) = X_1 X_0 h_2 \).

Hence, the feedback linearization control law is introduced by

\[
\begin{align*}
&u^*_f = G^{-1}(x,t) \left[ -F(x,t) - K_v \dot{e} - K_p e \right] \\
&\text{where } K_v \text{ and } K_p \text{ are } 2 \times 2 \text{ diagonal gain matrices whose entries are chosen as some positive constants. Substituting (13) into (12) yields}
\end{align*}
\]

\[
\dot{e} + K_v \dot{e} + K_p e = 0
\]

which implies that the guidance law (13) will asymptotically drive the tracking error to zero.

3. Design of Fuzzy Adaptive Controller

The purpose of designing the fuzzy adaptive controller is making tracking error to zero by using acceleration command \( u = [a_w, a_{\psi}]^T \), as control input. The proposed controller is schematically shown in Fig. 3.

3.1. General structure of the fuzzy controller

As depicted in Fig. 4, the fuzzy controller consists of some main parts as

1. Fuzzy rules that contain some IF-THEN rules.
2. Expert that contains fuzzy membership functions.
3. Decision unit which is defined as inference engine.
4. Defuzzifier for converting the fuzzy inference engine results to outputs.
5. Fuzzifier for converting the input of fuzzy system to a set of linguistic values.
### 3.2. Fuzzy Mamdani controller

The basic FLC should be viewed as a linguistic conditional statement symbolized in the form of a relation matrix \( R \) given by the Cartesian product

\[
R = E \times \hat{E} \times U
\]  

(15)

where \( R \) is the control rule base, \( E \) and \( \hat{E} \) are the fuzzified values of \( e \) and \( \dot{e} \), respectively, \( U \) denotes the fuzzy output of the controller and \( \times \) means the Cartesian product. The overall relation matrix \( R \) obtained from the fuzzy control rules is calculated as the union of \( m \) individual relation matrices

\[
R = R_1 \cup R_2 \cup \ldots \cup R_m = \bigcup_{i=1}^{m} R_i
\]  

(16)

Therefore, the output \( U \) from the fuzzy controller can be obtained from its inputs \( E \) and \( \hat{E} \). Zadeh's compositional rule is employed for rule inference:

\[
U = (E \times \hat{E}) \circ R
\]  

(17)

where \( \circ \) denotes the compositional rule of inference. The fuzzy control rules are in the form

\[
\text{rule } i : \text{If } e \text{ is } F^i_e \text{ and } \dot{e} \text{ is } F^i_{\dot{e}} \text{ then } u = p_i
\]  

(18)

in which \( F^i_e \) and \( F^i_{\dot{e}} \) represent the fuzzy sets; \( p_i, i = 1,2,\ldots,n \) are the singleton control actions. The defuzzification of the controller output is accomplished by the method of center-of-gravity as

\[
u (e, \dot{e}, p_i) = \frac{\sum_{i=1}^{n} w_i \times p_i}{\sum_{i=1}^{n} w_i}
\]  

(19)

where \( w_i \) is the firing weight of the \( i \)-th rule. The controller acceleration commands are obtained by (19) as \( u = [a_{zc}, a_{zz}]^T \).
3.3. Adaption of weights using the negative gradient

The objective is to bring the system from any initial state to a desired state, and the dynamic behaviour of the system should be insensitive to the variations of the system parameters and external disturbances. To this end, an iterative learning algorithm is adopted to adjust the coefficients \( p_i, i = 1, 2, ..., n \). The central part of the iterative learning algorithm for a FLC system is to change the control effort in the direction of the negative gradient of a performance index \( I \), defined as a function of \( e \) and \( \dot{e} \) by

\[
I = \sum_k \sqrt{e^2(k) + \lambda \dot{e}^2(k)}
\]  

(20)

where \( k \) stands for the index of time interval, and \( \lambda > 0 \) is a weighting factor. The partial derivatives of \( I \) with respect to \( e \) and \( \dot{e} \) can be obtained as

\[
\frac{\partial I}{\partial e(k)} = \frac{e(k)}{\sum_k \sqrt{e^2(k) + \lambda \dot{e}^2(k)}}
\]

(21)

\[
\frac{\partial I}{\partial \dot{e}(k)} = \frac{\lambda \dot{e}(k)}{\sum_k \sqrt{e^2(k) + \lambda \dot{e}^2(k)}}
\]

The negative gradient for the optimal performance can be expressed as

\[
[\nabla I] = \begin{bmatrix}
\frac{e(k)}{\sum_k \sqrt{e^2(k) + \lambda \dot{e}^2(k)}} \\
\frac{\lambda \dot{e}(k)}{\sum_k \sqrt{e^2(k) + \lambda \dot{e}^2(k)}}
\end{bmatrix}
\]  

(22)

Based on optimal control theory, the control signal \( \delta u \) is adjusted by an update mechanism as

\[
\delta u = \eta \left( -[\nabla I] \right) \begin{bmatrix}
e(k) \\
\dot{e}(k)
\end{bmatrix}
\]

(23)

where \( \eta \) is the learning rate with positive constant. The modification algorithm for each fuzzy control rule is proposed as

\[
\Delta p_i(k) = \delta u(k) \cdot \frac{W_i}{\sum_{i=1}^n W_i}
\]

(24)

in which \( \Delta p_i \) is a modification value, added to the \( i \)-th control rule, i.e.

\[
p_i(k + 1) = p_i(k) + \Delta p_i(k)
\]

(25)

Without normalizing, the dependence of update law for \( p(k) \) to \( \delta u(k) \), may result in fast changes in \( p(k) \) and lead to fluctuations in system response. In fact, such modification can improve the stability property of the system. Taking a
summary, the fuzzy rules of FLC are given in (18) with the control efforts $p_i$ is updated by (25). Then, the defuzzified control force is calculated in (19). In fact, choosing the structure and number of fuzzy rules together with update laws are performed such that the closed-loop stability is achieved.

4. Simulation Results

To show the effectiveness of the developed tracking control scheme, it’s assumed that the missile does not roll down any axial acceleration. The simplified model of the target in the inertial frame can be presented as

$$
\begin{align*}
\ddot{x}_t &= -a_y c \psi_t - a_z s \theta c \psi_t \\
\ddot{y}_t &= a_y c \psi_t - a_z s \theta s \psi_t \\
\ddot{z}_t &= a_z c \theta_t - g \\
\dot{\psi}_t &= \frac{a_y}{(v, c \theta)} \\
\dot{\theta}_t &= \frac{(a_z - gc \theta)}{v_t}
\end{align*}
$$

Moreover, the target speed is calculated by

$$
v_t = \left( x_t^2 + y_t^2 + z_t^2 \right)^{1/2}
$$

To evaluate the performance of the controller one scenario of flight should be specified. As demonstrated in Figs. 5 and 6, the target moves for 2.5 seconds with accelerations $a_y = 5g$ and $a_z = -g$. After 2.5 seconds, the target presents the maneuverability until the interception is changed and the new values of the accelerations are $a_y = -5g$ and $a_z = 5g$. The basic information of the missile, target and ground tracker are given in Tables 1 and 2. To avoid creating unreal accelerations and limit the maneuverability of the missile, take

$$
\begin{align*}
\tilde{a}_{yc} &= \text{sat} \left( a_{yc}, 30g \right) \\
\tilde{a}_{zc} &= \text{sat} \left( a_{zc}, 30g \right)
\end{align*}
$$

The fuzzy inference rules, used in the simulation, are given in Table 3. The fuzzy labels are negative big (NB), negative small (NS), zero (ZO), positive big (PB), positive small (PS). To design the rule modifier, the learning rate $\eta$ is taken as 0.1 and the weighting factor $\lambda$ as 10.

Applying the feedback linearization and fuzzy adaptive guidance law, the simulation results are respectively depicted in Figs. 7(a) and (b). Figures 8 and 9 show the engagement responses of CLOS missile guidance law and the acceleration command, respectively. The comparison of simulation results shows that the fuzzy adaptive guidance law can achieve smaller miss distance than the
feedback linearization one. Moreover, in the case of fuzzy adaptive control, the acceleration commands present the smoother control efforts.

![Fig. 5. Yaw acceleration command.](image)

![Fig. 6. Pitch acceleration command.](image)

### Table 1. Basic information for simulation.

<table>
<thead>
<tr>
<th>Parameter</th>
<th>Unit</th>
<th>Value</th>
</tr>
</thead>
<tbody>
<tr>
<td>$x_t(0), y_t(0), z_t(0)$</td>
<td>[m]</td>
<td>2500, 5361.9, 1000</td>
</tr>
<tr>
<td>$\dot{x}_t(0), \dot{y}_t(0), \dot{z}_t(0)$</td>
<td>[m/s]</td>
<td>0, -340, 0</td>
</tr>
<tr>
<td>$x_m(0), y_m(0), z_m(0)$</td>
<td>[m]</td>
<td>14.3, 39.3, 3.3</td>
</tr>
<tr>
<td>$\dot{x}_m(0), \dot{y}_m(0), \dot{z}_m(0)$</td>
<td>[m/s]</td>
<td>70.8, 151.9, 28</td>
</tr>
<tr>
<td>$\psi_t(0), \theta_t(0)$</td>
<td>[deg]</td>
<td>-90, 0</td>
</tr>
<tr>
<td>$\psi_m(0), \theta_m(0)$</td>
<td>[deg]</td>
<td>65, 9.59</td>
</tr>
<tr>
<td>$\Delta \sigma(0), \Delta \gamma(0)$</td>
<td>[deg]</td>
<td>5, -5</td>
</tr>
<tr>
<td>$a_x$</td>
<td>[m/s²]</td>
<td>340 for $0 &lt; t &lt; 2$</td>
</tr>
<tr>
<td></td>
<td></td>
<td>-44.1 for $t &gt; 2$</td>
</tr>
</tbody>
</table>

Table 2. Ground tracker and missile information.

<table>
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<th>Value</th>
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<tbody>
<tr>
<td>$\zeta, \omega_c$</td>
<td>$1/\sqrt{2}, 2\sqrt{2}$</td>
</tr>
<tr>
<td>$\phi_{\text{cmd}}$</td>
<td>0 deg</td>
</tr>
<tr>
<td>guidance command frequency</td>
<td>50 Hz</td>
</tr>
<tr>
<td>autopilot damping ratio</td>
<td>0.6</td>
</tr>
<tr>
<td>autopilot natural frequency</td>
<td>6$\pi$ rad/s</td>
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</table>

Table 3. Fuzzy inference rules.

<table>
<thead>
<tr>
<th>e</th>
<th>$\dot{e}$</th>
<th>NB</th>
<th>NS</th>
<th>ZO</th>
<th>PS</th>
<th>PB</th>
</tr>
</thead>
<tbody>
<tr>
<td>NB</td>
<td>-1.00</td>
<td>-1.00</td>
<td>-1.00</td>
<td>-0.43</td>
<td>0.00</td>
<td></td>
</tr>
<tr>
<td>NS</td>
<td>-1.00</td>
<td>-1.00</td>
<td>-1.00</td>
<td>0.00</td>
<td>0.43</td>
<td></td>
</tr>
<tr>
<td>ZO</td>
<td>-1.00</td>
<td>-0.43</td>
<td>0.00</td>
<td>0.43</td>
<td>1.00</td>
<td></td>
</tr>
<tr>
<td>PS</td>
<td>-0.43</td>
<td>0.00</td>
<td>0.43</td>
<td>1.00</td>
<td>1.00</td>
<td></td>
</tr>
<tr>
<td>PB</td>
<td>0.00</td>
<td>0.43</td>
<td>1.00</td>
<td>1.00</td>
<td>1.00</td>
<td></td>
</tr>
</tbody>
</table>

Fig. 7. Tracking error $e_t$, obtained by applying (a) feedback linearization, and (b) fuzzy adaptive design.

Fig. 8. Engagement responses of Clos missile guidance law, (a) Feedback linearization guidance, (b) Fuzzy adaptive guidance.
5. Conclusions

In this paper, a new fuzzy adaptive learning method is developed and applied for the CLOS guidance law design. A comparison between the fuzzy adaptive and feedback linearization guidance laws for one engagement scenario is made. Simulation results demonstrate that the proposed fuzzy adaptive guidance law can achieve satisfactory performance for engagement scenario. Furthermore, the proposed guidance law, with using the negative gradient method to determine weights of the adaptive mechanism, is found to perform better than the other guidance law in terms of the miss distance, tracking error and smooth acceleration commands. It is revealed that the proposed fuzzy adaptive learning algorithm is suitable for the CLOS guidance law design.

References


